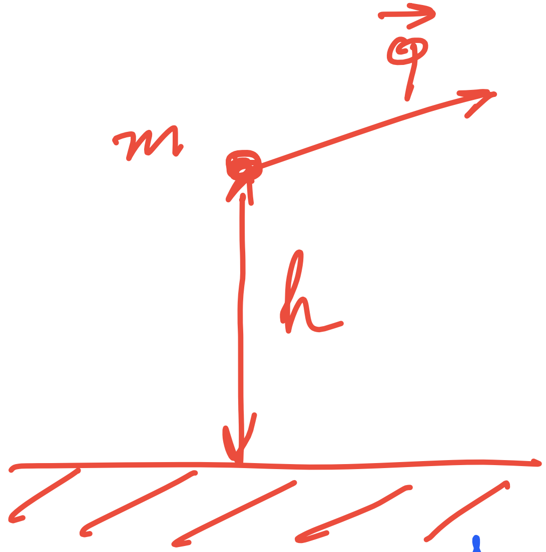
WAVE ENERGY

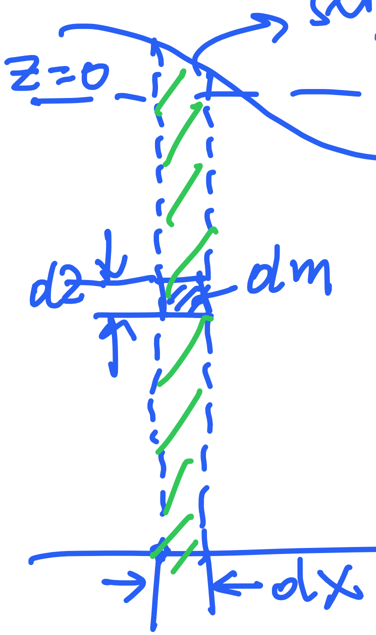


K.E. = Kinetic energy of m : $m\frac{q^2}{2}$

$q = |\vec{q}|$

P.E. = Potential energy of m : mgh

T.E. = K.E + P.E. = $m\frac{q^2}{2} + mgh$



slice of wave of width b

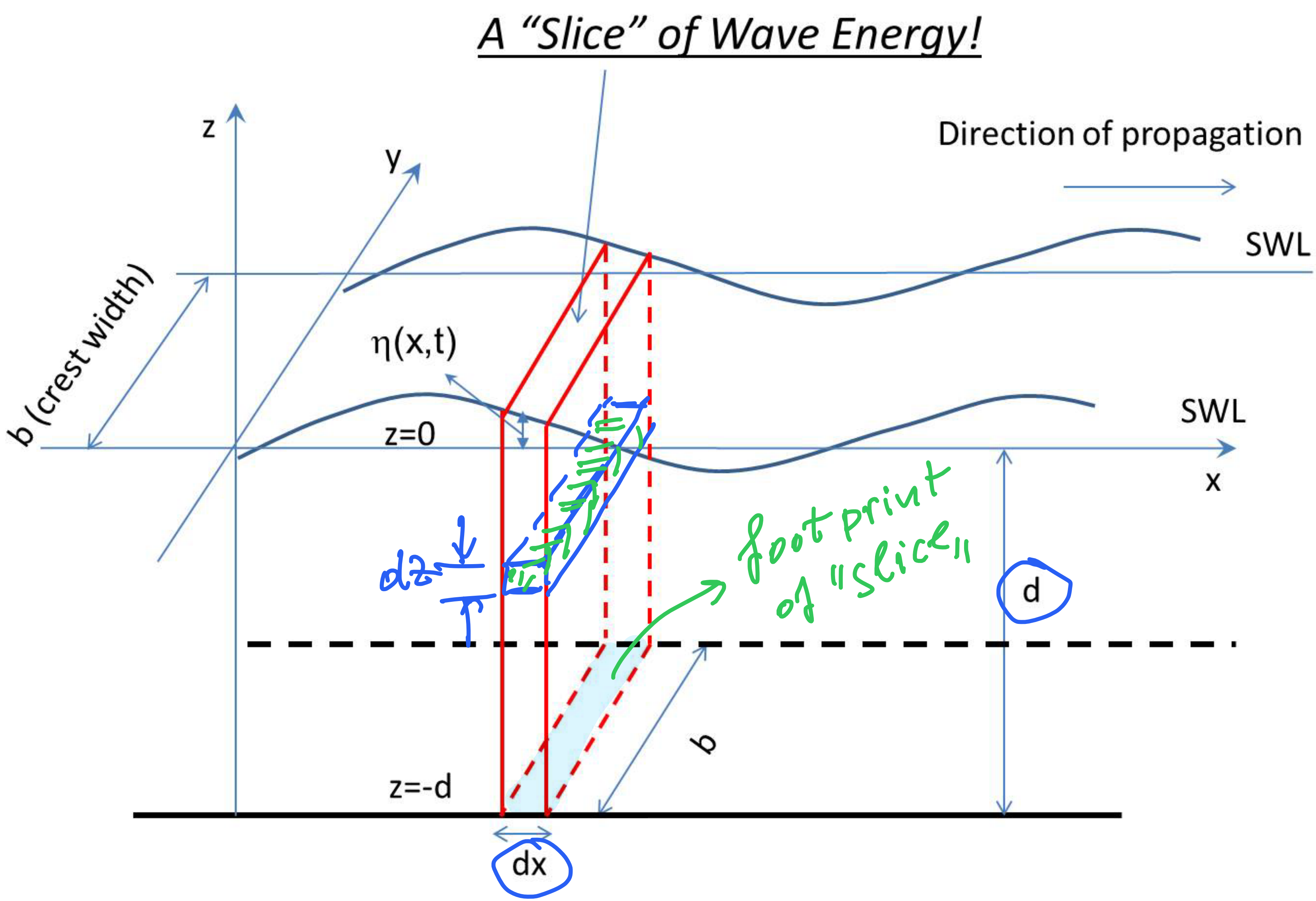
SWL

$\vec{q}$

m

$$T.E._{slice} = \int_{\forall_{slice}} \left(dm\,\frac{q^2}{2} + dm\, g\, z \right)$$

