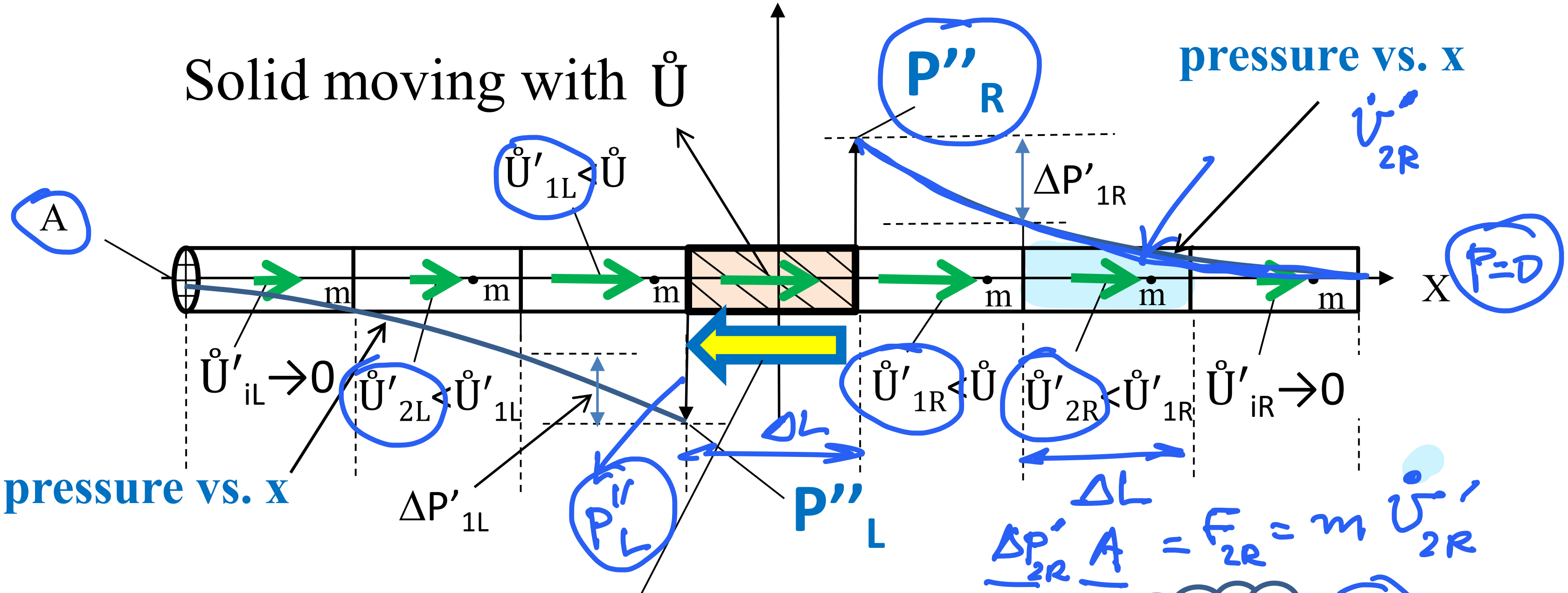
The concept of Added Mass (1/7)

Solid moving with acceleration $\mathring{U}$ inside quiescent fluid



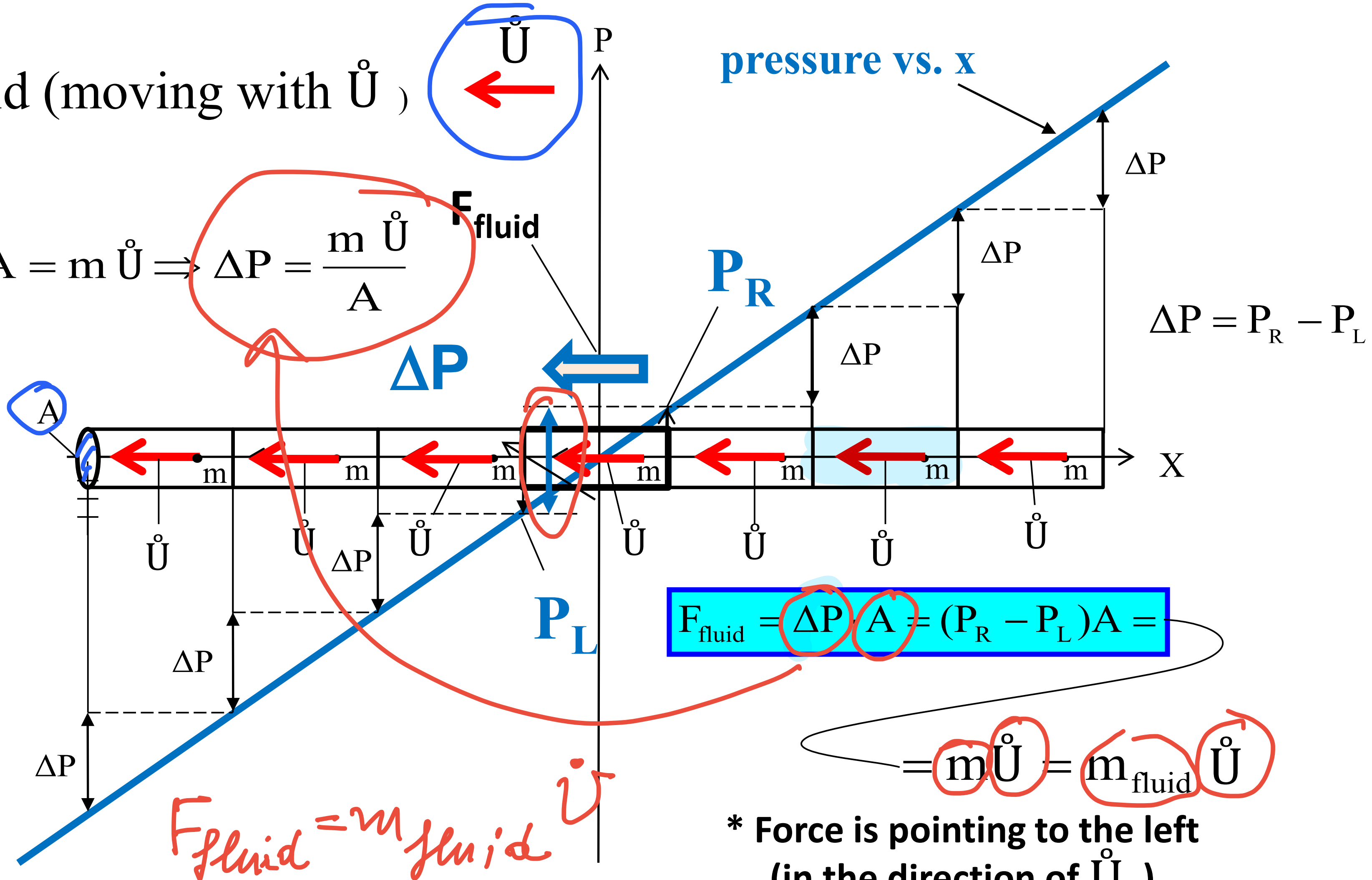
Resistive Force F_{res} : $\boxed{F_{res} = (P''_R - P''_L)\, A = m_{added}\, \dot{U}}$

$$F_{res} = (P''_R - P''_L)\, A = \sum \Delta P'_{iR} A + \sum \Delta P'_{iL} A = \sum m\, \dot{U}'_{iR} + \sum m\, \dot{U}'_{iL}$$

The resistive force ($F_{res} = m_{added}\, \mathring{U}$) is needed in order to accelerate parts of the surrounding fluid which move with the body.

The concept of Added Mass (2/7)

Fluid subject to acceleration **without a body** inside it



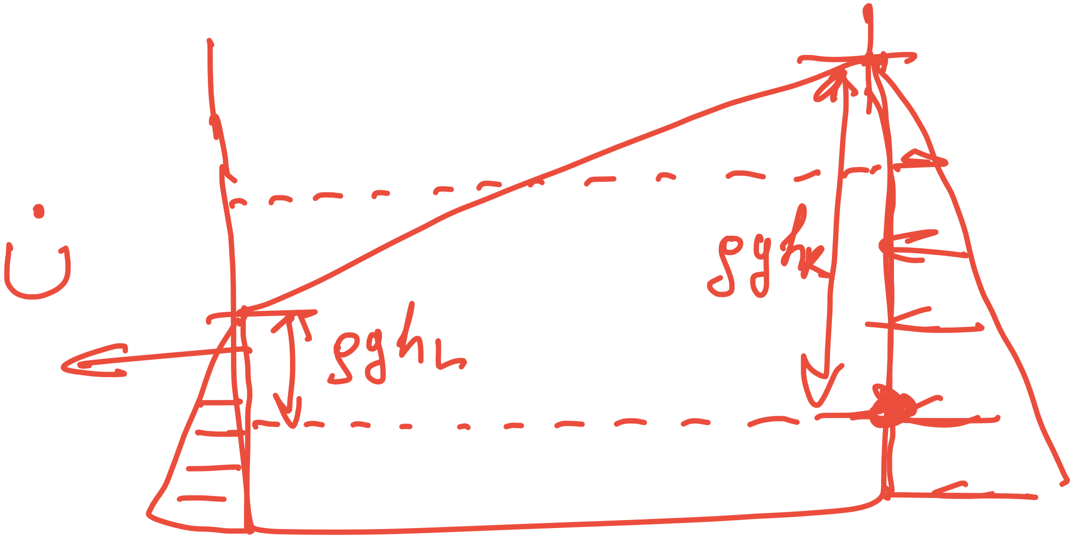
$$\Delta P \cdot A = m\mathring{U} \Rightarrow \Delta P = \frac{m\mathring{U}}{A}$$

$$\Delta P = P_R - P_L$$

$$F_{fluid} = \Delta P\, A = (P_R - P_L)A = m\mathring{U} = m_{fluid}\,\mathring{U}$$

