

Propagation of Uncertainty in Transportation-Land Use Models: An Investigation of  
DRAM-EMPAL and UTPP Predictions in Austin, Texas

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## ABSTRACT

This work examines the propagation of uncertainty in outputs of a standard integrated model of transportation and land use. Austin-calibrated DRAM-EMPAL predictions of residence and work locations are used as inputs to a UTPP-type four-step travel demand model (TDM), and the resulting travel times are fed forward into the future period's land use models. Covariance in inputs (including model parameters and demographic variables) was accommodated through multivariate Monte Carlo sampling of 200 scenarios. Variances in land use and travel predictions were then analyzed, over time, and as a function of input values. Results indicate that output variations were most sensitive to the exponent of the link performance function, the split of trips between peak and off-peak and several trip generation & attraction rates. 20 years in the future, final uncertainty levels (as measured by coefficients of variation) due solely to input and parameter estimation errors are on the order of 38% for total regional peak-period VMT, 45% for peak-period flows, and 50% and 37% for residential and employment densities, respectively. This means that central point estimates of key model outputs are very likely (more than 30%) to fall 38% to 50% below or above the mean value. In the Austin example, 15% of the 200 region's simulated peak-period VMT estimates fell below 3.7 million miles (per day) and 15%

exceeded 8.4 million miles. Such substantial variation is due solely to standard model parameter and input uncertainties. Other uncertainty about the future and human behavior also exists and will add further variation.

**Keywords**

Uncertainty propagation, integrated transportation-land use model, travel demand model

## INTRODUCTION

Uncertainty in transportation systems has recently received some attention (1,2, 3, 4). And over the past 30 years, many studies have highlighted uncertainty inherent in predictions of travel demand (e.g., 5, 6), but very few have attempted to quantify this uncertainty. The difficulty, as Mahmassani (5) explains, is that not all sources of uncertainty are suitable for empirical analysis.

Most Metropolitan Planning Organizations (MPOs) do not quantify uncertainty in their modeling process, even though they feel it is pervasive in their planning activities (7). Mehndiratta et al. (7) noted two main reasons for this reluctance: 1) MPOs feel that uncertainty analysis will add another layer of complexity to an already complex process. 2) The attitude among planners is that legislative directive dictates the planning process, and the current legislative mandate does not dictate uncertainty analysis. Yet for optimal decision-making, an appreciation of uncertainty is critical. (See, e.g., 8.) Rodier and Johnston (3) recently stressed the importance of acknowledging major areas of uncertainty in travel demand analysis, and they suggest such acknowledgement is necessary in order to maintain the credibility of transportation modeling.

Pradhan and Kockelman (1) argued that since many MPOs are now moving towards integrated land use-transportation models (largely in response to federal legislation), it is important to understand uncertainty propagation in these very complex models. They also suggested that reluctance of planners may be overcome by developing general results and a standard method of uncertainty analysis that can be applied to a variety of integrated models and settings.

This study furthers the work of Pradhan and Kockelman (1) by calibrating the entire suite of models based on a single region's data set; this approach provides all parameter covariance matrices, thus allowing more realistic simulation. We investigate uncertainty propagation in an integrated model known as ITLUP (9), and we highlight the potential for errors in both land use and transportation model outputs.

## BACKGROUND

Uncertainty in travel demand predictions derives from a host of sources, including model misspecification, imperfect input information, and innate randomness in events and behaviors. Mahmassani (5) categorized uncertainties affecting the evaluation of alternative transportation options into five classes. The first, and most difficult to represent, is uncertainty arising from major political upheavals or unexpected technological breakthroughs. The second class arises from political, economic, or social events and variables relatively independent of the transportation system being evaluated, but affecting the environment in which it operates. The third category derives from use of an imperfectly specified model as well as from measurement uncertainty in model inputs and parameters. This is also the category that is most suitable for empirical analysis. The fourth category is caused by ambiguity in evaluation criteria. (These include such attributes as 'aesthetics' and 'political desirability', which cannot be addressed using probabilistic logic.) The fifth category includes uncertainty regarding the basis of evaluation. This is especially critical because in many cases it ultimately determines the outcome of the planning and decision-making processes.

Pradhan and Kockelman's (1) investigations addressed only those sources of uncertainty that could be represented using probabilistic theory. The scope of their work, therefore, was limited

to an examination of randomness in the predictions of land use-transportation models arising from uncertainty in model inputs and parameters. Essentially all studies have similarly limited the scope of their investigations (3,4,10,11). Because of the complexities that arise should any of the other categories of uncertainty be considered, and the difficulties associated with the quantification of their impacts, the current study also focuses on uncertain predictions arising from variability in model inputs and model parameters.

### Computing Uncertainty

The method of moments and Monte Carlo simulation are two key methods for assessing the distribution of outputs, which are functions of random inputs. By relying on Taylor-series expansions of the output function, the method of moments approximates output moments using means, standard deviations, and other, higher-order moments of inputs.

Use of this method requires that outputs be specified as a clear, single function of inputs; this is extremely difficult, if not impossible, for almost all integrated model outputs (for reasons that will become clear in the model specification). Additionally, accuracy in approximation requires use of high-order derivatives, further complicating the analyses.

Monte Carlo techniques essentially draw input values from their multivariate distributions. These are used as model inputs, and their corresponding outputs are calculated. If inputs are drawn randomly, the resulting output values constitute a random sample from their respective probability distributions.

Due to the computational complexities inherent in integrated land use-transportation models (e.g., network equilibration), Monte Carlo methods were employed in this study. Such methods require substantial computer run time, and human input, but they produce more accurate and realistic results than method of moments. With 200 simulations of the full model, our outputs demonstrate most of their range; however, extreme events are rarely generated.

### Uncertainty Analysis

There are several relatively simple but valuable techniques for examining the effects of input uncertainty on outputs (12). The most useful here is multivariate sensitivity analysis, which helps identify the degree to which model outputs are affected by changes in inputs, while controlling for variations in other inputs. By eliminating dimensions or units, much like elasticities, standardized regression coefficients are very helpful here. After linearly regressing an output on inputs, a standardized regression coefficient  $SRC_i$  is calculated as follows:

$$SRC_i = \frac{\beta_i \times \sigma_i}{\sigma_y} \quad (1)$$

where  $SRC_i$  is the standardized regression coefficient,  $\beta_i$  is the regression coefficient,  $\sigma_i$  is the standard deviation of the independent variables, and  $\sigma_y$  is the standard deviation of the dependent variable. This measures the number of standard deviations in the output one may expect from a single standard deviation's increase in input  $i$ . These standardized coefficients are used to report results from this work's sensitivity analyses, in a later section.