Perspective view of a plane progressive wave



WAVE ENERGY

$$T.E._{slice} = \int_{\forall_{slice}} \left(\rho \frac{q^2}{2} + \rho g z\right) d\forall = b\, dx \int_{-d}^{\eta} \left(\rho \frac{q^2}{2} + \rho g z\right) dz = \bar{E}_{slice}$$

$$d\forall = b.dx\, dz$$

$$\rho = \frac{dm}{d\forall}$$

$$\bar{E}_{slice} = \text{specific energy} = \frac{T.E._{slice}}{b.dx}$$

$b.dx$: footprint of slice

$$\bar{E}_{slice} = \underbrace{\int_{-d}^{\eta} \rho \frac{q^2}{2}\, dz}_{\bar{E}^{k}_{slice}} + \underbrace{\int_{-d}^{\eta} \rho g z\, dz}_{\bar{E}^{P}_{slice}}$$

WAVE ENERGY (Assume deep H_2O)

$$\bar{E}^{k}_{slice} = \int_{-d}^{\eta} \rho \frac{q^2}{2}\, dz = \int_{-d}^{\eta} \rho \frac{a^2\omega^2}{2} e^{2kz}\, dz =$$

$$q = a\omega e^{kz}$$

$$= \rho \frac{a^2\omega^2}{2} \int_{-d}^{\eta} e^{2kz}\, dz = \rho \frac{a^2\omega^2}{2} \int_{-d}^{\eta} \frac{e^{2kz}\, d(2kz)}{2k} =$$

$$= \rho \frac{a^2\omega^2}{2} \frac{1}{2k} \cdot e^{2kz} \Big|_{-d}^{\eta} = \rho \frac{a^2\omega^2}{4k} \left[e^{2k\eta} - \cancel{e^{-2kd}} \right]$$

linear Theory → $e^{0} = 1$ $\qquad = 0$

$$\bar{E}^{k}_{slice} = \rho \frac{a^2\omega^2}{4k} = \boxed{\frac{\rho a^2 g}{4}} = \boxed{\frac{\rho H^2 g}{16}} \qquad a = \frac{H}{2}$$

$\frac{\omega^2}{k} = g$ dispersion relationship in deep H_2O!

$$\bar{E}^{p}_{slice} = \int_{-d}^{\eta} \rho g z\,dz = \rho g \int_{-d}^{\eta} z\,dz = \rho g \left.\frac{z^2}{2}\right|_{-d}^{\eta} =$$

$$= \rho g \frac{\eta^2}{2} - \rho g \frac{(-d)^2}{2} = \rho g \frac{\eta^2}{2} - \rho g \frac{d^2}{2}$$

potential energy associated with waves

Potential Energy of the standing H_2O (without waves)

$$\bar{E}^{p}_{slice} = \rho g \frac{\eta^2}{2} = \rho g \frac{a^2 \cos^2(kx - \omega t)}{2}$$