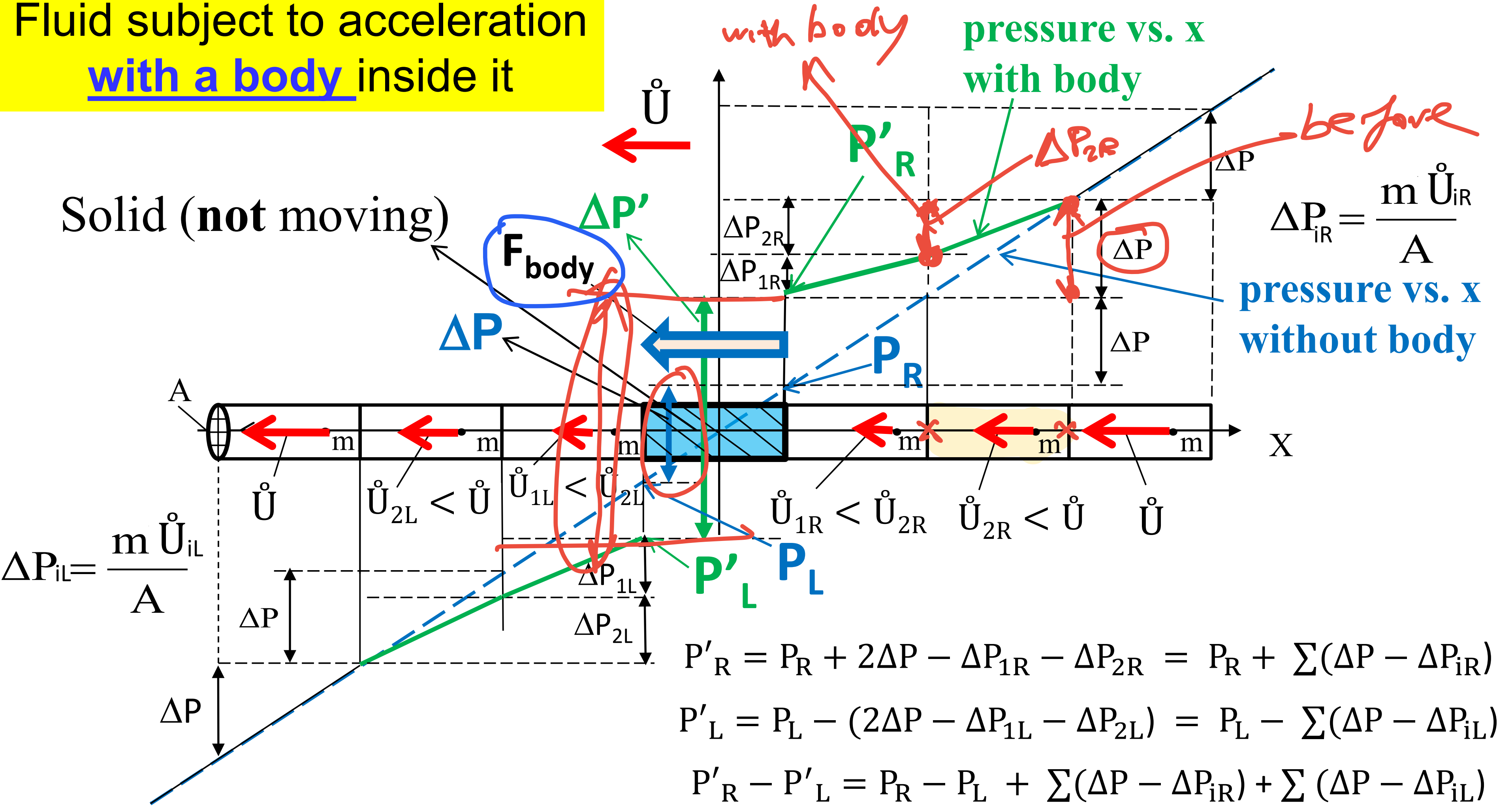
$$F_{fluid} = m_{fluid}\,\mathring{U}$$

* **Force is pointing to the left**
(in the direction of Ů)


u̇
ρgh_L
ρgh_K

The concept of Added Mass (3/7)

Fluid subject to acceleration **with a body** inside it



$$P'_R = P_R + 2\Delta P - \Delta P_{1R} - \Delta P_{2R} = P_R + \sum(\Delta P - \Delta P_{iR})$$

$$P'_L = P_L - (2\Delta P - \Delta P_{1L} - \Delta P_{2L}) = P_L - \sum(\Delta P - \Delta P_{iL})$$

$$P'_R - P'_L = P_R - P_L + \sum(\Delta P - \Delta P_{iR}) + \sum(\Delta P - \Delta P_{iL})$$

$$F_{body} = (P'_R - P'_L)\,A = (P_R - P_L)\,A + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})$$

$$\underline{F_{body}} = \underline{m_{fluid}\,\dot{U}} + \boxed{\underline{\sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})}}$$

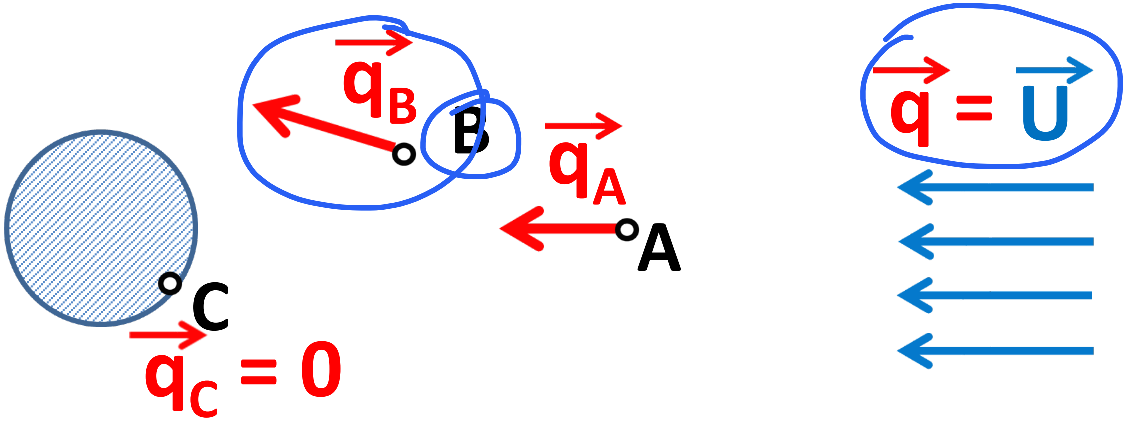
additional force

The concept of Added Mass (4/7)

Two Different Frames of Reference (for the same problem!):

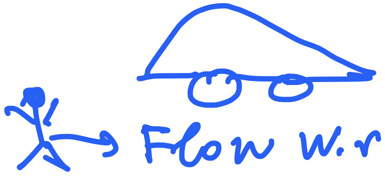
Body is stationary and flow is moving to the left with speed U $\vec{U}$

Flow w.r.t. driver

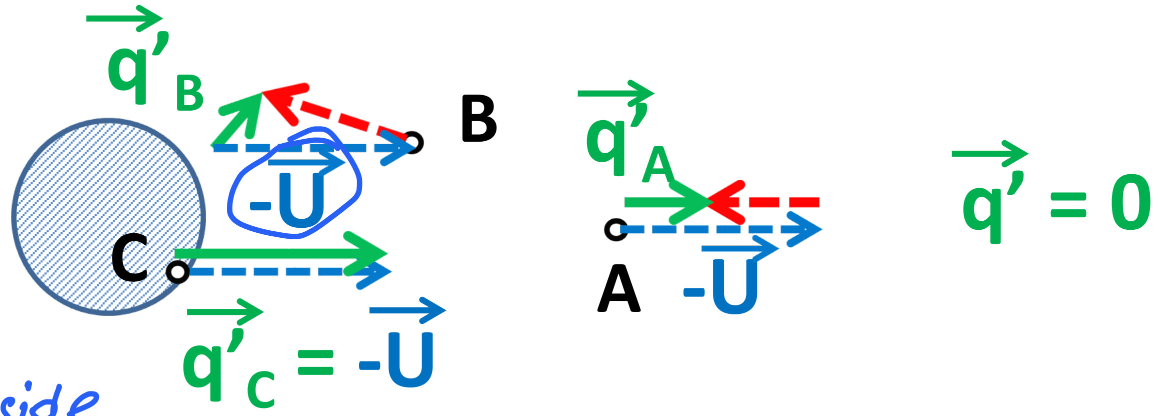


$$\vec{q}' = \vec{q} - \vec{U}$$

Flow is stationary (far upstream) and body is moving to the right with speed U



Flow w.r.t. outside observer



The concept of Added Mass (5/7)

$$F_{res} = (P''_R - P''_L)\,A = \sum m\,\dot{U}'_{iR} + \sum m\ \dot{U}'_{iL} = m_{added}\dot{U}$$

$$\dot{U}'_{iR} = -\dot{U}_{iR} - (-\dot{U}) = \dot{U} - \dot{U}_{iR}$$

$$\dot{U}'_{iL} = -\dot{U}_{iL} - (-\dot{U}) = \dot{U} - \dot{U}_{iL}$$

Thus:

$$\sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\ (\dot{U} - \dot{U}_{iL}) = m_{added}\dot{U}$$

$$F_{body} = m_{fluid}\,\dot{U} + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\ (\dot{U} - \dot{U}_{iL})$$

Thus:

$$\mathbf{F_{body} = m_{fluid}\,\dot{U} + m_{added}\,\dot{U} = (m_{fluid} + m_{added})\,\dot{U}}$$

$$\boxed{\mathbf{F_{body} = F_{inertial} = C_M\,m_{fluid}\,\dot{U}}}$$

Inertia coefficient C_M

$$\boxed{C_M = \frac{F_{body}}{F_{fluid}}}$$

$$\boxed{C_M = \frac{m_{fluid} + m_{added}}{m_{fluid}}}$$

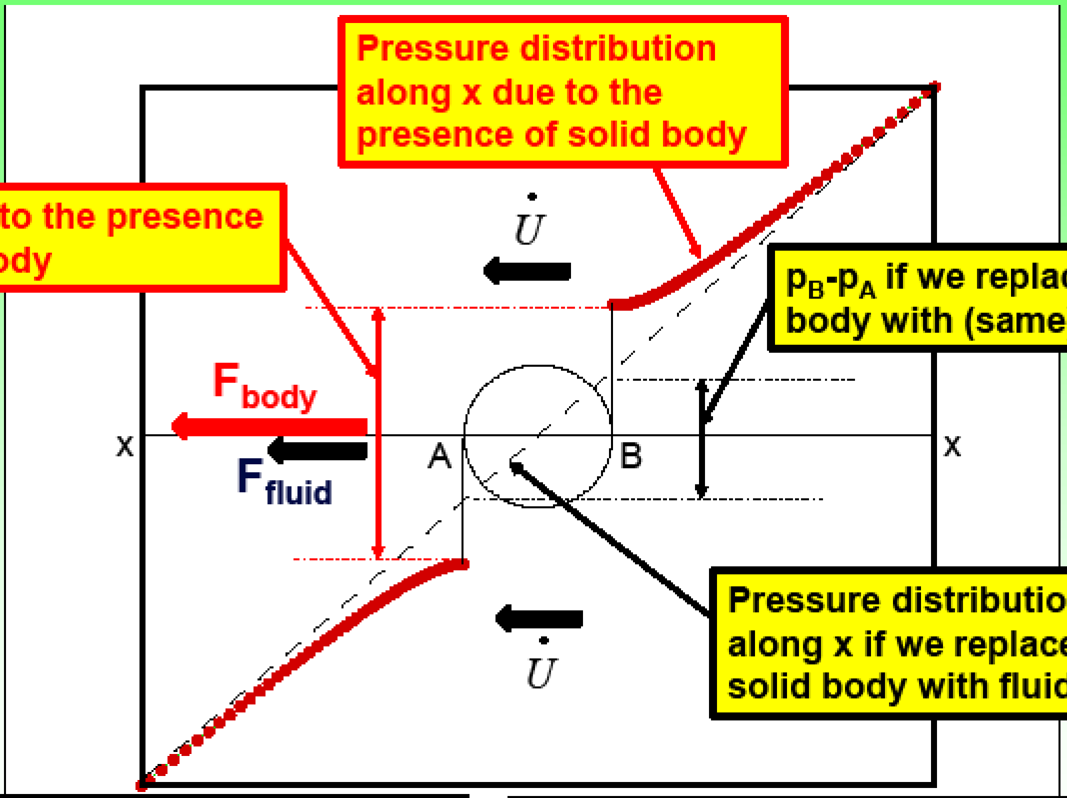
- **m_{fluid}=mass of fluid displaced by body**
- **m_{added}=added mass; function of body shape and inflow direction**

The concept of Added Mass (6/7)

Cylinder subject to accelerated inflow.