## DATA DESCRIPTION

The current application was undertaken for the Austin, Texas, region, which is mid-size metropolitan area of 700,000 people, spread across three counties (Hays, Travis and Williamson). The region is divided into 1074 traffic serial zones (TSZ's), on the basis of aggregations of census block groups; and the network is comprised of 16,966 links. The great majority of data used for calibrating the models was obtained from the Capital Area Metropolitan Planning Organization (CAMPO). These include population and employment by type and by zone, as well as 24-hour and two-hour peak travel times for 1997 (which were used to calibrate various land use and travel demand sub-models).

The 1990 Census of Population data also were used, in order to estimate the mean and standard deviation of household incomes by zone, as a function of Census-provided median incomes. The results of these regressions permitted predictions of household distributions across four income categories for each zone, these were then used in the land-use and trip-generation models.

Austin's 1996 Travel Survey (the ATS) was used to calibrate the four-step travel demand model (TDM). This survey consisted of all members above 5 years of age across 1939 households reporting all trips made on a single weekday.

## MODEL DESCRIPTION

The current application of Putman's Integrated Transportation and Land-Use Package (ITLUP) consists of: a Disaggregate Residential Allocation Model (DRAM) and an Employment Allocation Model (EMPAL) (9). Both are modified forms of Lowry's model. DRAM uses the attractiveness of a zone and the accessibility of a zone's workers to jobs in other zones as the principal factors in allocating households to zones. EMPAL allocates employment based on the employment in the previous time period and the attractiveness of the zone for households. Upon close inspection, their specifications may appear somewhat counter to expectations. For example, households are assumed to locate where employers would be most attracted to them (rather than in locations where they are most attracted to employers). And jobs locate in positions where households would find them most accessible (rather than in positions where they find households to be most accessible). Both perspectives make sense. Our work strives to follow the traditional paradigm as closely as possible.

### Disaggregate Residential Allocation Model

The original DRAM and EMPAL equations (9) have undergone several changes over the years, primarily due to data differences. In the current application households were allocated to zones based on the following DRAM formula:

$$\hat{N}_{i,t} = \sum_j E_{j,t} r_t \left[ \frac{W_{i,t} e^{\beta_p c_{ji,t}^p + \beta_{op} c_{ji,t}^{op}}}{\sum_k W_{k,t} e^{\beta_p c_{ki,t}^p + \beta_{op} c_{ki,t}^{op}}} \right] \quad (2)$$

where  $\hat{N}_{i,t}$  is the estimated number of households in zone  $i$  at time  $t$ ,  $W_{i,t}$  is an attractiveness measure for zone  $i$  at time  $t$ ,  $c_{ji,t}^p$  is the peak travel time between zones  $j$  and  $i$  at time  $t$ ,  $c_{ji,t}^{op}$

and is the off-peak travel time between zones  $j$  and  $i$  at time  $t$ ,  $r_t$  is the region-wide ratio of households per employee at time  $t$ , and both  $\beta_{op}$  and  $\beta_p$  are empirically derived parameters. The attractiveness of a zone was calculated as follows:

$$W_{i,t} = (L_{i,t})^\theta \prod_{k=1}^4 \left( 1 + \frac{N_{i,t}^k}{\sum_{k=1}^4 N_{i,t}^k} \right)^{\gamma_k} \quad (3)$$

where  $L_{i,t}$  is the total land area of zone  $i$  at time  $t$ ,  $N_{i,t}^k$  is the number of residents in zone  $i$  who are in the  $k^{th}$  income quartile at time  $t$ , and  $\theta$  and  $\gamma_1 \dots \gamma_4$  are empirically derived parameters. There are 4 household categories (as defined by annual income) and, therefore, four versions of equations 2 and 3. Thus, there are 28 parameters used in the DRAM portion of the integrated model.

### Employment Allocation Model

In this application the EMPAL equation used for allocating employment to zones is as follows:

$$E_{j,t} = \left( \frac{e^\delta}{1 + e^\delta} \right) r_t^h \sum_i H_{i,t-1} \left[ \frac{W_{j,t-1} e^{\beta_p c_{ki,t} + \beta_{op} t_{ki,t}}}{\sum_k W_{k,t-1} e^{\beta_p c_{ki,t} + \beta_{op} t_{ki,t}}} \right] + \left( \frac{1}{1 + e^\delta} \right) r_t^e E_{j,t-1} \quad (4)$$

where  $E_{j,t}$  is the employment in the relevant sector (basic, retail, and service) in zone  $j$  at time  $t$ ,  $H_{i,t-1}$  is the number of households in zone  $i$  at time  $t$ ,  $W_{j,t-1}$  is the attractiveness function for zone  $j$  at time  $t-1$ ,  $r_t^h$  is the region-wide ratio of employment at time  $t$  to households at time  $t-1$ ,  $r_t^e$  is the region-wide ratio of employment at time  $t$  to employment at time  $t-1$ , and  $\delta$  is an empirically derived parameter.

The attractiveness of a zone was calculated as follows:

$$W_{j,t-1} = (E_{j,t-1})^{\delta_1} \times (L_j)^{\delta_2} \quad (5)$$

where  $L_j$  is the total land area of zone  $j$ , and  $\delta_1$  and  $\delta_2$  are empirically derived parameters. There are 3 sectors defined by employment type and, therefore, three versions of equations 4 and 5. Thus, there are 15 parameters used in the EMPAL portion of the integrated model.

The DRAM model parameters were calibrated using the 1997 household and employment allocations across Austin's zones. Households were categorized into four groups based on estimates of average zonal annual income and its standard deviation (both as a function of available median income data); these groups were defined as having an annual household income less than or equal to \$30,000, between \$30,000 and \$42,500, between \$42,500 and \$55,000, and

greater than \$55,000.

EMPAL's parameters were calibrated using the 1997 household data, as well as 1997 and CAMPO-predicted 2007 employment data. The implicit assumption of this strategy is that job locations respond first (to prior employment and household allocations), and households respond second (to current employment allocations). Employment was categorized into three types: basic (Standard Industrial Classification (SIC) code 1-5199), retail (SIC code 5200-5999) and service (SIC code 6000-9799). Maximum likelihood estimation of all the parameters was performed using Gauss software (version 3.2.34 (13)). The parameter estimates and their associated t-statistics are shown in Table 1. All the parameter values are consistent with expectations. The positive peak travel time parameters suggest that households and jobs are attracted to regions, which are congested during the peak period but this effect is compensated to a slight extent off-peak effects.