$$\eta = a \cos(kx - \omega t)$$

From trig: $\cos^2\theta = \dfrac{1 + \cos(2\theta)}{2}$

$$\bar{E}^{P}_{slice} = \frac{\rho g a^2}{4}\left[1+\cos(2kx-2\omega t)\right]$$

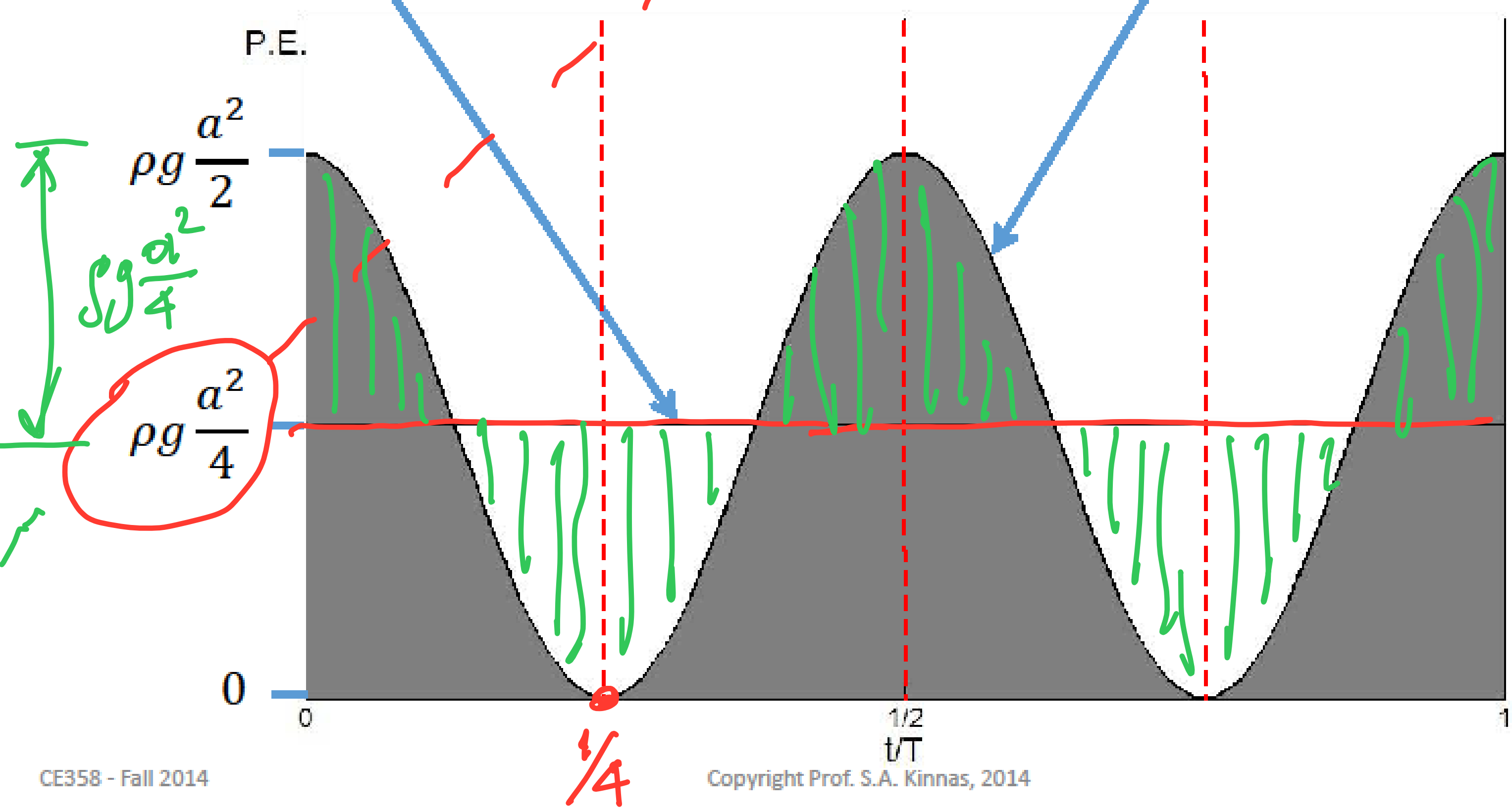
Let's take $x=0$

$$\bar{E}^{P}_{slice} = \frac{\rho g a^2}{4}\left[1+\cos(2\omega t)\right]$$

The potential energy (P.E.) under a sinusoidal wave

mean value of $P.E. = \rho g \frac{a^2}{4}$

$P.E.(t) = \rho g \frac{a^2}{4}\cos(2\omega t) + \rho g \frac{a^2}{4}$



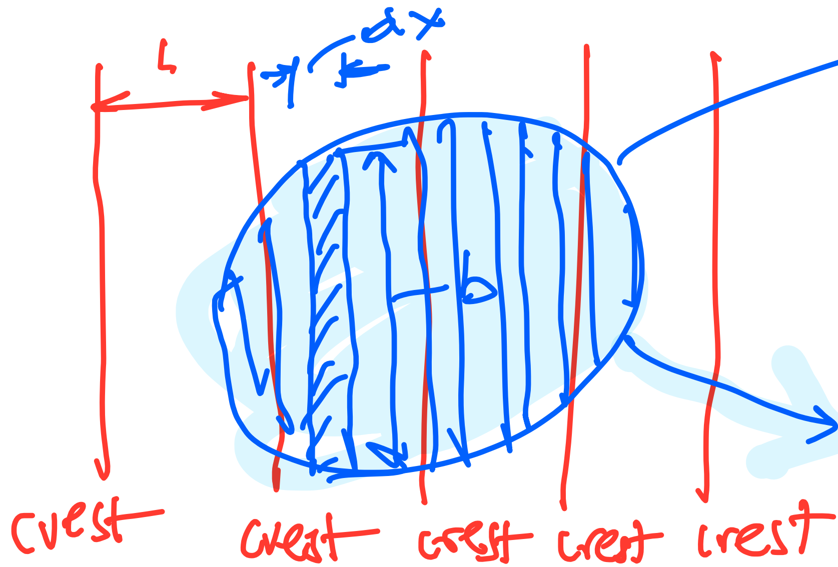
$\omega = \frac{2\pi}{T}$

Mean value of $\bar{E}^{p}_{slice}$ (over one period) is $\dfrac{\rho g a^2}{4}$

$$\boxed{\bar{E}_{slice}} = \bar{E}^{K}_{slice} + \bar{E}^{p}_{slice} = $$

$$= 2\,\bar{E}^{K}_{slice} = 2\,\bar{E}^{p}_{slice} = \boxed{\frac{\rho g a^2}{2}}$$

$$\boxed{\frac{\rho g H^2}{8}} \qquad \left(a = \frac{H}{2}\right)$$



top view of waves

column of water from the sea floor and through the free-surface

sectional area A

$$E_{slice} = b\,dx\,\bar{E}_{slice} = \underline{b\,dx}\,\frac{\rho g H^2}{8}$$

$$\sum_{\substack{All\\ slices}} E_{slice} = \frac{\rho g H^2}{8} \sum_{\substack{All\\ slices}} b\,dx$$

A = sectional area of the column of water

$$\boxed{E_{column} = A\,\frac{\rho g H^2}{8}}$$

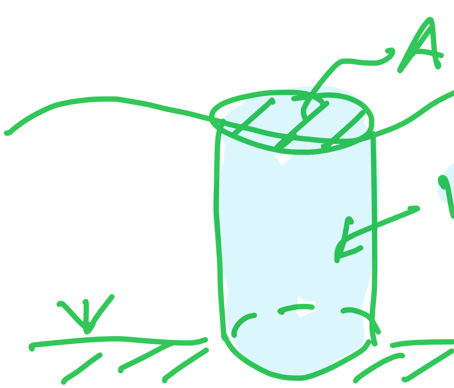
(in Joules in SI)

$\boxed{\bar{E}}$ = specific energy or energy density $\Big\}$ $\boxed{\dfrac{\rho g H^2}{8}}$