Results from Inviscid flow simulation

Definition of inertia coefficient C_M



Pressure distribution along x due to the presence of solid body

p_B-p_A due to the presence of solid body

p_B-p_A if we replaced solid body with (same) fluid

Pressure distribution along x if we replaced solid body with fluid

$$F_{fluid} = \rho(\pi D^2/4)\dot{U}$$

$$F_{body} = C_M \rho(\pi D^2/4)\dot{U}$$

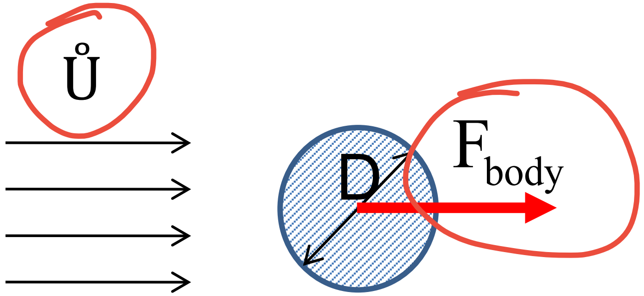
$$C_M = \frac{F_{body}}{F_{fluid}}$$

$$C_M = \frac{m_{fluid} + m_{added}}{m_{fluid}}$$

For inviscid flow around 2-D cylinder: C_M=2

The concept of Added Mass (7/7)

Summary: (all quantities are per unit width in 2-D)



$$F_{body} = C_M m_{fluid} \mathring{U}$$

$$m_{fluid} = \rho_{fluid} V_{fluid} = \text{mass of diplaced fluid}$$

$$V_{fluid} = \text{volume of displaced fluid}$$

$$C_M = \text{inertia coefficient} = \frac{m_{fluid} + m_{added}}{m_{fluid}} = 1 + \frac{m_{added}}{m_{fluid}} = 1 + a$$

$$a = \frac{m_{added}}{m_{fluid}} = \text{added mass coefficient (depends on shape + direction of flow)}$$

and inviscid flow

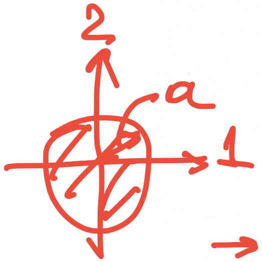
For a cylinder (circle in 2-D) V_{fluid}=area of cross section = $\pi D^2/4$

$m_{fluid} = \rho V_{fluid} = \rho \frac{\pi D^2}{4}$

For a cylinder in **inviscid unbounded flow**: C_M=2 (a=1)

(Inviscid) Added Mass for other shapes:

m_{11} and **m_{22}** are the added masses when the flow is accelerated in the horizontal or the vertical axis, respectively. **m_{66}** is the added moment of inertial when the body rotates around an axis normal to the paper.



Assumes inviscid flow

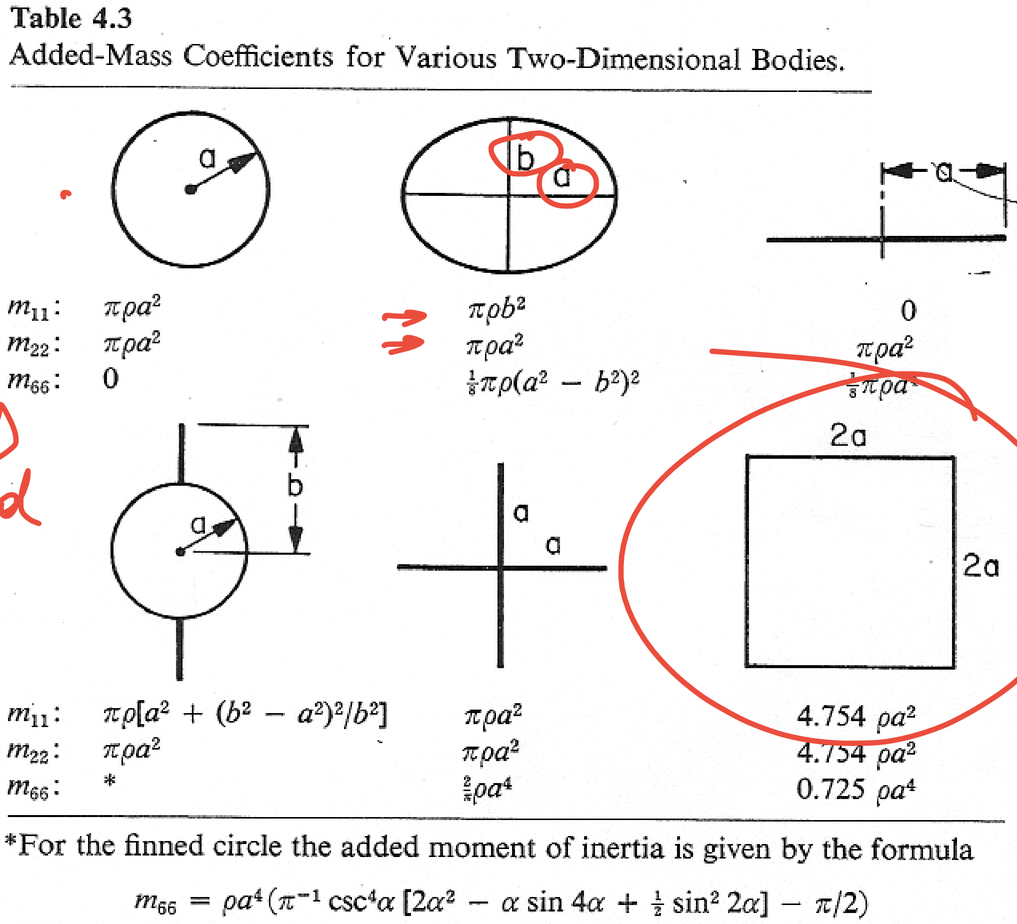


Table 4.3
Added-Mass Coefficients for Various Two-Dimensional Bodies.

m_{11}:	$\pi\rho a^2$	$\pi\rho b^2$	0
m_{22}:	$\pi\rho a^2$	$\pi\rho a^2$	$\pi\rho a^2$
m_{66}:	0	$\frac{1}{8}\pi\rho(a^2 - b^2)^2$	$\frac{1}{8}\pi\rho a^4$
m_{11}:	$\pi\rho[a^2 + (b^2 - a^2)^2/b^2]$	$\pi\rho a^2$	$4.754\ \rho a^2$
m_{22}:	$\pi\rho a^2$	$\pi\rho a^2$	$4.754\ \rho a^2$
m_{66}:	*	$\frac{2}{\pi}\rho a^4$	$0.725\ \rho a^4$

*For the finned circle the added moment of inertia is given by the formula

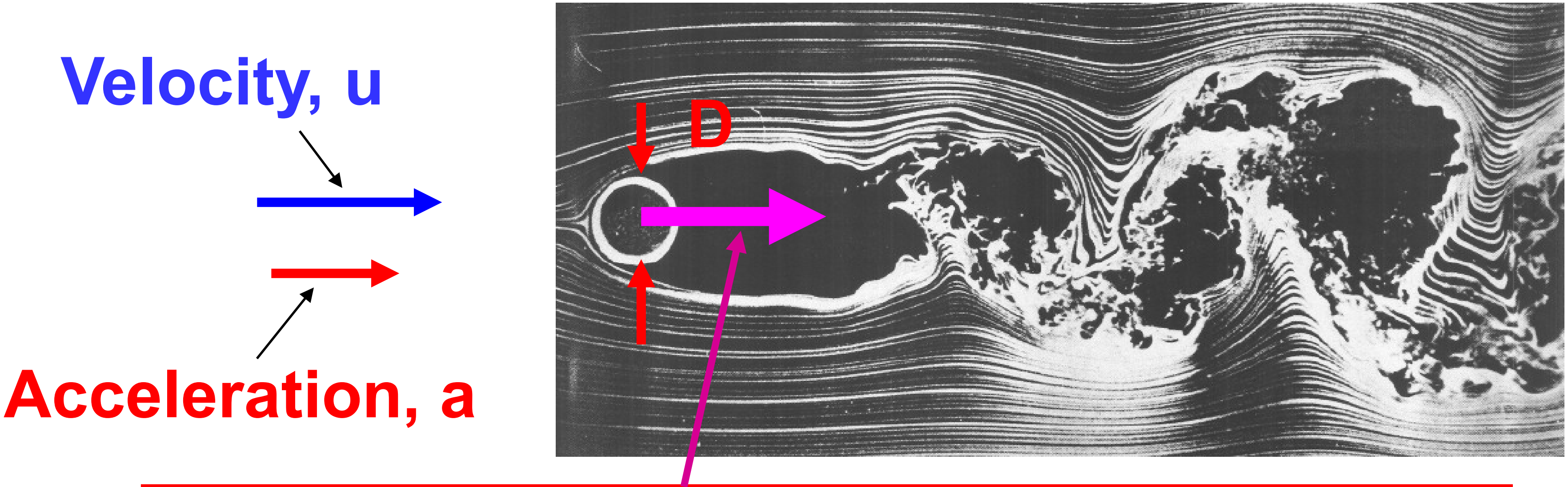
$$m_{66} = \rho a^4(\pi^{-1}\csc^4\alpha\,[2\alpha^2 - \alpha\sin 4\alpha + \tfrac{1}{2}\sin^2 2\alpha] - \pi/2)$$

From Marine Hydrodynamics, Newman, J.N., 1977

Copyright: Prof. S.A. Kinnas, 2022

Morison's equation for total force in the direction of wave propagation

[Morison, J. R.; O'Brien, M. P.; Johnson, J. W.; Schaaf, S. A. (1950), "The force exerted by surface waves on piles", Petroleum Transactions (American Institute of Mining Engineers) 189: 149–154]



Velocity, u

Acceleration, a

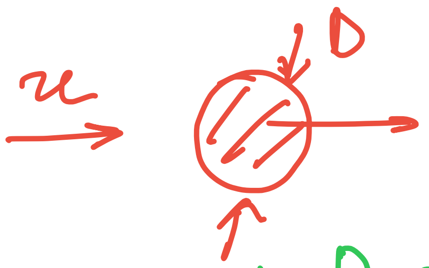
Total force = Viscous force + Inertial force