The standard ITLUP formulation has several limitations. A major one is that it does not account for land use intensity constraints. ITLUP models assign jobs and households to zones even if they do not have the capacity to accommodate more jobs or households. The current application deals with this limitation by reallocating excess allocation to zones which have the area to absorb more jobs and households. Maximum allowable residential and commercial densities used in this work are 25 households per residential acre and 100 jobs per commercial acre. These two maximum densities were calculated based on an envelope analysis of CAMPO's 1995 land use data set. Unfortunately, the land use data were only available for 549 out of the 1074 TSZ's, so a multinomial logit model for fraction of land by category (residential, commercial, and vacant), as a function of zonal area and network distance to the CBD, was developed to estimate the land use distribution in the remaining zones in the year 1995. The household and job allocations obtained from EMPAL and DRAM are first used to fill up the commercial and residential area in a zone. If, the numbers require more area, any vacant area is allocated to jobs and households in proportion of their demand for land. If a zone cannot accommodate more development, that zone is removed from consideration and the remaining jobs and/or households are allocated to the other zones by re-running the DRAM and EMPAL models.

Another limitation of ITLUP is that DRAM and EMPAL models are applied sequentially, neglecting simultaneous interactions between jobs and households. In addition, ITLUP does not consider land prices and commodity flows in allocating jobs and households; so it lacks important relationships and variables of great interest to planners, policymakers, and the public. However, its relative simplicity permits a fairly transparent analysis of uncertainty. In contrast, for example, Waddell et al.'s UrbanSim (14) requires on the order of 1,000 parameters and tens of thousands of input values that extremely few regions possess (such as development history and land prices by hectare). Analysis of uncertainty under that model was performed by Pradhan and Kockelman (1), but only with limited input combinations and reliance on given parameter sets (without covariance matrices).

### **Travel Demand Model**

The Urban Transportation Planning Package's (UTPP) traditional four-step travel demand model (TDM) was adopted to link the land use allocations of jobs and households to the Austin transportation network. The trip purposes considered in this application are Home-Based Work (HBW), Home-Based Non Work (HBNW) and Non Home-Based (NHB) trips. Details of the

four basic model steps follow here.

### ***Trip Generation***

Linear regression models for trip production and attraction were developed based on the 1996 ATS data. Trip purpose and mode split proportions in the ATS are as follows: 20.1% of trips were HBW, 49.3% were HBNW, and 30.6% were NHB; 88.6% were by automobile, 6.1% by transit, 3.8% by walking, and 1% by bike. Trip production models for HBW and HBNW trips were developed at the household level, whereas the trip production model for NHB trips was developed at the zonal level.

The model specifications are given below:

$$P_{HBW} = f(Inc_1, Inc_2, Inc_3, Inc_4)$$

$$P_{HBNW} = f(Inc_1, Inc_2, Inc_3, Inc_4)$$

$$P_{NHB} = f(Basic, Retail, Service)$$

where  $P_{HBW}$  and  $P_{HBNW}$  are the number of person trips (per day) produced by a household, and  $P_{NHB}$  are the numbers of NHB trips produced/generated by a zone.  $Inc_i$  is the indicator variable for the  $i^{th}$  income category, and *Basic*, *Retail* and *Service* are the numbers of these jobs in a zone.

Trip-attraction models also were developed at the zonal level, and their general functional specifications are the same as for the production of NHB trips (i.e., as a function of numbers by basic, retail, and service jobs per attractive zone). The parameter estimates for both production and attraction models are shown in Table 2.

### ***Trip Distribution***

Multinomial logit models of destination choice were calibrated for each trip purpose and for peak and off-peak times (15). The natural log of the total number of attracted trips (as estimated using Table 2's values) and travel times were used as explanatory variables. The coefficient on the total number of trips attracted term was constrained to equal one, so that the model form is the same as a gravity model; and size is accommodated proportionally, which is consistent with probabilistic theory. The model specification is shown below:

$$p_{ij} = \frac{A_j e^{\beta t_{ij}}}{\sum_k A_k e^{\beta t_{ik}}} \quad (6)$$

where  $p_{ij}$  is the percentage of trips produced in zone  $i$  that are attracted to zone  $j$ ,  $A_j$  is +the number of person-trips attracted to zone  $j$ , and  $t_{ij}$  is the travel time from zone  $i$  to zone  $j$  (for both peak and off-peak periods, depending on the time of day). Calibration results are given in Table 3.

### ***Time of Day Characteristics***

Peak and off-peak times of day were used in this study. According to CAMPO, the peak period lasts two hours in the morning (from 7:15 a.m. to 9:15 a.m.). The trip production-attraction (PA)



matrices were converted to trip origin-destination (OD) matrices by time of day (peak and off-peak) using departure and arrival rates during the peak period, as shown in equation 7.

$$OD^{Peak} = PA \times \Psi + PA^T \times \Theta \quad (7)$$

where  $\Psi$  is the departure rate for trips in the peak period,  $\Theta$  is the return rate for trips in the peak period. This equation was used to calculate the OD matrices for HBNW, HBW, and NHB trips. These rates were calculated from the 1996 ATS data.

### ***Mode Split***

Binary logit models of mode choice were calibrated to assign person-trips to automobile and non-automobile modes. Six different models were estimated: one for each trip purpose and time of day. But each model form's probability for auto choice is structured the same, as follows:

$$P_{auto} = \frac{e^{\alpha_{auto} + \beta_m t_{ij}^{auto}}}{e^{\alpha_{auto} + \beta_m t_{ij}^{auto}} + e^{\beta_m t_{ij}^{non-auto}}} \quad (8)$$

The parameters and goodness-of-fit measures for these 6 mode-choice (MC) models are given in Table 3.

All parameter estimates are consistent with expectations. But the low goodness-of-fit measures suggest that the preference for automobile use is independent of the CAMPO-provided travel times (which may be measured with significant error). In other words, simply a mode-specific constant will do almost as well in explaining mode choice as the available travel time data. As a result of this, the  $\beta_m$  was not found to be important in sensitivity analysis, as described later in this paper.

### ***Vehicle Occupancy***

Vehicle occupancy for each trip purpose was calculated from the 1996 ATS data. Average vehicle occupancy levels of 1.20, 1.99, and 1.85 were for HBW, HBNW, and NHB trips, respectively. (Unlike all other parameters, these parameters were not varied.)

### ***Traffic Assignment***

All automobile trips were assigned to the Austin network using the Stochastic User Equilibrium (SUE) (16) assignment method available in TransCAD (17). The following settings were chosen for the assignment: a probit route-choice model with 5% error (this represents the percentage error for the error term used in stochastic user equilibrium assignment), a convergence criterion of 0.01 (convergence criterion of 1% refers to the maximum percentage flow difference between successive iterations), and a maximum of 30 iterations. Only 30 iterations were performed due to the time required. An average of 18 minutes was needed for each traffic assignment. Two assignments (peak and off-peak) were done for each run, and four runs were performed for each of the 200 simulations (i.e., for the future years 2002, 2007, 2012, and 2017). Thus, on average 2.4 hours were required to perform essentially just the traffic-assignment step for *each* of the 200 simulations. Travel times were not fed back to trip distribution stage; this would have required additional time.