Energy per unit sectional area of water column

$$\boxed{E_{column} = \bar{E} A}$$

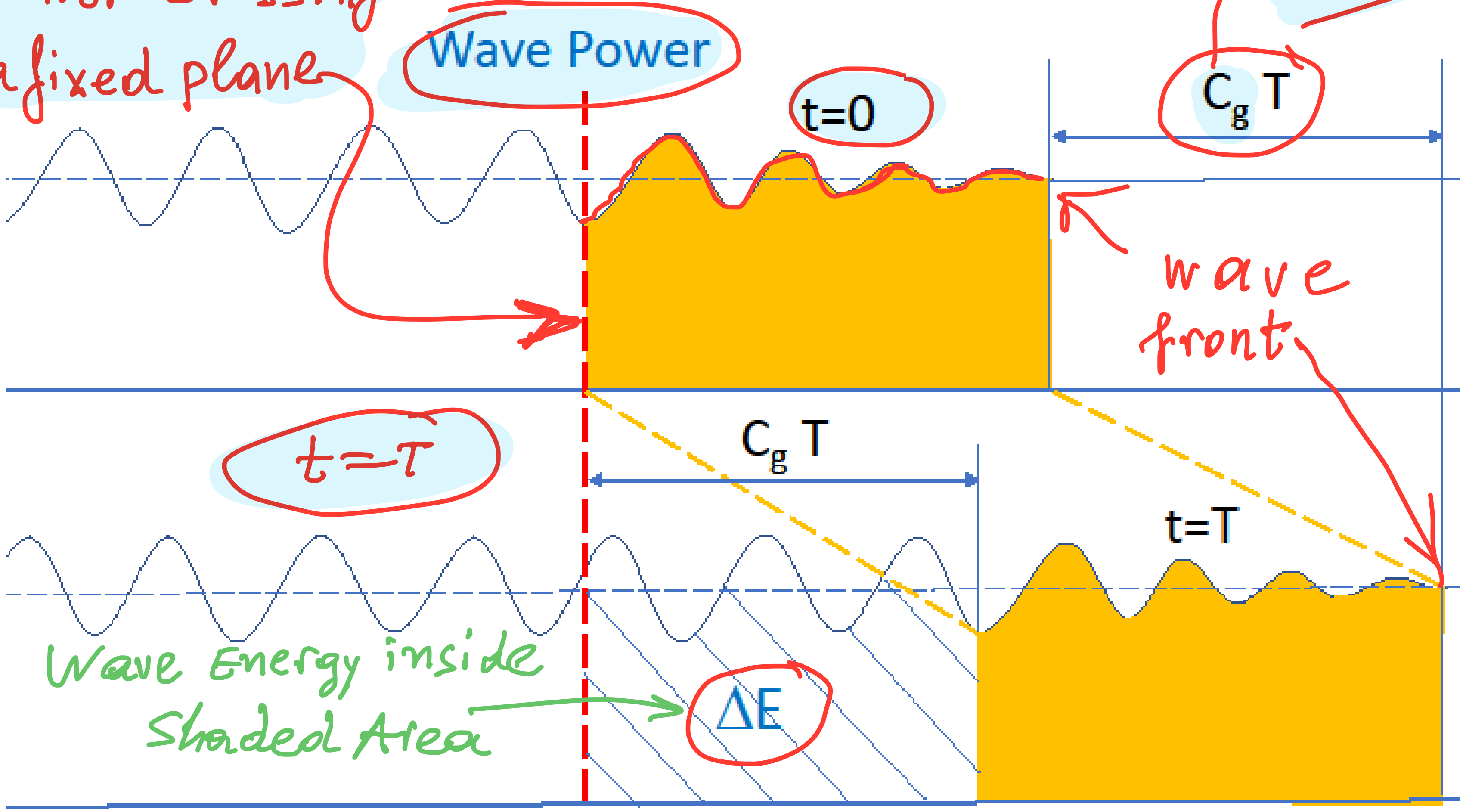
VALID FOR DEEP, TRANSITIONAL & SHALLOW WATER!



water column extending from sea-floor through free surface

WAVE POWER

Defined as wave power crossing a fixed plane



we consider a width $\underline{\underline{b}}$

$$\boxed{\text{Power;}}\ P=\Delta E/T = \bar{E}\, C_g\, \cancel{T}\, b\, /\, \cancel{T} = \boxed{\bar{E}\, C_g\, b}$$

distance perpendicular to screen

WAVE POWER

$\bar{P}$ = Power (of the wave) $= \bar{E}\, C_g\, b = \boxed{\frac{\rho g H^2}{8} C_g\, b}$ (in kW in SI)

Power per unit crest width " b

Watt = $\frac{\text{Joule}}{\text{sec}}$

$$\boxed{\bar{P} = \frac{\rho g H^2}{8} C_g}$$ (kW/m in SI)