Total force (per unit width)

$$\text{Total force} = C_D \frac{1}{2} \rho D u\,|\,u\,| + C_M \rho \frac{\pi D^2}{4} a$$

Drag coefficient

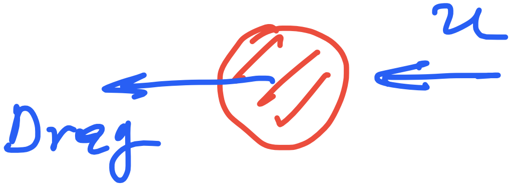
Inertia coefficient



$$\text{Drag} = C_D \frac{1}{2} \rho u^2 \cdot D$$

$\longrightarrow$ + direction

$u > 0 \Rightarrow \text{Drag} > 0$

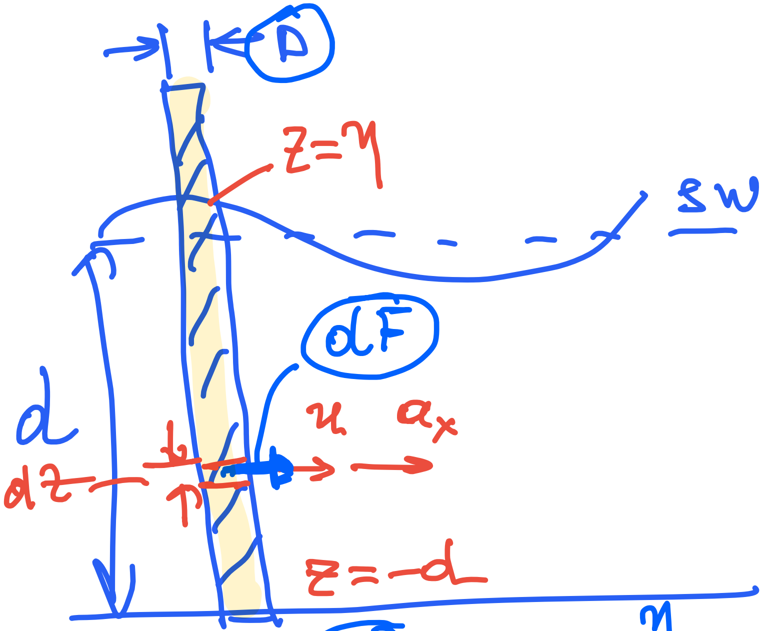


$(= C_D \frac{1}{2} \rho u^2 \cdot D$

$u < 0 \Rightarrow \text{Drag} < 0$

$$\text{Drag} = C_D \frac{1}{2} \rho D \boxed{u|u|}$$

Using $\underline{u|u|}$ instead of $\underline{u^2}$ guarantees that the Drag force is in the same direction as the inflow!



$$f_D = C_D \frac{\rho}{2} u|u| D$$ viscous force

$$f_i = C_M \rho \frac{\pi D^2}{4} a_x$$ inertial force

forces per unit depth

$$dF = f_D\,dz + f_i\,dz$$

$$F_t(t) = \int_{-d}^{\eta} dF = \int_{-d}^{\eta} f_D\,dz + f_i\,dz = \underbrace{\int_{-d}^{\eta} f_D\,dz}_{\substack{\text{viscous force} \\ F_D(t)}} + \underbrace{\int_{-d}^{\eta} f_i\,dz}_{\substack{\text{inertial Force} \\ F_i(t)}}$$

$$F_t(t) = F_D(t) + F_i(t)$$

$$F_i(t) = \int_{-d}^{\eta} C_M \rho \frac{\pi D^2}{4} a_x\,dz =$$

due to linear wave theory $\eta \to 0$

$$= C_M \rho \frac{\pi D^2}{4} \int_{-d}^{\eta} a_x\,dz = C_M \rho \frac{\pi D^2}{4} \int_{-d} \frac{g\pi H}{L} \frac{\cosh[k(z+d)]}{\cosh[kd]} \sin\theta\,dz$$

RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos \left[\frac{2\pi x}{L} - \frac{2\pi t}{T} \right] = \frac{H}{2} \cos \theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2} \left[1 + \frac{4\pi d / L}{\sinh(4\pi d/L)} \right] C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos \theta$
(b) Vertical	$w = \frac{H\pi}{T} \left(1 + \frac{z}{d} \right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin \theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$a_x = 2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$a_z = -2H \left(\frac{\pi}{T} \right)^2 \left(1 + \frac{z}{d} \right) \cos \theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \cos \theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$\zeta = \frac{H}{2} \left(1 + \frac{z}{d} \right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos \theta$
8. Subsurface Pressure	$p = \rho g (\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

Figure 2-6. Summary of linear (Airy) wave theory--wave characteristics.

$$\hookrightarrow = C_M \rho \frac{\pi D^2}{4} H g \boxed{\int_{-d}^{0} \frac{\pi}{L} \frac{\cosh[k(z+d)]}{\cosh[kd]} dz} \sin\theta$$

$$K_{im} = \frac{1}{2} \tanh\left(\frac{2\pi d}{L}\right)$$

$$\Rightarrow \underline{F_i(t) = F_{im} \sin\theta} \qquad \underline{\theta = kx - \omega t}$$

$$F_{im} = \underline{C_M}\ \underline{\rho}\ \underline{g}\ \underline{\frac{\pi D^2}{4}}\ \underline{H} \cdot \underline{K_{im}}$$

$\hookrightarrow$ maximum inertial force

Similarly it can be derived that:

$$F_D(t) = \int_{-d}^{0} f_D \, dz = F_{Dm} |\cos\theta| \cos\theta$$

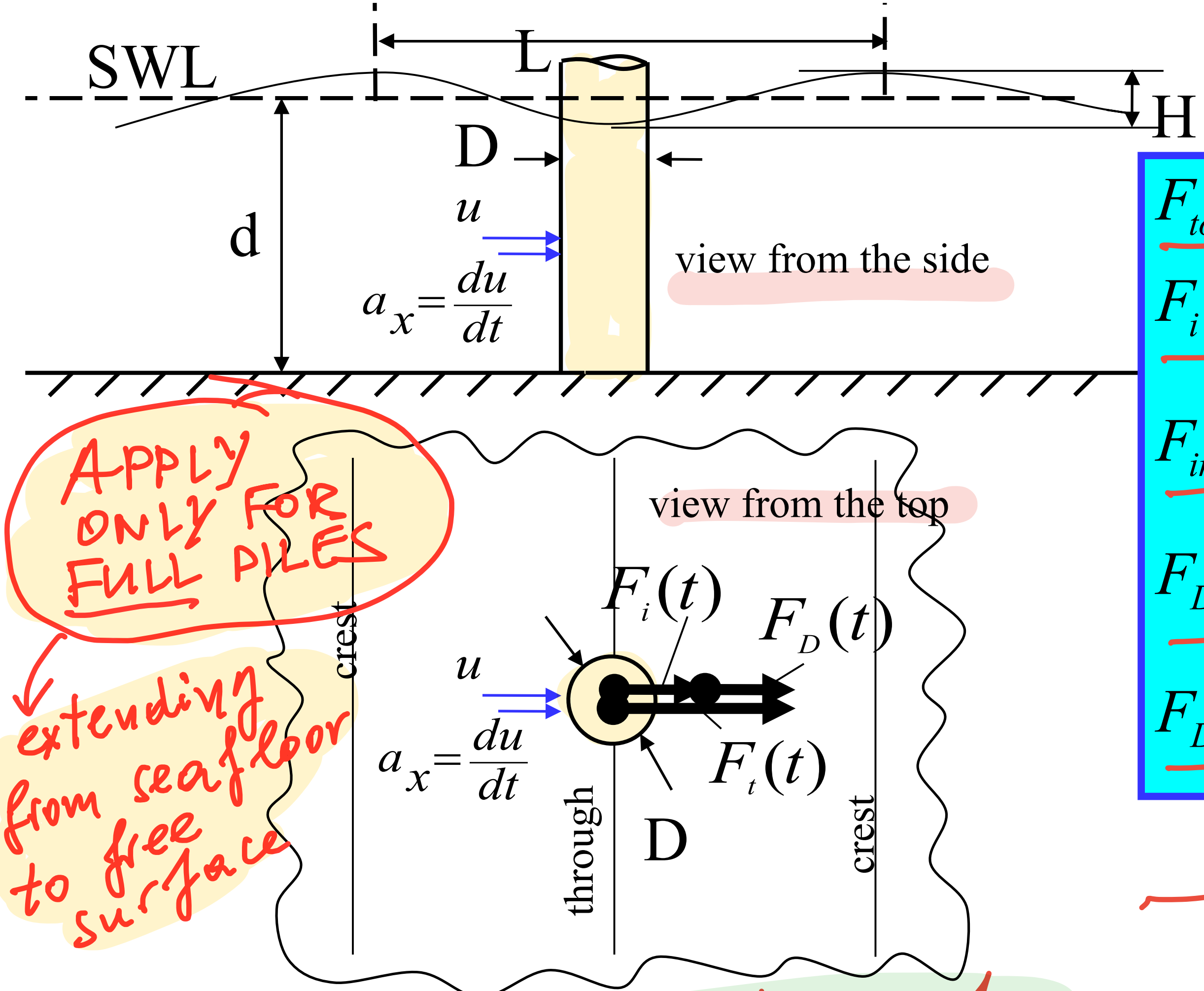
$$F_{Dm} = C_D \frac{1}{2} \rho g D H^2 K_{Dm} \quad ; \quad K_{Dm} = \frac{1}{8}\left[1 + \frac{\frac{4\pi d}{L}}{\sinh\left[\frac{4\pi d}{L}\right]}\right]$$

$\hookrightarrow$ maximum Drag force

$$\hookrightarrow = \frac{n}{4} \qquad n: C_g/C$$

Application of Morison's equation to determine forces on vertical piles

Morison's equation is integrated over the length of the pile, after the values for **u** and **a_x** have been determined by either using linear or **non-linear wave theories**



APPLY ONLY FOR FULL PILES extending from seafloor to free surface

$$\vartheta = \overset{kx}{} - \omega\, t$$

$$F_{total}(t) = F_i(t) + F_D(t)$$

$$F_i(t) = F_{im} \cdot \sin(\vartheta)$$

$$F_{im} = C_M \cdot \rho g \cdot \frac{\pi D^2}{4} H \cdot K_{im}$$

$$F_D(t) = F_{Dm} \cdot \left|\cos\vartheta\right| \cos\vartheta$$

$$F_{Dm} = C_D \frac{1}{2} \rho g\ D H^2 \cdot K_{Dm}$$

$$\rightarrow \quad K_{im} = \frac{1}{2}\tanh\left(\frac{2\pi d}{L}\right)$$

$$\rightarrow \quad K_{DM} = \frac{1}{8}\left(1 + \frac{4\pi d / L}{\sinh\left[4\pi d / L\right]}\right) = \frac{n}{4}$$