The link performance function is of the following type (based on the original Bureau of Public

Roads (18) formula), but using different parameter values for  $\alpha$  and  $\beta_{link}$ ):

$$t = t_f \left[ 1 + \alpha \left( \frac{v}{c} \right)^{\beta_{link}} \right] \quad (9)$$

where  $t_f$  is the free-flow travel time, and  $\alpha$  and  $\beta_{link}$  are link performance parameters, provided by CAMPO. There are 8 sets of these parameters, depending on road type. No information on these parameters' uncertainty was available, so they were assumed to have standard deviations that are 30% of their given values and assumed to follow independent normal distributions.

### **Background Flows**

Austin's 43 external zones also attract and produce trips, which congest the network. Counts of trips with at least one external zone (as the origin, destination, or both) were obtained from CAMPO's 2007 OD matrix predictions, and these were loaded as "background flows" onto the network. For future model applications (i.e., 2012 and beyond), these year-2007 trip counts were assumed to grow at the (randomly drawn) population growth rate.

## **SIMULATIONS**

200 full Monte Carlo simulations of input sets (including both model parameters and starting distributions of jobs and households, by type and zone) were performed. The demographic variables varied were population and employment growth rates. The means and the standard deviations of these rates were taken to be 3.3% +/- 0.5% (population growth rate), and 3.1% +/- 0.5% (employment growth rate) (19). The mean growth rates were obtained from CAMPO's newsletter, and standard deviation values were assumed.

All model parameters were varied assuming multivariate normal distributions, based on their estimated variance-covariance matrices (an output of the software codes used to calibrate the various ITLUP and TDM submodels). The correlation matrices of parameters for DRAM and EMPAL models are shown in Tables 4 and 5. The link-performance parameters, and the peak/off-peak splits were also varied, assuming independent normal distributions and coefficients of variation (mean divided by standard deviation) of 0.3.

In all 95 parameters and 2 demographic variables were varied, and 200 simulations were performed. To observe the evolution of uncertainty over time, the land use and travel demand model were run every 5 years, and four such runs were performed for each simulation, for a total of 20 years of forecasts of population, employment, and travel, across the region.

## **RESULTS**

Sensitivity analysis was performed by regressing various key outputs of the integrated model on input parameters. Standardized coefficients and p-values were used to gauge the practical and statistical impact of variables on the model outputs. (Note: A p-value provides a sense of the significance level for the two-tailed null hypothesis test that an explanatory variable has no effect on dependent variable (i.e. the regression coefficient is zero). Variables with p-value less than or

equal to 0.05 are considered very significant.)

The outputs analyzed explicitly were weighted residential density, weighted commercial density, VMT and VHT for both peak and off-peak periods and link flows during peak and off-peak periods. The final model specifications were obtained by removing the variables, which were not significant in any model. Different combinations of the remaining variables were considered to arrive at the specifications shown in Tables 6, 7, and 8.

The results for off-peak and peak VMT and VHT are shown in Table 6. The results indicate that the employment and population growth rates do not impact VMT and VHT in a significant way. The exponent ( $\beta_{link}$ ) on the volume-to-capacity term in the link performance function is the most significant variable for predicting both peak and off-peak VHT. This makes sense, since VHT is very dependent on link travel times. The coefficients in the trip production and trip attraction models are also highly significant.

Peak-period VHT and VMT are estimated to be highly sensitive to peak-off peak splits, which is consistent with our expectation. Peak and off-peak VMTs also are sensitive to  $\alpha_i$ , the coefficient on the volume-to-capacity term in the link performance function.

The three links chosen for analysis are: (1) IH35 Northbound, near Cameron Road; (2) Loop 1 Southbound, south of 5<sup>th</sup> and 6<sup>th</sup> Streets; and (3) IH35 Southbound, south of US290 and SH 71. These are critical links for the network, and each carried close to capacity flow in each simulation. Averages of flows on these three links were used to perform sensitivity analysis. The link flow results shown in Table 8 indicate that  $\beta_{link}$  is the most significant parameter in the initial years for off-peak period link flows. In the long term, however population growth rate has a significant impact on the link flows. As with peak period VHT and VMT, peak period link flows are also significantly dependent on peak-off peak splits.

Sensitivity analyses also were performed on land use results, in the form of population- and jobs-weighted averages of residential and commercial densities. The regression results are shown in Table 7, and results indicate that commercial density is significantly influenced by various DRAM and EMPAL model parameters, as well as trip production and attraction rates. Residential density is significantly impacted by EMPAL parameters, trip production and attraction rates, and mode choice model parameters.

The “evolution” of uncertainty (as measured by coefficient of variation in several outputs) over time is shown in Figure 1. Since VHT is a final model output and is highly sensitive to congestion (e.g.,  $v/c$  ratios near or above 1.0), it is not surprising that it was found to be the most variable of outputs studied. Uncertainty can compound itself across all the intermediate sub-models, though there certainly are opportunities for variability reductions (such as in aggregation of individual choices and assignments). Peak-period outputs also were found to exhibit greater variation than off-peak outputs, which is probably due to the exponential effects of congestion on travel times. These findings are consistent with Pradhan and Kockelman’s (20,21) investigations of ITLUP for Eugene-Springfield.

## CONCLUSIONS

This work investigated the dependence of future location and travel choice predictions on integrated-model inputs and parameters. Results indicate that output variations were most sensitive to the exponent of the link performance function, the split of trips between peak and

off-peak periods, and several trip generation and attraction rates. 20 years in the future, final uncertainty levels (as measured by coefficients of variation) due solely to input and parameter estimation errors were found to be on the order of 38% for total regional peak-period VMT, 45% for peak period flows, and 50% and 37% for residential and employment densities, respectively. This means that central point estimates of key model outputs are very likely (more than 30%) to fall 38% to 50% below or above the mean value. In the Austin example, 15% of the 200 region's simulated peak-period VMT estimates fell below 3.7 million miles (per day) and 15% exceeded 8.4 million miles. Such substantial variation is due solely to standard model parameter and input uncertainties. Other uncertainty about the future and human behavior also exists and will add further variation.

This work builds on Zhao and Kockelman's investigations of four-step travel demand models (4) and Pradhan and Kockelman's investigations of integrated models by adding realism to parameter distributions (through controlled calibrations) and model specification (through detailed submodel assembly and an integration of land use and travel behaviors) (1,21). In contrast to Zhao and Kockelman's work, it examples the evolution of prediction uncertainties over time and across model stages, and travel conditions are permitted to impact location choices. Due to these distinctions, simulation results indicate that the link performance parameter  $\beta_{link}$  is a key source of uncertainty in outputs; this is probably due to significant travel-time feedbacks to location decisions, which are fundamental to travel patterns. Population and employment growth rates only seem to have an effect in the long run. However, it should be emphasized that these results may be specific to ITLUP; in UrbanSim Pradhan and Kockelman observed that demographic inputs were principal sources of uncertainty in the short and long terms (1).