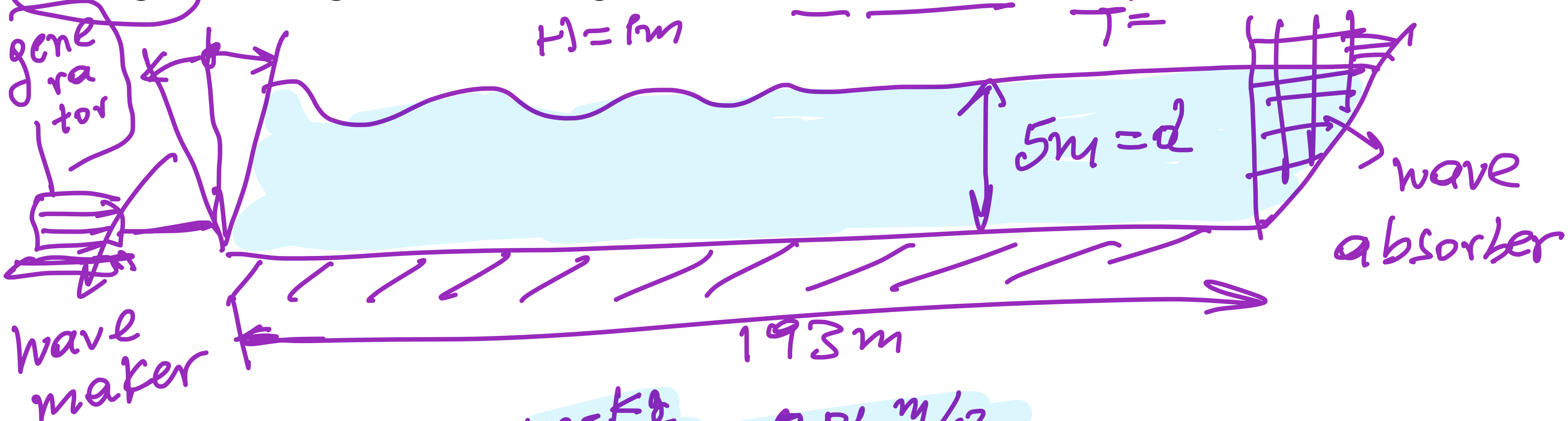
$$\boxed{P = \bar{P} \cdot b}$$

Wave Energy Converters (WEC)

wave crests (top view)

WAVE POWER/EXAMPLE

A sea-water wave tank at the U.S. Army Coastal Engineering Research Center is 193 m long, 4.57 m wide, and 5 m deep. Assume that the wave energy is fully absorbed at the opposite end of the wave generator. What power is required to generate 4-second waves which are 1m high, assuming that the wave generator is 40% efficient?



$b = 4.57\,m$

$$P_{waves} = \frac{\rho g H^2}{8} C_g\, b$$

ρ: $1{,}025\ \frac{kg}{m^3}$; g: $9.81\ m/s^2$; $H = 1m$; $b \rightarrow 4.57m$

$$C_g = n \cdot C\ ; \quad T = 4sec \Rightarrow L_0 = \frac{gT^2}{2\pi} = 24.88m$$

WAVE POWER/EXAMPLE

A sea-water wave tank at the U.S. Army Coastal Engineering Research Center is 193 m long, 4.57 m wide, and 5 m deep. Assume that the wave energy is fully absorbed at the opposite end of the wave generator. What power is required to generate 4-second waves which are 1m high, assuming that the wave generator is 40% efficient?

$$\frac{d}{L_0} = \frac{5}{24.88} = 0.2002 \xrightarrow[C-1]{\text{Table}} \frac{d}{L} = 0.2252 \Rightarrow$$

$$\Rightarrow \boxed{L} = \frac{d}{0.2252} = \boxed{22.2\,m}$$

$$\boxed{C} = \frac{L}{T} = \frac{22.2}{4} = \boxed{5.55\,\frac{m}{s}}$$

$$\boxed{C_g} = n\,C = 0.6675 \times 5.55 = \boxed{3.705\,\frac{m}{s}}$$

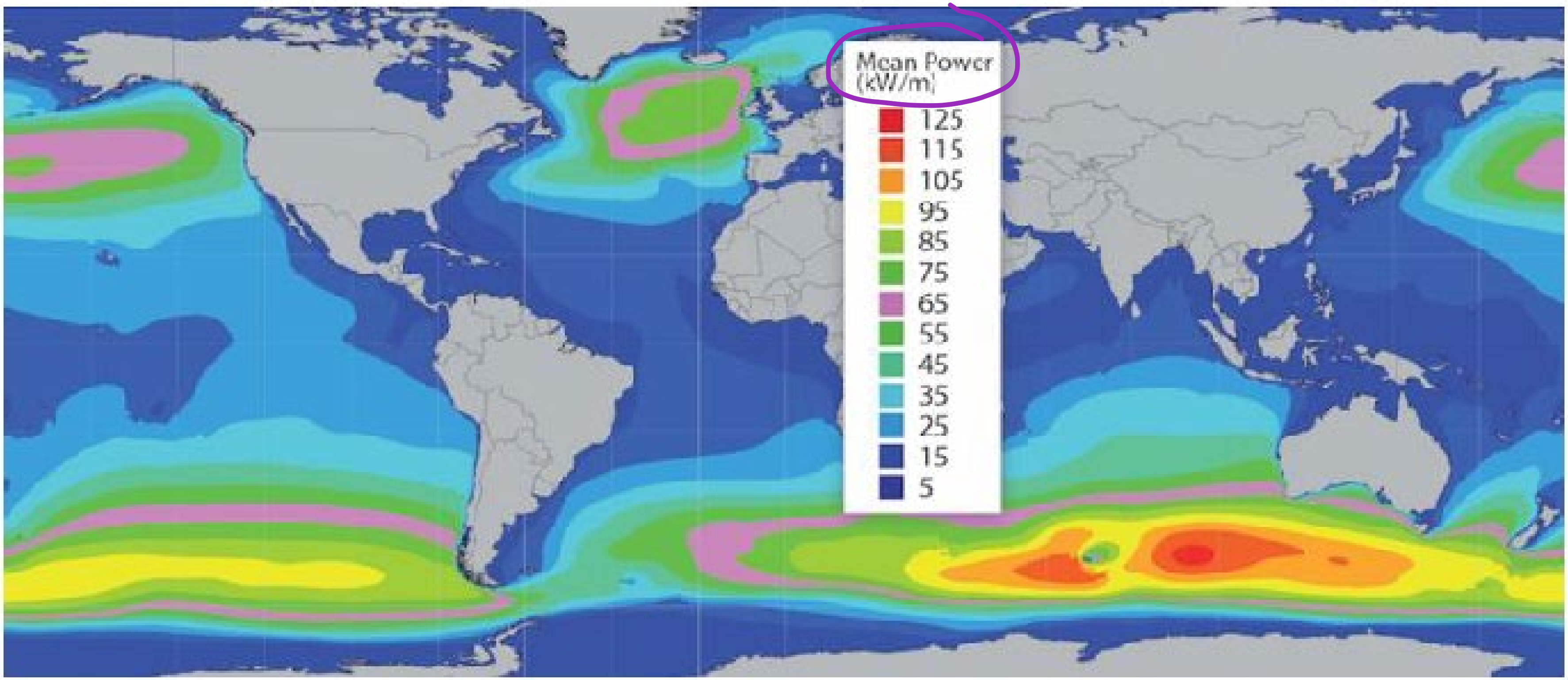
From Table C-1 with $\frac{d}{L_0} = 0.2002 \Rightarrow n = 0.6675$