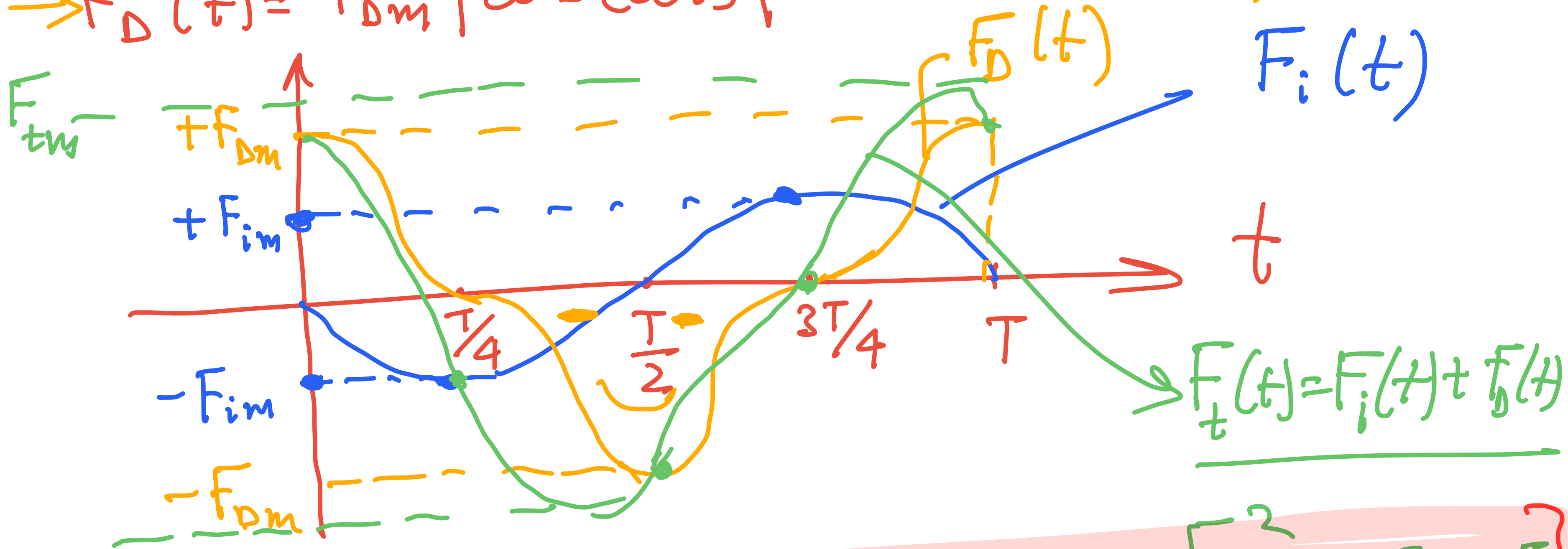
$n = c_g / C$

For deep H_2O: $K_{im} = \frac{1}{2}$; $K_{Dm} = \frac{1}{8}$

$x = 0$, $\theta = kx - \omega t = -\omega t$

$\rightarrow F_i(t) = F_{im} \sin(-\omega t) = \underline{-F_{im}} \, \underline{\sin(\omega t)}$

$\rightarrow F_D(t) = F_{Dm} \, |\cos(\omega t)| \, \cos(\omega t)$

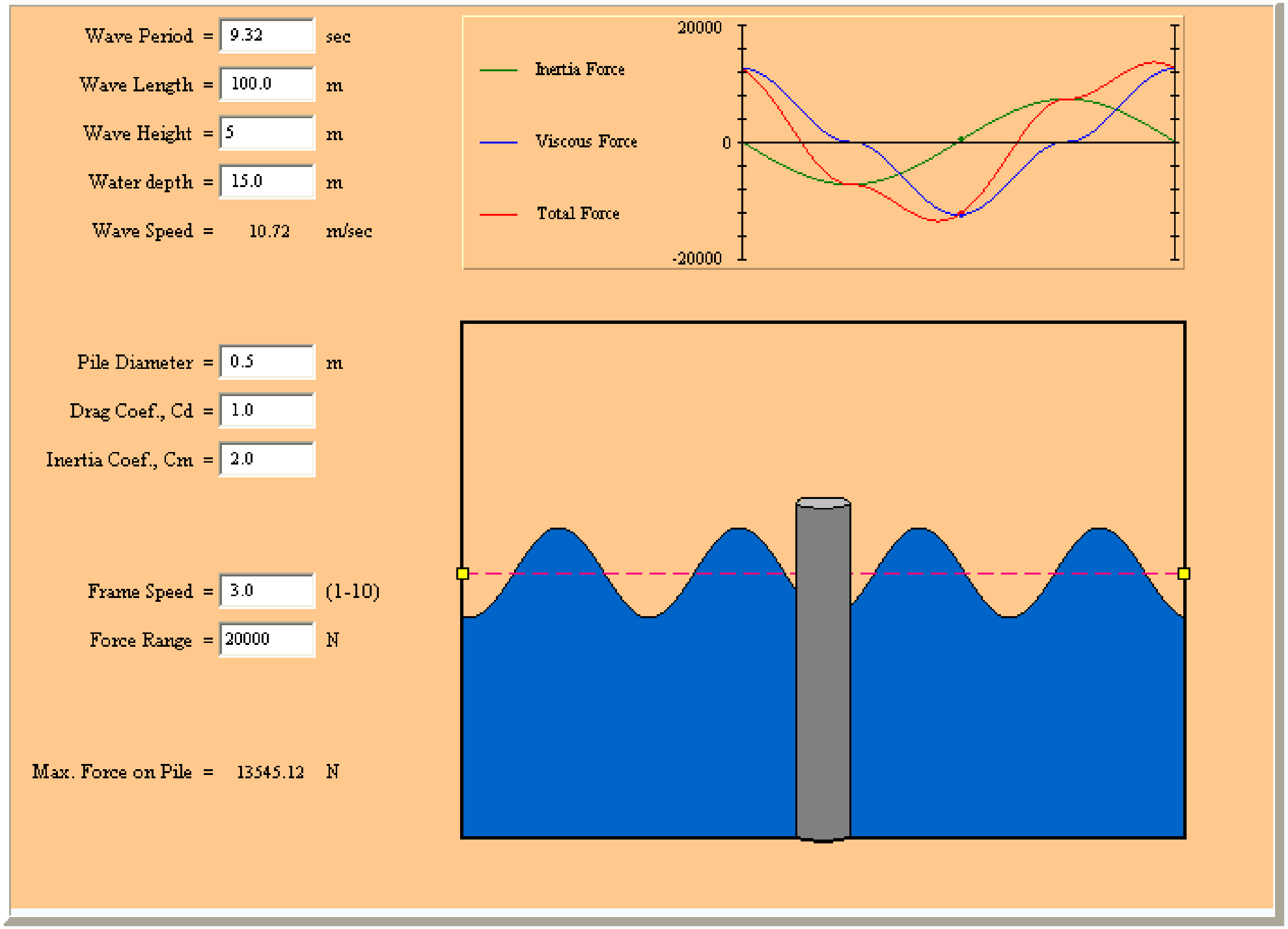


$F_t(t) = F_i(t) + F_D(t)$

- F_{tm} = max total force $= F_{Dm} + \frac{F_{im}^2}{4F_{Dm}}$ if $\underline{F_{im}} < \underline{2F_{Dm}}$
- " " $= F_{im}$ if $\underline{F_{im}} > \underline{2F_{Dm}}$

The wave forces applet sums-up the forces per pile slice over the pile length

http://cavity.ce.utexas.edu/kinnas/wow/public_html/waveroom/Applet/WaveForces/WaveForces.html



Wave Period = 9.32 sec

Wave Length = 100.0 m

Wave Height = 5 m

Water depth = 15.0 m

Wave Speed = 10.72 m/sec

Pile Diameter = 0.5 m

Drag Coef., Cd = 1.0

Inertia Coef., Cm = 2.0

Frame Speed = 3.0 (1-10)

Force Range = 20000 N

Max. Force on Pile = 13545.12 N

Typical values of the drag and inertia coefficients
From API's (American Petroleum Institute) Recommended Practice 2A-WSD (Dec. 2000)

- C_D = 0.65 and C_M=1.6 for smooth piles
- C_D = 1.05 and C_M=1.2 for rough piles (due to marine growth)

C_D ↑ since rougher surface

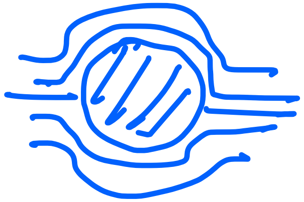
Note: The diameter of the pile, D, also increases with marine growth

Q: Why C_M for smooth piles is less than 2, and why is becoming even smaller for rough piles?

$$C_M = 1 + \boxed{a}$$

$$a = \frac{m_{added}}{m_{Fluid}} = \text{added mass coeff.}$$

inviscid

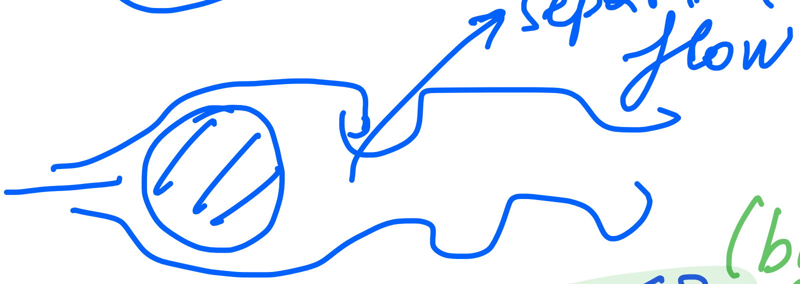


(no separation)

($a=1$ for cylinder in inviscid flow)

$\Rightarrow C_M = 2$

viscous

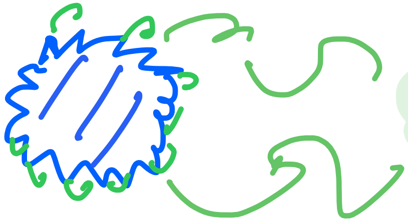


$m_{added} \downarrow \Rightarrow a \downarrow \Rightarrow C_M \downarrow$

(body "accelerates" less fluid mass due to separated flow)

$C_M < 2$

Rough viscous



$C_M' \ll C_M$ $m_{added} \downarrow\downarrow \Rightarrow a \downarrow\downarrow \Rightarrow C_M \downarrow\downarrow$

(body "accelerates even less fluid mass due to additional separated flow at rough edges)

Total Force on Pile
(in the direction of wave propagation)