This study is of value for MPO's, which are using or hoping to use Integrated Transportation–Land Use models in their forecasting and planning activities. MPOs can perform multiple runs of their models under different scenarios and then gauge the impact of inputs on the outputs. This provides a sense of which variables are greatly mis-predicted and which inputs or model parameters should be carefully observed or estimated, to minimize the uncertainty in model predictions. If MPOs do not have the capability to run multiple simulations under varying scenarios, they can make use of the key parameters identified in this study and the related estimates of output variation to forecast output variance and improve the model design and overall planning processes.

The time-consuming nature of the simulations and a lack of data affected the investigations. For example, the lack of employment data for two past time periods required we use 2007's predicted employment data for calibrating the EMPAL models, which could have some bearing on the parameter estimates and sensitivity analysis. Also, correlation information for certain variables (like peak/off-peak split and link-performance parameters) were not available, so these were assumed. External trips were not modeled explicitly; instead we relied on CAMPO's estimates to load the network with growing background flows. And vehicle occupancy rates were not randomly varied.

Further work would be helpful for more fully understanding the growth in prediction uncertainties over time and across different model frameworks. Instead of random simulations, experiments could be performed by varying only one variable at a time (e.g., the population

growth rate), and gauging its marginal impact on outputs. This can be time-consuming, but it certainly can assist in drawing crisper conclusions about the impacts of individual parameters and inputs. Also, such work should be done with other land use transportation models, to draw general conclusions on the impact of certain variables and parameters on uncertainty in other models' outputs.

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**Table 1. Parameter Estimates: DRAM and EMPAL Models**

|                  | <b>Group 1</b>     | <b>Group 2</b>     | <b>Group 3</b>    | <b>Group 4</b>    | <b>Basic</b>       | <b>Retail</b>      | <b>Service</b>     |
|------------------|--------------------|--------------------|-------------------|-------------------|--------------------|--------------------|--------------------|
| $\beta_p$        | 0.0490<br>(20.3)   | 0.0264<br>(7.55)   | 0.018<br>(2.72)   | 0.0170<br>(8.92)  | 0.0937<br>(18.2)   | 0.0980<br>(19.4)   | 0.0322<br>(14.8)   |
| $\beta_{op}$     | -0.3782<br>(-94.6) | -0.2859<br>(-56.5) | -0.236<br>(-28.9) | -0.1842<br>(-102) | -0.2781<br>(-29.4) | -0.3078<br>(-38.1) | -0.1308<br>(-35.9) |
| $\theta$         | 0.6548<br>(157)    | 0.6340<br>(104)    | 0.5985<br>(96.6)  | 0.5491<br>(112)   |                    |                    |                    |
| $\gamma_1$       | 71.49<br>(47.2)    | 56.33<br>(30.9)    | 44.81,<br>(13.70) | 12.60<br>(0.999)  |                    |                    |                    |
| $\gamma_2$       | 58.16<br>(54.7)    | 49.19<br>(34.8)    | 35.96<br>(14.02)  | 14.13<br>(1.55)   |                    |                    |                    |
| $\gamma_3$       | 13.21<br>(34.5)    | 17.16<br>(35.1)    | 21.02<br>(34.1)   | 14.67<br>(12.9)   |                    |                    |                    |
| $\gamma_4$       | 67.19<br>(45.9)    | 54.35<br>(30.6)    | 44.22<br>(13.8)   | 18.16<br>(1.48)   |                    |                    |                    |
| $\delta$         |                    |                    |                   |                   | -1.6350<br>(-82.5) | -1.5762<br>(-64.5) | 0.633<br>(-33.44)  |
| $\delta_1$       |                    |                    |                   |                   | 0.2743<br>(36.7)   | 0.2018<br>(19.9)   | 0.4056<br>(74.9)   |
| $\delta_2$       |                    |                    |                   |                   | 0.8725<br>(49.7)   | 1.2590<br>(56.0)   | 0.5969<br>(56.8)   |
| Log Lik          | 731.6              | 272.3              | 238.4             | 801.2             | 1618               | 617.6              | 1698               |
| LRI*             | 0.556              | 0.531              | 0.511             | 0.513             | 0.941              | 0.856              | 0.907              |
| N <sub>obs</sub> | 1074               | 1074               | 1074              | 1074              | 1074               | 1074               | 1074               |

\*LRI = Likelihood Ratio Index

**Table 2. Parameter Estimates: Trip Production and Attraction Models**

| <b>Production Models</b> |               |              |               |
|--------------------------|---------------|--------------|---------------|
|                          | <b>HBW</b>    | <b>HBNW</b>  | <b>NHB</b>    |
| <i>Inc<sub>1</sub></i>   | 1.325 (21.32) | 4.049 (25.3) |               |
| <i>Inc<sub>2</sub></i>   | 2.092 (23.6)  | 4.423 (19.4) |               |
| <i>Inc<sub>3</sub></i>   | 2.115 (26.8)  | 4.881 (24.0) |               |
| <i>Inc<sub>4</sub></i>   | 2.443 (30.4)  | 5.547(26.8)  |               |
| <i>Basic</i>             |               |              | 0.4621 (4.12) |
| <i>Retail</i>            |               |              | 5.186 (12.5)  |
| <i>Service</i>           |               |              | 1.481 (10.0)  |
| Adj. R <sup>2</sup>      | 0.559         | 0.525        | 0.538         |
| N <sub>obs</sub>         | 1939          | 1939         | 586           |
| <b>Attraction Models</b> |               |              |               |
| <i>Basic</i>             | 0.4892 (8.79) |              | 0.4093 (3.33) |
| <i>Retail</i>            | 1.668 (7.26)  | 5.419 (8.41) | 5.659 (12.4)  |
| <i>Service</i>           | 1.020 (12.5)  | 1.723 (7.69) | 1.287 (7.94)  |
| Adj. R <sup>2</sup>      | 0.526         | 0.303        | 0.491         |
| N <sub>obs</sub>         | 613           | 665          | 568           |