$$\boxed{P_{waves}} = \frac{1{,}025 \times 9.81 \times 1^2}{8} \times 3{,}705 \times 4.57 = 21{,}282 \text{ W} = \boxed{21.3 \text{ kW}}$$

$$0.4 = \text{efficiency} = \frac{\text{usefull power}}{\text{given power}} = \frac{P_{waves}}{P_{gen}}$$

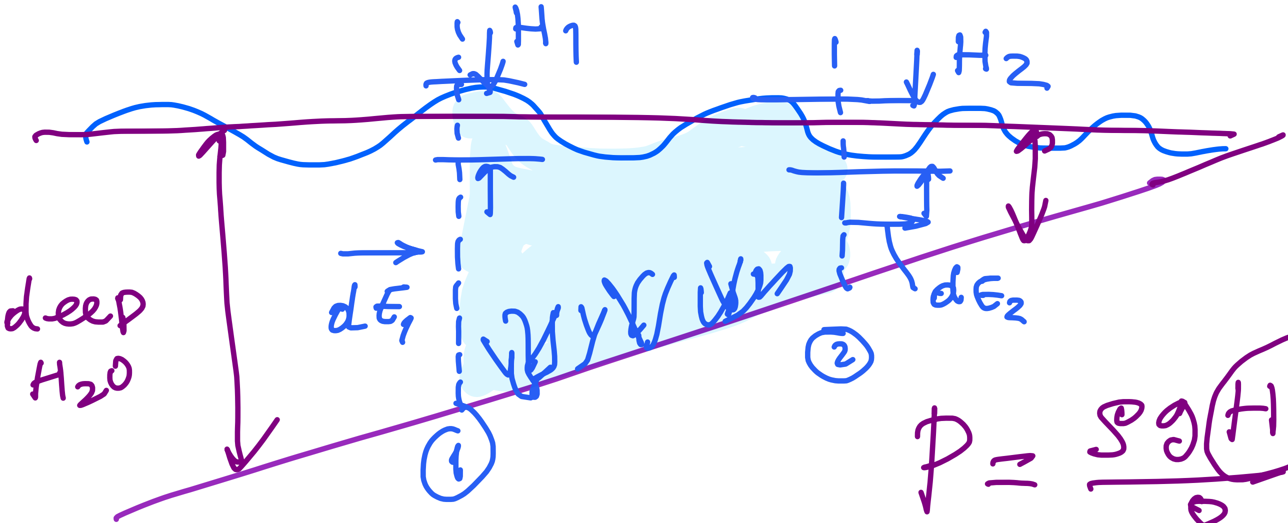
$$\Rightarrow \boxed{P_{generator}} = \frac{P_{waves}}{0.4} = \frac{21.3}{0.4} = \boxed{53.2 \text{ kW}}$$

Mean Wave Power (KW/m) $= \bar{P}$



Lewis, A., S. Estefen, J. Huckerby, W. Musial, T. Pontes, J. Torres-Martinez, 2011: Ocean Energy. In IPCC Special Report on Renewable Energy Sources and Climate Change Mitigation [O. Edenhofer, R. Pichs-Madruga, Y. Sokona, K. Seyboth, P. Matschoss, S. Kadner, T. Zwickel, P. Eickemeier, G. Hansen, S. Schlömer, C. von Stechow (eds)], Cambridge University Press. Figure 6.1

WAVE SHOALING



T (period) stays the same

L↓ and C↓ as d↓

$$P = \frac{\rho g H^2}{8} \cdot C_g \cdot b$$

$$\frac{dE_1}{dt} = \frac{dE_2}{dt} \Rightarrow P_1 = P_2$$

$$\frac{\cancel{\rho}\cancel{g} H_1^2}{\cancel{8}} C_{g1} \cancel{b_1} = \frac{\cancel{\rho}\cancel{g} H_2^2}{\cancel{8}} C_{g2} \cancel{b_2}$$

assumes no energy losses!

$$\boxed{C_{g1} H_1^2 = C_{g2} H_2^2} \leadsto \frac{H_2}{H_1} = \sqrt{\frac{C_{g1}}{C_{g2}}}$$

If energy losses as the wave propagates

WAVE SHOALING

from 1 $\rightarrow$ 2 then $P_2 = x\, P_1$ where $0 < x < 1$

For example if 10% energy losses then $P_2 = 0.9\, P_1$

Special application

if ① $\rightarrow$ ⓪ $\leftarrow$ deep H_2O

② $\rightarrow$ corresponds to depth d.

$$C_{g0} H_0^2 = C_g H^2$$

$$\boxed{K_s} = \boxed{\frac{H}{H_0}} = \sqrt{\frac{c_{g0}}{c_g}} = \boxed{\sqrt{\frac{C_0}{2nC}}}$$

$C_{g0} = \frac{C_0}{2}$ (deep H_2O)

$C_g = nC$

$K_s \rightarrow$ shoaling coefficient

$\rightarrow$ relates H at certain depth to that in deep water, H_0, in the absence of energy losses

WAVE SHOALING COEFFICIENT

shoaling coefficient K_s

Table C-1. Continued.