Total force = Viscous force + Inertial force

Morison's equation → $\sim C_D \rho D H^2 + \sim C_M \rho D^2 H$

Drag coefficient

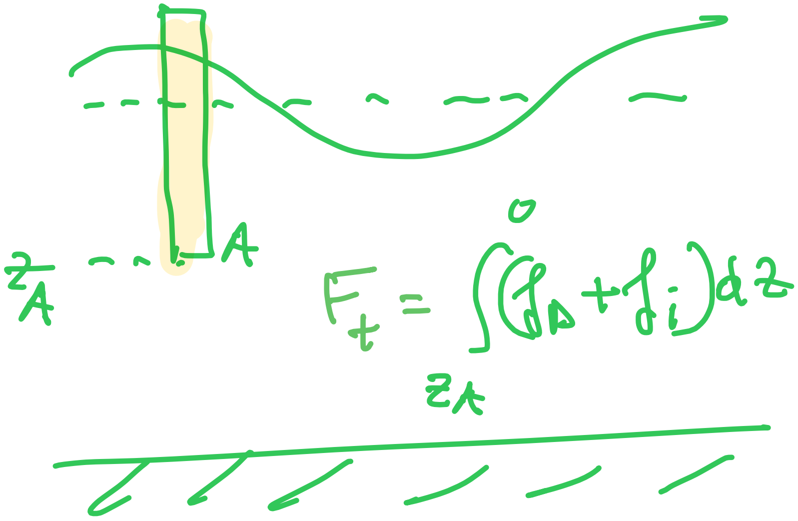
Inertia coefficient

Drag Force

$$\frac{\text{Viscous force}}{\text{Inertial force}} \sim \frac{C_D}{C_M} \frac{H}{D}$$

As H↑ or D↓ or H/D↑ the viscous forces become more important

Short pile theory

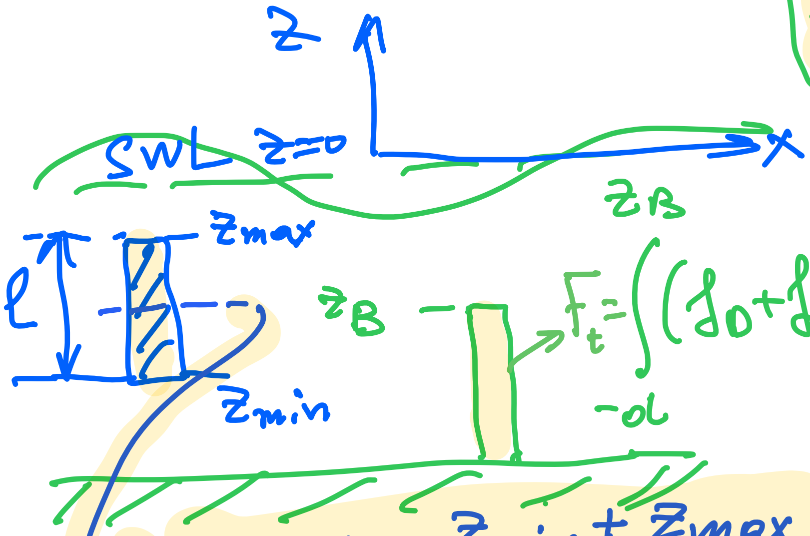


$$F_t = \int_{z_A}^{0} (f_D + f_i)\, dz$$

$$F_i(t) = \int_{z_{min}}^{z_{max}} f_i \, dz \approx f_{i,mid} \cdot l$$

$$F_D(t) = \int_{z_{min}}^{z_{max}} f_D \, dz \approx f_{D,mid} \cdot l$$

$$\boxed{F_t(t) = F_i(t) + F_D(t)}$$



$$F_t = \int_{-d}^{z_B} (f_D + f_i)\, dz$$

$$f_{i,mid} = C_M \rho \frac{\pi D^2}{4} a_{x,mid}$$

$$f_{D,mid} = C_D \frac{1}{2} \rho u_{mid} |u_{mid}| D$$

$$z_{mid} = \frac{z_{min} + z_{max}}{2}$$

z of midpoint of pile.

$a_{x,mid}$ and u_{mid} are values at the midpoint of the piles.

$$F_i(t) = F_{im} \sin\theta$$

$$F_D(t) = F_{Dm} |\cos\theta| \cos\theta$$

$$\underline{F_{im}} = f_{i,mid} \cdot \ell = \underline{C_M \rho \frac{\pi D^2}{4} a_{x,mid} \cdot \ell}$$

$$\underline{F_{Dm}} = f_{D,mid} \cdot \ell = \underline{C_D \frac{1}{2} \rho u_{mid} |u|_{mid} \cdot D \cdot \ell}$$

The formulas for F_{tm}, maximum total force, still apply!

The values for $a_{x,mid}$ and u_{mid} can be found in Fig. 2-6

$$\Downarrow$$

u_{mid} at $z=z_{mid}$

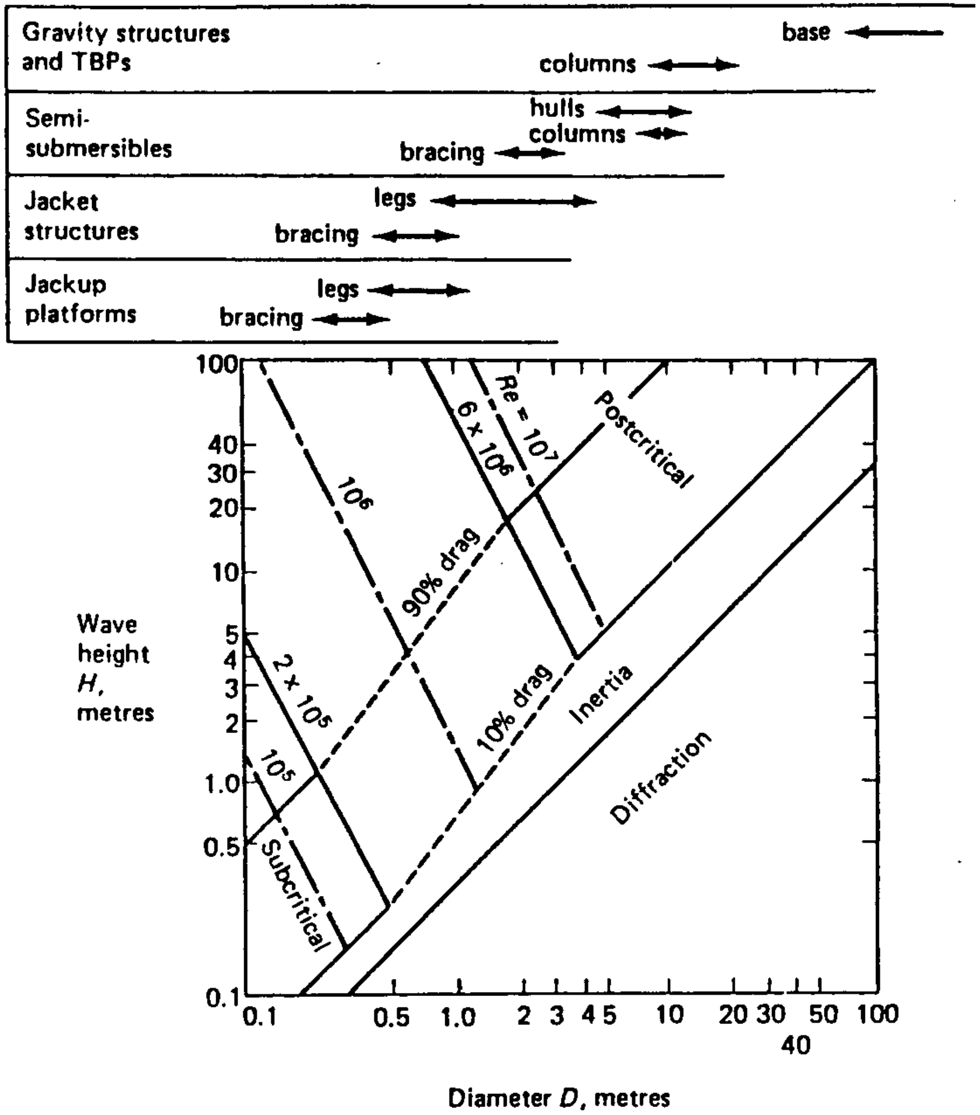
$a_{x,mid}$ at $z=z_{mid}$

RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2}\cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2}\cos\theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]\cdot C$	$C_g = \frac{1}{2}C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}}\cos\theta$	$u = \boxed{\frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}}\cos\theta$	$u = \frac{\pi H}{T}e^{\frac{2\pi z}{L}}\cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1+\frac{z}{d}\right)\sin\theta$	$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$w = \frac{\pi H}{T}e^{\frac{2\pi z}{L}}\sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}}\sin\theta$	$a_x = \boxed{\frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}}\sin\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1+\frac{z}{d}\right)\cos\theta$	$a_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}}\sin\theta$	$\xi = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta$	$\xi = -\frac{H}{2}e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1+\frac{z}{d}\right)\cos\theta$	$\zeta = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta$	$\zeta = \frac{H}{2}e^{\frac{2\pi z}{L}}\cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g\eta\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho gz$	$p = \rho g\eta e^{\frac{2\pi z}{L}} - \rho gz$

Figure 2-6. Summary of linear (Airy) wave theory--wave characteristics.

MATERIAL BELOW THIS LINE WAS NOT COVERED AND WILL NOT BE INCLUDED IN TEST II!

Effect of wave height H and diameter of element D on importance of viscous forces

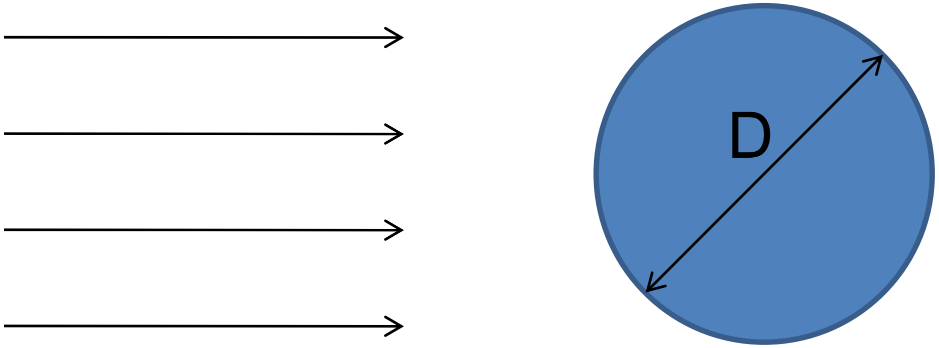


Loading regimes at still water level (from Hogben[4])

An assessment of Morison’s equation using CFD (Computational Fluid Dynamics)

Two Dimensional Cylinder in Oscillatory Flow

$$U = U_m \cdot \cos(\omega t)$$



Two important numbers:

- **Re (Reynolds No)=$U_m D/\nu$**

- **KC (Keulegan-Carpenter No)=$U_m T/D$ ($T=2\pi/\omega$)**
  ~(distance the particles travel in T)/D