**Table 3. Parameter Estimates: Destination and Mode Choice Models**

| Destination Choice Models |                    |                    |                    |                    |                    |                    |
|---------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
|                           | HBW                |                    | HBNW               |                    | NHB                |                    |
|                           | Off Peak           | Peak               | Off Peak           | Peak               | Off Peak           | Peak               |
| $\beta_I$                 | -0.0804<br>(-22.3) | -0.0637<br>(-22.4) | -0.1449<br>(-42.9) | -0.1201<br>(-40.0) | -0.1401<br>(-44.6) | -0.1299<br>(-28.4) |
| Adj. R <sup>2</sup>       | 0.374              | 0.349              | 0.500              | 0.515              | 0.446              | 0.475              |
| N <sub>obs</sub>          | 971                | 1037               | 2571               | 2001               | 3307               | 1142               |
| Mode Choice Models        |                    |                    |                    |                    |                    |                    |
| $\alpha_{avro}$           | 1.944<br>(6.15)    | 1.935<br>(6.61)    | 1.421<br>(11.4)    | 1.442<br>(8.75)    | 2.011<br>(9.36)    | 2.046<br>(5.25)    |
| $\beta_m$                 | -0.0225<br>(-2.58) | -0.0306<br>(-2.99) | -0.0091<br>(-2.62) | -0.0004<br>(-0.08) | -0.0216<br>(-3.12) | -0.0264<br>(-1.88) |
| Adj. R <sup>2</sup>       | 0.0154             | 0.0182             | 0.0022             | 0.0015             | 0.0084             | 0.0086             |
| N <sub>obs</sub>          | 1085               | 1145               | 3154               | 1374               | 2507               | 792                |

**Table 4. EMPAL Parameter Estimates: Correlation Matrix**

| <b>Basic</b>   | $\delta_2$ | $\delta_1$ | $\delta$ | $\beta_{op}$ | $\beta_p$ |
|----------------|------------|------------|----------|--------------|-----------|
| $\delta_2$     | 1.000      | -0.714     | -0.751   | -0.682       | 0.507     |
| $\delta_1$     | -0.714     | 1.000      | 0.65     | 0.802        | -0.487    |
| $\delta$       | -0.751     | 0.65       | 1.000    | 0.566        | -0.898    |
| $\beta_{op}$   | -0.682     | 0.802      | 0.566    | 1.000        | -0.475    |
| $\beta_p$      | 0.507      | -0.487     | -0.898   | -0.475       | 1.000     |
| <b>Retail</b>  | $\delta_2$ | $\delta_1$ | $\delta$ | $\beta_{op}$ | $\beta_p$ |
| $\delta_2$     | 1.000      | -0.62      | -0.585   | -0.701       | 0.216     |
| $\delta_1$     | -0.627     | 1.000      | 0.611    | 0.771        | -0.366    |
| $\delta$       | -0.585     | 0.61       | 1.000    | 0.547        | -0.858    |
| $\beta_{op}$   | -0.701     | 0.77       | 0.547    | 1.000        | -0.367    |
| $\beta_p$      | 0.216      | -0.36      | -0.858   | -0.367       | 1.000     |
| <b>Service</b> | $\delta_2$ | $\delta_1$ | $\delta$ | $\beta_{op}$ | $\beta_p$ |
| $\delta_2$     | 1.000      | -0.723     | -0.642   | -0.772       | -0.151    |
| $\delta_1$     | -0.723     | 1.000      | 0.563    | 0.841        | 0.137     |
| $\delta$       | -0.642     | 0.563      | 1.000    | 0.443        | -0.551    |
| $\beta_{op}$   | -0.772     | 0.841      | 0.443    | 1.000        | 0.197     |
| $\beta_p$      | -0.151     | 0.137      | -0.551   | 0.197        | 1.000     |

**Table 5. DRAM Parameter Estimates: Correlation Matrix**

|                |          |            |            |            |            |              |           |
|----------------|----------|------------|------------|------------|------------|--------------|-----------|
| <b>Group 1</b> | $\theta$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\beta_{op}$ | $\beta_p$ |
| $\theta$       | 1.000    | 0.026      | 0.037      | -0.026     | 0.024      | -0.358       | 0.066     |
| $\gamma_1$     | 0.026    | 1.000      | 0.98       | 0.837      | 0.999      | -0.327       | 0.366     |
| $\gamma_2$     | 0.037    | 0.98       | 1.000      | 0.723      | 0.986      | -0.326       | 0.359     |
| $\gamma_3$     | -0.026   | 0.837      | 0.723      | 1.000      | 0.819      | -0.275       | 0.329     |
| $\gamma_4$     | 0.024    | 0.999      | 0.986      | 0.819      | 1.000      | -0.326       | 0.365     |
| $\beta_{op}$   | -0.358   | -0.327     | -0.326     | -0.275     | -0.326     | 1.000        | -0.843    |
| $\beta_p$      | 0.066    | 0.366      | 0.359      | 0.329      | 0.365      | -0.843       | 1.000     |
| <b>Group 2</b> | $\theta$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\beta_{op}$ | $\beta_p$ |
| $\theta$       | 1.000    | -0.014     | -0.001     | -0.059     | -0.016     | -0.305       | -0.017    |
| $\gamma_1$     | -0.014   | 1.000      | 0.941      | 0.636      | 0.998      | -0.063       | 0.071     |
| $\gamma_2$     | -0.001   | 0.941      | 1.000      | 0.361      | 0.959      | -0.066       | 0.071     |
| $\gamma_3$     | -0.059   | 0.636      | 0.361      | 1.000      | 0.592      | -0.049       | 0.067     |
| $\gamma_4$     | -0.016   | 0.998      | 0.959      | 0.592      | 1.000      | -0.063       | 0.072     |
| $\beta_{op}$   | -0.305   | -0.063     | -0.066     | -0.049     | -0.063     | 1.000        | -0.852    |
| $\beta_p$      | -0.017   | 0.071      | 0.071      | 0.067      | 0.072      | -0.852       | 1.000     |
| <b>Group 3</b> | $\theta$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\beta_{op}$ | $\beta_p$ |
| $\theta$       | 1.000    | 0.009      | 0.016      | -0.041     | 0.008      | -0.159       | -0.029    |
| $\gamma_1$     | 0.009    | 1.000      | 0.974      | 0.741      | 0.999      | -0.665       | 0.684     |
| $\gamma_2$     | 0.016    | 0.974      | 1.000      | 0.585      | 0.982      | -0.648       | 0.666     |
| $\gamma_3$     | -0.041   | 0.741      | 0.585      | 1.000      | 0.717      | -0.527       | 0.547     |
| $\gamma_4$     | 0.008    | 0.999      | 0.982      | 0.717      | 1.000      | -0.664       | 0.683     |
| $\beta_{op}$   | -0.159   | -0.665     | -0.648     | -0.527     | -0.664     | 1.000        | -0.955    |
| $\beta_p$      | -0.029   | 0.684      | 0.666      | 0.547      | 0.683      | -0.955       | 1.000     |
| <b>Group 4</b> | $\theta$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\beta_{op}$ | $\beta_p$ |
| $\theta$       | 1.000    | 0.692      | 0.691      | 0.664      | 0.692      | -0.002       | -0.536    |
| $\gamma_1$     | 0.692    | 1.000      | 0.998      | 0.966      | 0.999      | 0.279        | -0.708    |
| $\gamma_2$     | 0.691    | 0.998      | 1.000      | 0.95       | 0.999      | 0.282        | -0.709    |
| $\gamma_3$     | 0.664    | 0.966      | 0.95       | 1.000      | 0.964      | 0.249        | -0.671    |
| $\gamma_4$     | 0.692    | 0.999      | 0.999      | 0.964      | 1.000      | 0.28         | -0.708    |
| $\beta_{op}$   | -0.002   | 0.279      | 0.282      | 0.249      | 0.28       | 1.000        | -0.755    |
| $\beta_p$      | -0.536   | -0.708     | -0.709     | -0.671     | -0.708     | -0.755       | 1.000     |