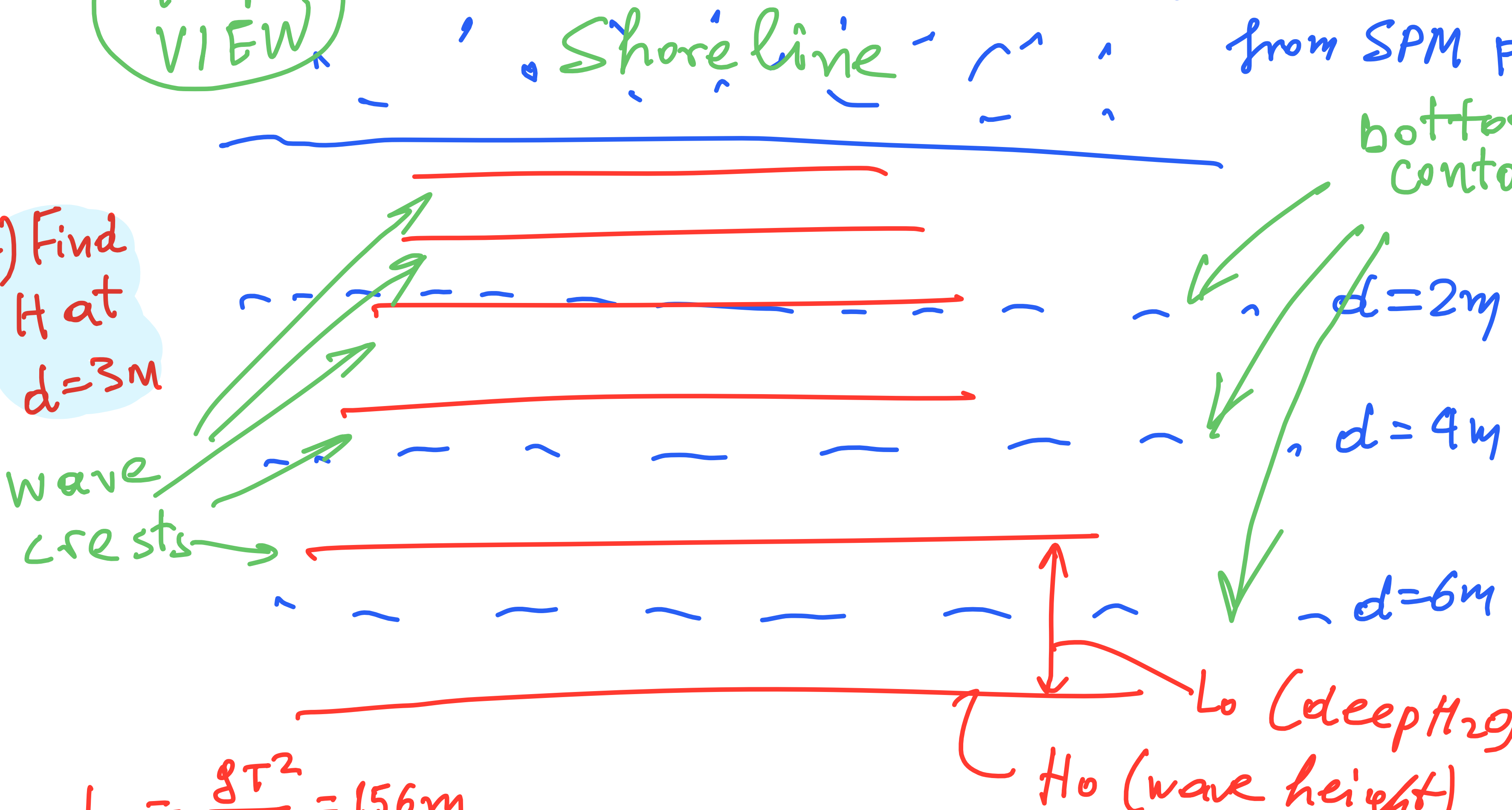
d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H_o'	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
.1500	.1833	1.152	.8183	1.424	1.740	.9133	.5748	2.303	4.954	5.054	.7325	.5994	7.369
.1510	.1841	1.157	.8200	1.433	1.747	.9133	.5723	2.314	5.007	5.106	.7311	.5994	7.339
.1520	.1850	1.162	.8217	1.442	1.755	.9132	.5699	2.324	5.061	5.159	.7296	.5995	7.309
.1530	.1858	1.167	.8234	1.451	1.762	.9132	.5675	2.335	5.115	5.212	.7282	.5996	7.279
.1540	.1866	1.173	.8250	1.460	1.770	.9132	.5651	2.345	5.169	5.265	.7268	.5996	7.250
.1550	.1875	1.178	.8267	1.469	1.777	.9131	.5627	2.356	5.225	5.320	.7254	.5997	7.221
.1560	.1883	1.183	.8284	1.479	1.785	.9130	.5602	2.366	5.283	5.376	.7240	.5998	7.191
.1570	.1891	1.188	.8301	1.488	1.793	.9129	.5577	2.377	5.339	5.432	.7226	.5999	7.162
.1580	.1900	1.194	.8317	1.498	1.801	.9130	.5552	2.387	5.398	5.490	.7212	.5998	7.134
.1590	.1908	1.199	.8333	1.507	1.809	.9130	.5528	2.398	5.454	5.544	.7198	.5998	7.107
.1600	.1917	1.204	.8349	1.517	1.817	.9130	.5504	2.408	5.513	5.603	.7184	.5998	7.079
.1610	.1925	1.209	.8365	1.527	1.825	.9130	.5480	2.419	5.571	5.660	.7171	.5998	7.052
.1620	.1933	1.215	.8381	1.536	1.833	.9130	.5456	2.429	5.630	5.718	.7157	.5998	7.026
.1630	.1941	1.220	.8396	1.546	1.841	.9130	.5432	2.440	5.690	5.777	.7144	.5998	7.000
.1640	.1950	1.225	.8411	1.555	1.849	.9130	.5409	2.450	5.751	5.837	.7130	.5998	6.975
.1650	.1958	1.230	.8427	1.565	1.857	.9131	.5385	2.461	5.813	5.898	.7117	.5997	6.949
.1660	.1966	1.235	.8442	1.574	1.865	.9132	.5362	2.471	5.874	5.959	.7103	.5996	6.924
.1670	.1975	1.240	.8457	1.584	1.873	.9132	.5339	2.482	5.938	6.021	.7090	.5996	6.900
.1680	.1983	1.246	.8472	1.594	1.882	.9133	.5315	2.492	6.003	6.085	.7076	.5995	6.876
.1690	.1992	1.251	.8486	1.604	1.890	.9133	.5291	2.503	6.066	6.148	.7063	.5994	6.853