Morison’s Equation

The inline force (force in the direction of the flow) is the sum of the drag force and the inertia force (per unit width)

$$F = \frac{1}{2}\rho C_D D\left|U\right|U + \frac{1}{4}\rho\pi D^2 C_M \frac{dU}{dt}$$

$$U = U_m \cdot \cos(\omega t)$$

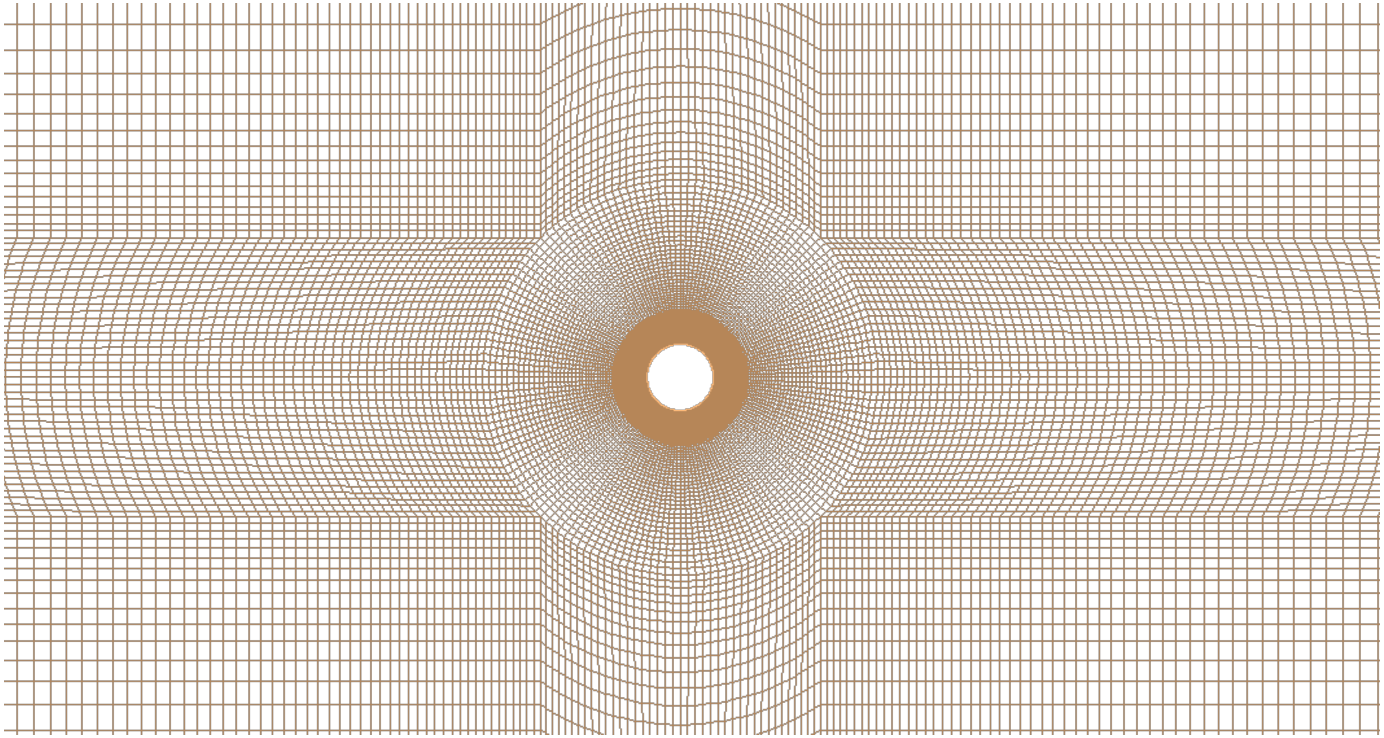
C_M is the inertia coefficient

C_D is the drag coefficient

- we also define: $$C_x = \frac{F}{\frac{\rho}{2} U_m{}^2 D}$$

Grid in Fluent

Structured mesh is used in the calculation domain



Mesh Info:

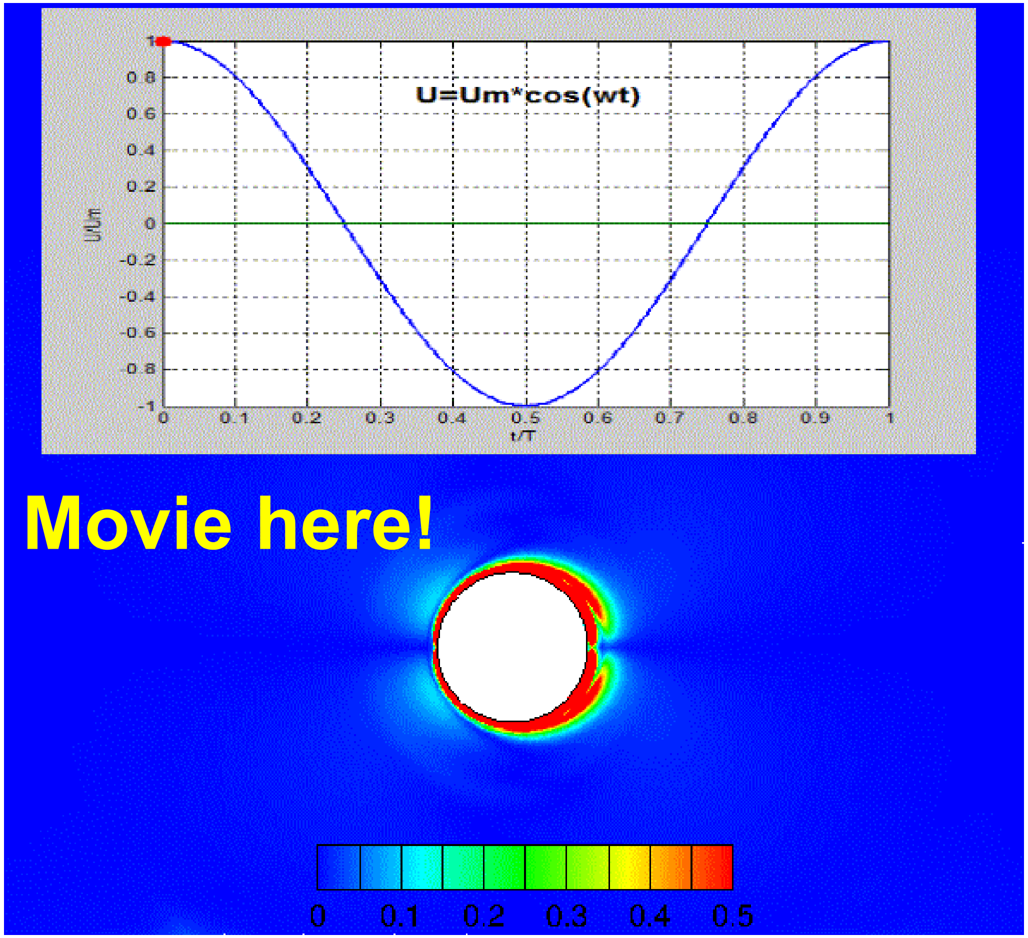
Cells: 76680

Faces: 154190

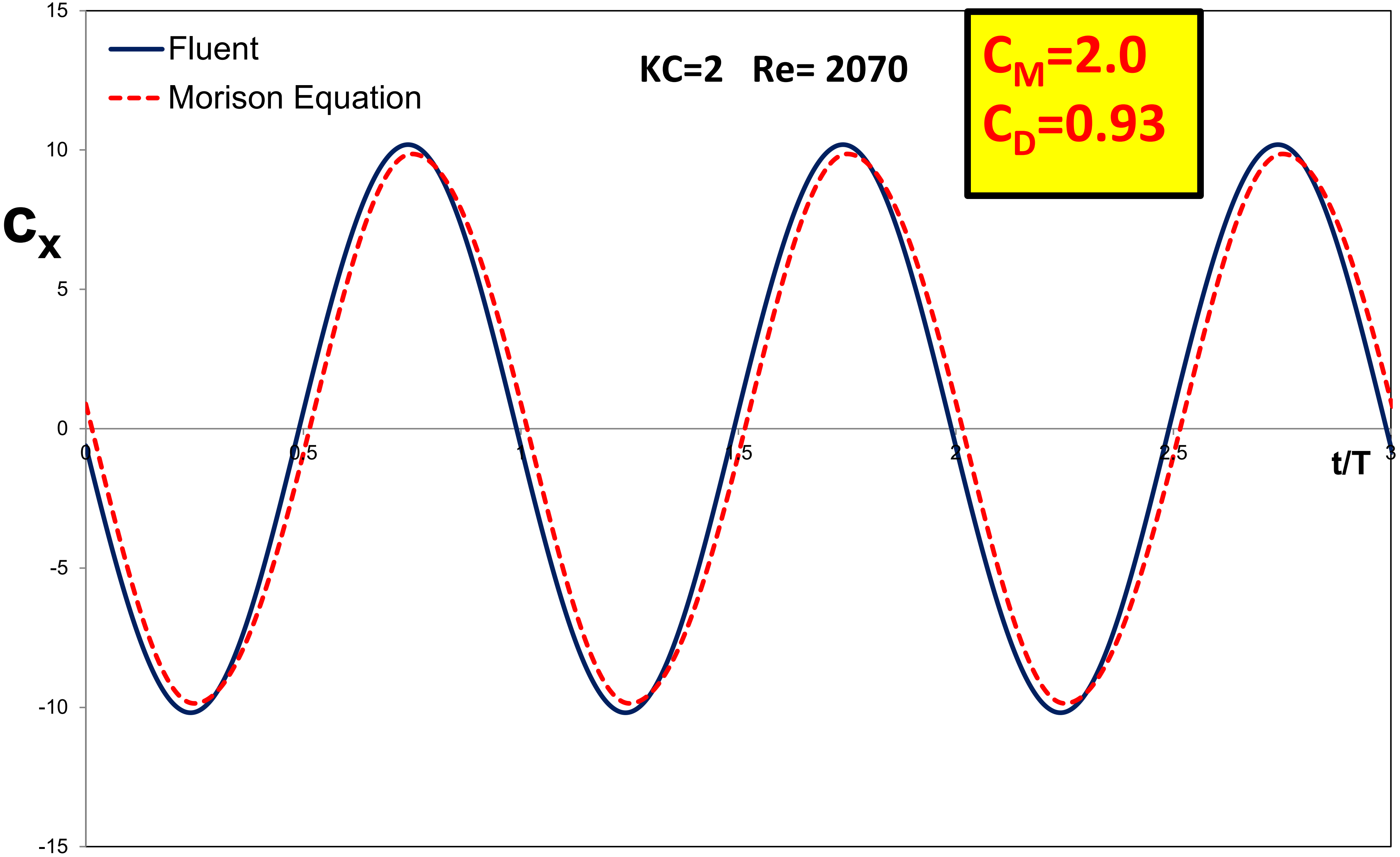
Nodes: 77510

Predicted flow (vorticity) by Fluent: KC=2, Re=1070

(click on the movie to play)