**Table 6. Sensitivity Analysis Results: VMT and VHT**

| Parameters                    | Description                         | Offpeak VHT |           | Peak VHT   |           |
|-------------------------------|-------------------------------------|-------------|-----------|------------|-----------|
|                               |                                     | 2017        |           | 2017       |           |
|                               |                                     | Std Coeff.  | p - value | Std Coeff. | p - value |
| <i>Emprate</i>                | Employment Rate                     | 0.033       | 0.610     | 0.032      | 0.633     |
| <i>Poprate</i>                | Population Rate                     | 0.098       | 0.136     | -0.049     | 0.450     |
| $\beta_{link}$                | Exponent Link Performance Function  | 0.289       | 0.000     | 0.283      | 0.000     |
| $\alpha_6$                    | Parameter Link Performance Function | 0.323       | 0.000     |            |           |
| $\Psi_{HBNW}$                 | Peak Off-peak Split                 |             |           | 0.304      | 0.000     |
| $\Psi_{HBW}$                  | Peak Off-peak Split                 |             |           | 0.169      | 0.011     |
| $\theta$ (Group 4)            | DRAM Parameter                      | 0.118       | 0.072     |            |           |
| <i>Inc<sub>3</sub></i> (HBNW) | Trip Production Parameter           | -0.073      | 0.266     |            |           |
| <i>Inc<sub>3</sub></i> (HBW)  | Trip Production Parameter           |             |           | 0.088      | 0.187     |
| <i>Basic</i> (NHB)            | Trip Production Parameter           | -0.118      | 0.071     |            |           |
| <i>Retail</i> (HBNW)          | Trip Attraction Parameter           |             |           | -0.131     | 0.102     |
| <i>Service</i> (HBNW)         | Trip Attraction Parameter           |             |           | -0.132     | 0.100     |
| <i>Retail</i> (HBW)           | Trip Attraction Parameter           | 0.154       | 0.022     |            |           |
| <i>Basic</i> (NHB)            | Trip Attraction Parameter           | -0.080      | 0.225     | -0.039     | 0.559     |
| Adj. R <sup>2</sup>           |                                     | 0.181       |           | 0.162      |           |
|                               |                                     | Offpeak VMT |           | Peak VMT   |           |
| <i>Emprate</i>                | Employment Rate                     | 0.079       | 0.232     | 0.001      | 0.984     |
| <i>Poprate</i>                | Population Rate                     | 0.176       | 0.009     | 0.077      | 0.254     |
| $\beta_{link}$                | Exponent Link Performance Function  | 0.226       | 0.001     |            |           |
| $\alpha_6$                    | Parameter Link Performance Function | 0.313       | 0.000     |            |           |
| $\alpha_8$                    | Parameter Link Performance Function |             |           | 0.197      | 0.004     |
| $\Psi_{HBNW}$                 | Peak Off-peak Split                 |             |           | 0.297      | 0.000     |
| <i>Inc<sub>3</sub></i> (HBNW) | Trip Production Parameter           | -0.080      | 0.224     |            |           |
| <i>Retail</i> (HBNW)          | Trip Attraction Parameter           | -0.060      | 0.371     |            |           |
| <i>Basic</i> (HBW)            | Trip Attraction Parameter           |             |           | -0.072     | 0.286     |
| <i>Retail</i> (HBW)           | Trip Attraction Parameter           | 0.190       | 0.005     |            |           |
| <i>Basic</i> (NHB)            | Trip Attraction Parameter           | -0.105      | 0.113     |            |           |
| $\beta_1$ (HBNW Offpeak)      | Destination Choice Parameter        | 0.196       | 0.004     |            |           |
| Adj. R <sup>2</sup>           |                                     | 0.164       |           | 0.105      |           |
| N <sub>obs</sub>              |                                     | 200         |           | 200        |           |