WAVE SHOALING - EXAMPLE

(From Ex. Prob. 6 from SPM p 2-27)

TOP VIEW

a) Find H at d=3m



$H_o = 2m$, $T = 10 sec$

$$L_o = \frac{gT^2}{2\pi} = 156m$$

$$\frac{d}{L_o} = \frac{3}{156} = 0.01923 \xrightarrow[C-1]{\text{Table}} \frac{H}{H_o'} = 1.237 = K_s \quad (H_o' \equiv H_o)$$

WAVE SHOALING - EXAMPLE

$$\frac{H}{H_0} = 1.237 \rightarrow \boxed{H} = 1.237 \times (2m) = \boxed{2.474m}$$

b) Determine the rate at which energy per unit crest width is transported towards the shore line.

They are asking for: $\bar{P} = \frac{\rho g H^2}{8} \cdot C_g$

$\bar{P}$ can be evaluated in deep water or at $d=3m$. However $\bar{P}_0 = \bar{P} \rightarrow$ at $d=3m$

$$\bar{P}_0 = \frac{\rho g H_0^2}{8} C_{g_0} = \frac{1{,}025 \times 9.81 \times (2m)^2}{8} \cdot \frac{15.6}{2}$$

$$C_{g_0} = \frac{C_0}{2} = \frac{L_0}{T}\frac{1}{2} = \frac{156}{10} \cdot \frac{1}{2} = 15.6/2$$

$$\boxed{\bar{P} = \bar{P}_0 = 39.2\ kW/m}$$

In case there are 10% losses ← due to friction with sea-floor or vegetation

$$P = (0.9) P_0$$

Note: In the solution of part (b) we could use $\bar{P} = \frac{\rho g H^2}{8} \cdot C_g$ at $d = 3m$

That would involve $C_g = n \cdot C$ with $C = \frac{L}{T}$. So we would need to find $\frac{d}{L_0} \xrightarrow{C-1} \frac{d}{L} \rightarrow L \rightarrow C = \frac{L}{T}$

$\xrightarrow{C-1} n$

$$C_g = n \frac{L}{T}$$

The answer would be the same, but determining $\bar{P}_0$ is easier

TSUNAMIS

Harbor   Wave

$L = 100\,km$ in $d = 4\,km$

$\frac{d}{L} = 0.04 \Rightarrow$ shallow H_2O!!

In characterizing a wave as deep, transitional, or shallow $\frac{d}{L}$ and NOT d is the critical parameter!

TSUNAMIS - EXAMPLE

An earthquake at a water depth of 4 km generates a tsunami with a length of 100 km and a height of 1m. Determine the speed of the front (in km/h and mph) of this tsunami at the depth of 4 km and at a depth of 10 m. What is the period (in minutes), the speed of its front, and the height of this tsunami at a depth of 10 m? Ignore refraction and reflection.

Remember $C_g = C$ for shallow H_2O!

at 4,000 m $\quad C_{g,4000} = \sqrt{g\,d} = \sqrt{9.81 \times 4{,}000} = 198\ \frac{m}{s} = 713\ \frac{km}{h} = 444\ mph$

at 10 m $\quad$ assume shallow } $C_{g,10} = \sqrt{9.81 \times 10} = 9.9\ \frac{m}{s}$

VERY HIGH SPEEDS!!

TSUNAMIS - EXAMPLE

An earthquake at a water depth of 4 km generates a tsunami with a length of 100 km and a height of 1m. Determine the speed of the front (in km/h and mph) of this tsunami at the depth of 4 km and at a depth of 10 m. What is the period (in minutes), the speed of its front, and the height of this tsunami at a depth of 10 m? Ignore refraction and reflection.

Find T from d=4,000

$$C_{g,4000} = C_{4,000} = 198\ \frac{m}{s} = \frac{L}{T} = \frac{100{,}000\ m}{T}$$

$$\Rightarrow T = \frac{100{,}000}{198} = 505\ sec = \underline{8.41\ min}$$

Now I can find L at d=10 m

$$L = C_{10} \times T = 9.9 \times 500 = 5{,}000\ m$$

At d=10 m $\quad \frac{d}{L} = \frac{10}{5{,}000} = 2 \times 10^{-3} < 0.04$

Shallow assumption was correct!!

$$\boxed{\bar{P}_{4,000} = \bar{P}_{10}}$$ (assuming no losses)

$$\frac{\cancel{\rho g} H_{4,000}^2}{\cancel{8}} \cdot C_{g,4000} = \frac{\cancel{\rho g} H_{10}^2}{\cancel{8}} \cdot C_{g,10}$$

$$H_{10} = \underbrace{H_{4,000}}_{=1m} \times \sqrt{\frac{C_{g,4000}}{C_{g,10}}} = 1m \times \sqrt{\frac{198}{9.9}} \Rightarrow$$

$$\boxed{H_{10} = 4.5m}$$

Significant wave height !! magnification !.