Case I: KC=2 Re=1070



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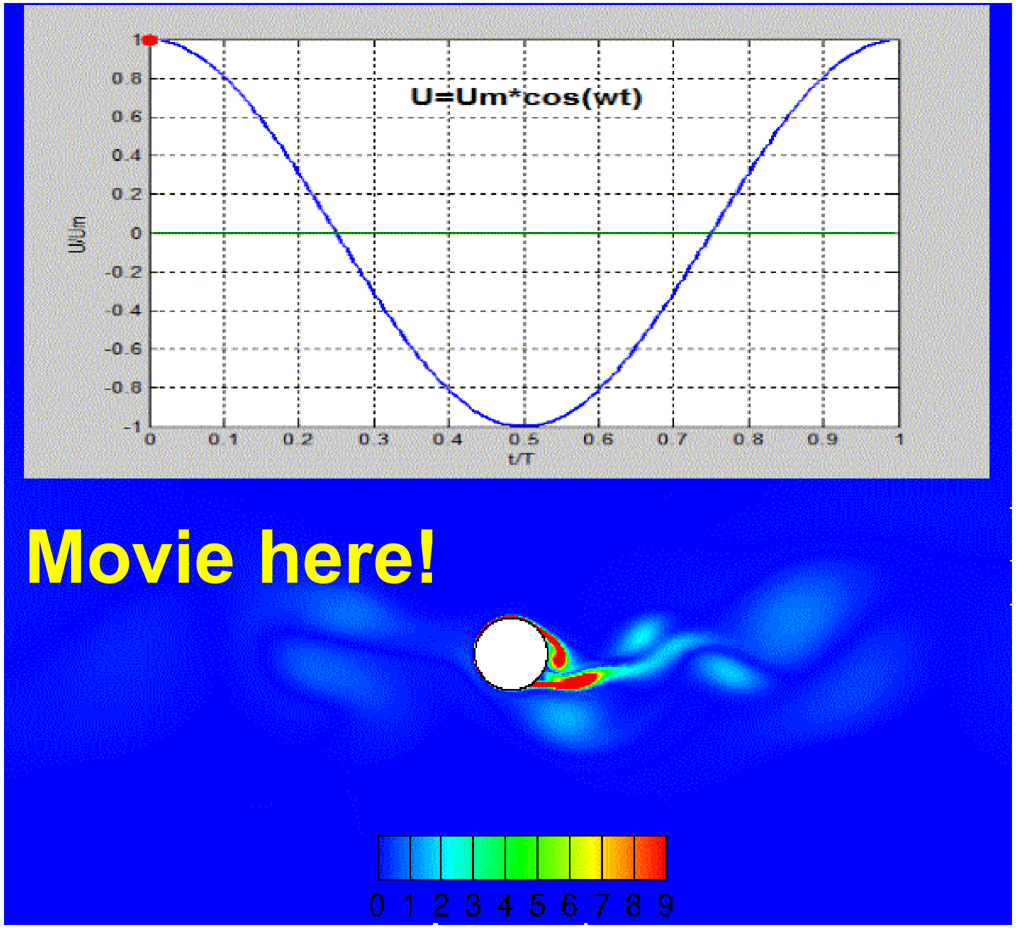
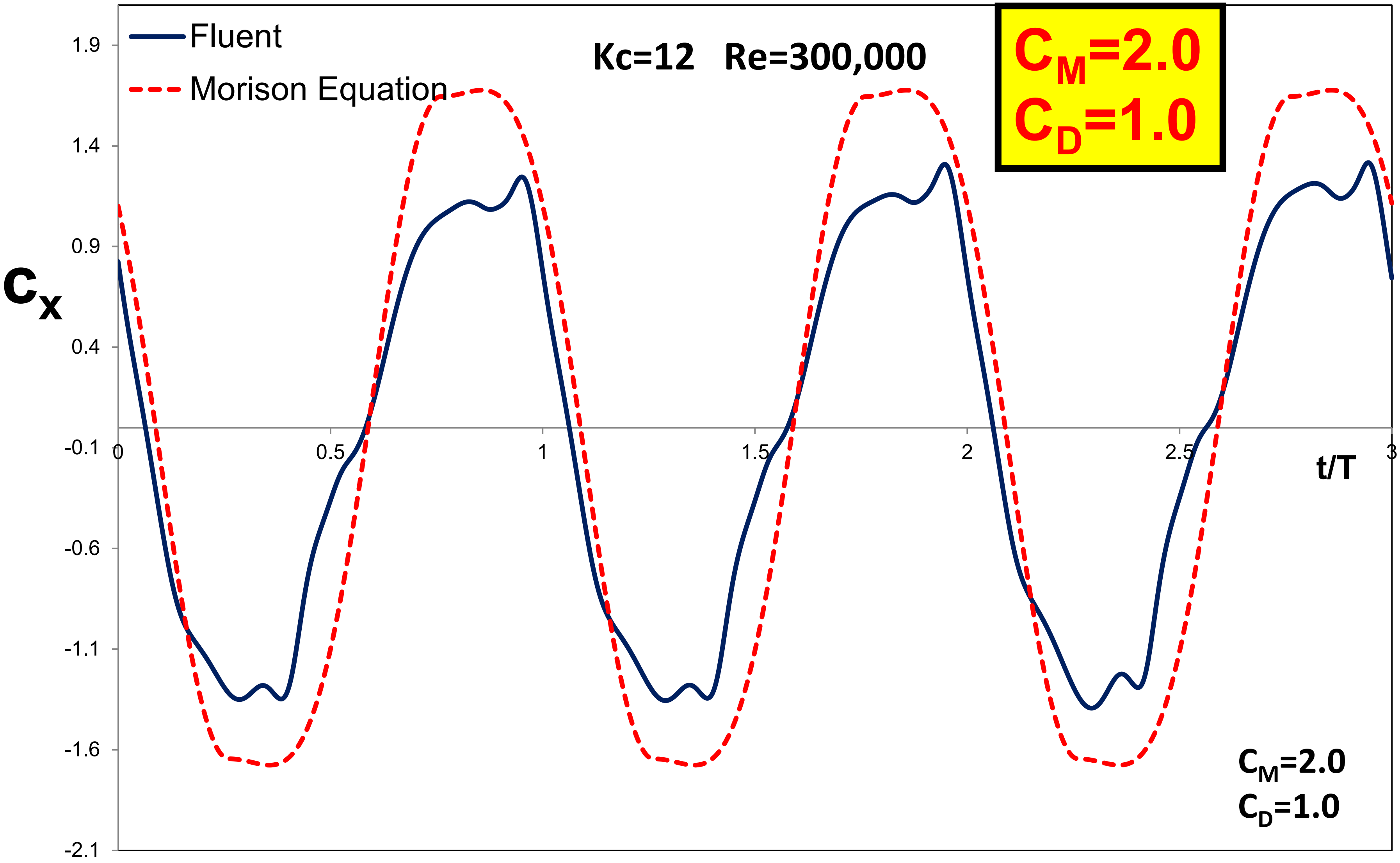
Predicted flow (vorticity) by Fluent: KC=12 Re=300,000

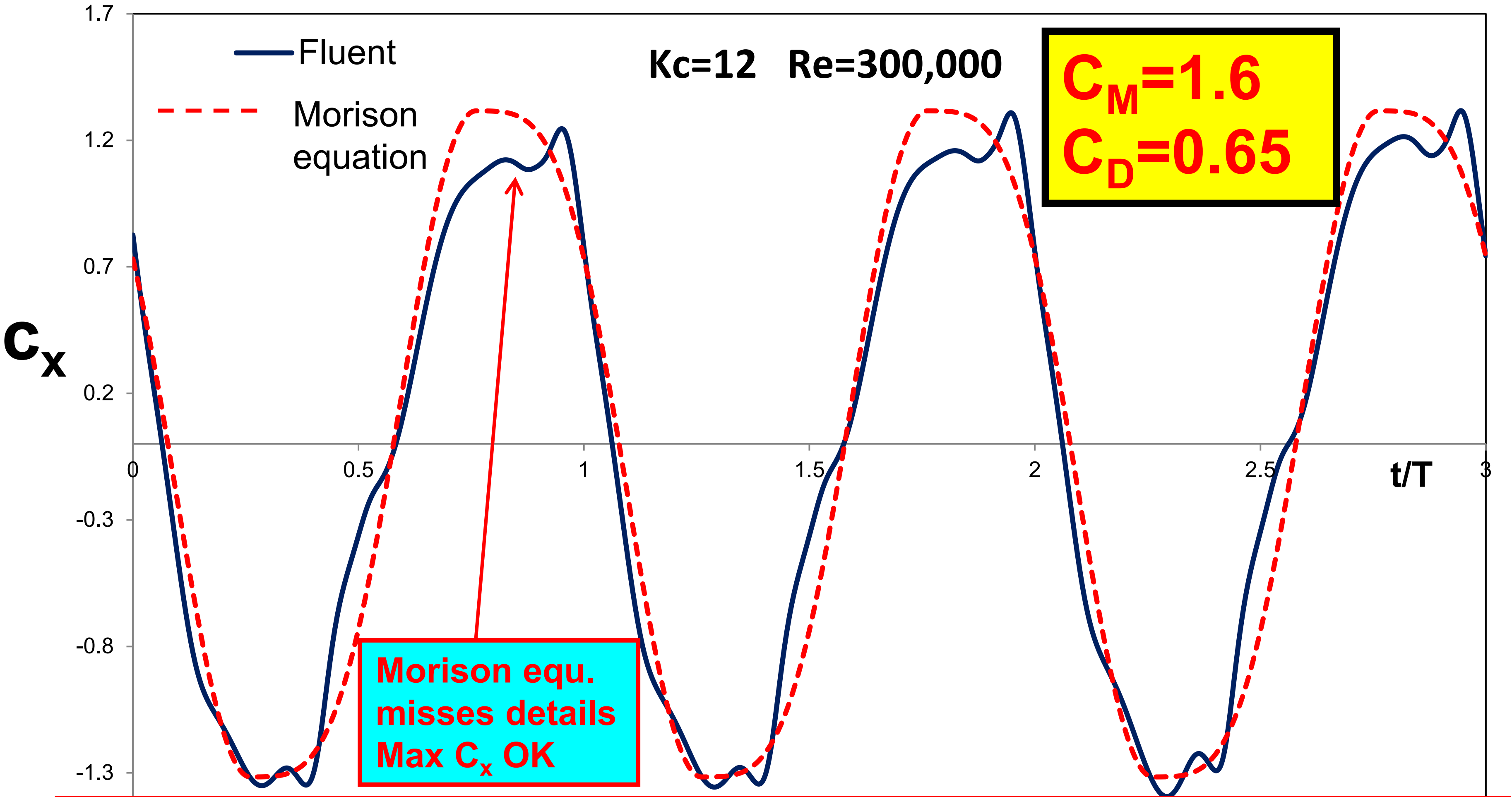
(click on the movie to play)



Case II: KC=12 Re=300,000



Case II: KC=12 Re=300,000



C_M=1.6
C_D=0.65

Morison equ. misses details Max C_x OK

Maybe due to luck…more needs to be done!!! Some newer simulations will be shown in the lecture on Computational Hydrodynamics