**Table 7. Sensitivity Analysis Results: Weighted Residential and Commercial Density**

| Parameters                     |  | Weighted Commercial Density |           | Weighted Residential Density |           |
|--------------------------------|--|-----------------------------|-----------|------------------------------|-----------|
|                                |  | 2017                        |           | 2017                         |           |
|                                |  | Std Coeff.                  | p - value | Std Coeff.                   | p - value |
| $Emprate$                      | Employment Rate                              | 0.037                       | 0.549     | 0.059                        | 0.394     |
| $Poprate$                      | Population Rate                              | 0.048                       | 0.447     | 0.031                        | 0.661     |
| $\alpha_2$                     | Parameter Link Performance Function          | 0.156                       | 0.017     |                              |           |
| $\alpha_3$                     | Parameter Link Performance Function          | 0.099                       | 0.113     |                              |           |
| $\Psi_{NHB}$                   | Peak Off-Peak Split                          | -0.094                      | 0.134     |                              |           |
| $\theta$ (Group 1)             | Exponent on Total Land DRAM                  | 0.179                       | 0.004     | 0.122                        | 0.091     |
| $\gamma_1$ (Group 3)           | Parameter DRAM                               | -3.293                      | 0         | -1.152                       | 0.166     |
| $\gamma_3$ (Group 3)           | Parameter DRAM                               | 0.21                        | 0.021     |                              |           |
| $\gamma_4$ (Group 3)           | Parameter DRAM                               | 3.081                       | 0         | 0.998                        | 0.231     |
| $\theta$ (Group 4)             | Exponent on Total Land DRAM                  | 0.24                        | 0.02      |                              |           |
| $\gamma_3$ (Group 4)           | Parameter DRAM                               | 0.746                       | 0.001     |                              |           |
| $\gamma_4$ (Group 4)           | Parameter DRAM                               | -0.758                      | 0.001     |                              |           |
| $\beta_{op}$ (Group 4)         | Off-peak time Coefficient DRAM               | 0.176                       | 0.219     |                              |           |
| $\beta_p$ (Group 4)            | Peak time Coefficient DRAM                   | 0.383                       | 0.031     |                              |           |
| $\delta_2$ (Basic)             | Exponent on Total Land EMPAL                 |                             |           | 0.181                        | 0.041     |
| $\delta_1$ (Basic)             | Exponent on Previous Employment EMPAL        |                             |           | 0.209                        | 0.071     |
| $\beta_{op}$ (Basic)           | Off-peak time Coefficient EMPAL              | -0.174                      | 0.066     | -0.266                       | 0.05      |
| $\delta_1$ (Retail)            | Exponent on Previous Employment EMPAL        | 0.222                       | 0.011     | 0.14                         | 0.153     |
| $\beta_{op}$ (Retail)          | Off-peak time Coefficient EMPAL              | -0.189                      | 0.052     | -0.208                       | 0.056     |
| $\beta_p$ (Retail)             | Peak time Coefficient EMPAL                  | -0.282                      | 0.001     | -0.142                       | 0.123     |
| $\delta$ (Basic)               | Previous Period Impact EMPAL                 | -0.18                       | 0.056     | -0.156                       | 0.142     |
| $\delta_2$ (Service)           | Exponent on Total Land EMPAL                 |                             |           | -0.124                       | 0.139     |
| $\beta_{op}$ (Service)         | Off-peak time Coefficient EMPAL              | 0.073                       | 0.231     | -0.165                       | 0.045     |
| $Inc_3$ (HBNW)                 | Trip Production Parameter                    |                             |           | 0.157                        | 0.023     |
| $Inc_1$ (HBW)                  | Trip Production Parameter                    |                             |           | 0.097                        | 0.151     |
| $Retail$ (HBW)                 | Trip Attraction Parameter                    | 0.21                        | 0.001     | 0.138                        | 0.056     |
| $Service$ (HBW)                | Trip Attraction Parameter                    |                             |           | 0.121                        | 0.092     |
| $Retail$ (NHB)                 | Trip Attraction Parameter                    | 0.09                        | 0.158     | 0.188                        | 0.009     |
| $Basic$ (NHB)                  | Trip Attraction Parameter                    | 0.17                        | 0.024     |                              |           |
| $Service$ (NHB)                | Trip Attraction Parameter                    | 0.122                       | 0.096     |                              |           |
| $B_1$ (HBW Offpeak)            | Destination Choice Parameter                 |                             |           | 0.11                         | 0.111     |
| $\alpha_{auto}$ (HBNW Offpeak) | Mode Constant                                | 0.156                       | 0.013     | 0.126                        | 0.068     |
| $\alpha_{auto}$ (NHB Offpeak)  | Mode Constant                                |                             |           | -0.219                       | 0.02      |
| $\beta_m$ (NHB Offpeak)        | Travel Time Coefficient in Mode Choice Model |                             |           | 0.189                        | 0.044     |
| $\alpha_{auto}$ (HBW Peak)     | Mode Constant                                | -0.232                      | 0.01      | -0.247                       | 0.01      |

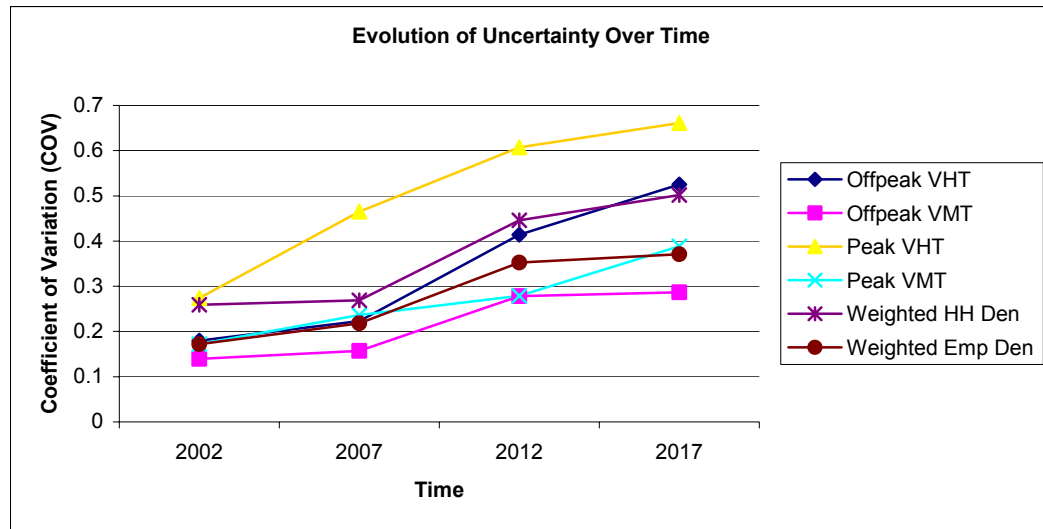
|                      |  |       |       |       |       |
|----------------------|--|-------|-------|-------|-------|
| $\beta_m$ (HBW Peak) | Travel Time Coefficient in Mode Choice Model | 0.293 | 0.001 | 0.247 | 0.011 |
| Adj. R <sup>2</sup>  |  | 0.32  |       | 0.171 |       |
| N <sub>obs</sub>     |  | 200   |       | 200   |       |

**Note:** Commercial density is employment divided by commercially used land area, per zone.

**Table 8. Sensitivity Analysis Results: Average Flow**

| Parameters                    | Description                                  | Off-peak Average Link Flow |           | Peak Average Link Flow |           |
|-------------------------------|--|----------------------------|-----------|------------------------|-----------|
|                               |  | 2017                       |           | 2017                   |           |
|                               |  | Std Coeff.                 | p - value | Std Coeff.             | p - value |
| <i>Emprate</i>                | Employment Rate                              | 0.135                      | 0.049     | 0.011                  | 0.872     |
| <i>Poprate</i>                | Population Rate                              | 0.225                      | 0.001     | 0.072                  | 0.295     |
| $\beta_{link}$                | Exponent Link Performance Function           | 0.194                      | 0.005     |                        |           |
| $\psi_{HBNW}$                 | Peak Off-Peak Split                          |                            |           | 0.206                  | 0.000     |
| $\psi_{HBW}$                  | Peak Off-Peak Split                          |                            |           | 0.208                  | 0.000     |
| <i>Basic</i> (NHB)            | Trip Production Parameter                    | -0.119                     | 0.079     |                        |           |
| <i>Retail</i> (HBW)           | Trip Attraction Parameter                    | 0.175                      | 0.011     |                        |           |
| $\beta_m$ (HBW Offpeak)       | Travel Time Coefficient in Mode Choice Model | -0.123                     | 0.071     |                        |           |
| $\alpha_{auto}$ (NHB Offpeak) | Mode Constant                                | -0.020                     | 0.767     |                        |           |
| $\alpha_{auto}$ (HBW Peak)    | Mode Constant                                | -0.087                     | 0.201     |                        |           |
| Adj. R <sup>2</sup>           |  | 0.115                      |           | 0.068                  |           |
| N <sub>obs</sub>              |  | 200                        |           | 200                    |           |

**Figure 1. Evolution of Output Uncertainty Over Time: Travel and Land Use**



Note: HH Den = Household Density; Emp Den = Employment Density