A Simulation-based Approximation Algorithm for Dynamic Marginal Cost Pricing

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Abstract

This work offers a simulation-based approximation algorithm for dynamic marginal cost pricing (MCP) that is a direct extension of static MCP. The algorithm approximates the time-dependent marginal costs, and is incorporated into the inner approximation dynamic user equilibrium algorithm to evaluate the results of dynamic MCP, which are then compared to static assignment results with MCP from previous study. The status quo and dynamic MCP-on-freeways scenarios are simulated (and then compared) on Dallas-Fort Worth 35,732-link network. Due to computational requirements for such large-scale DTA application, the dynamic MCP scenario is simulated without feedback, and only route choices are permitted to vary. When prices are imposed, some minor system benefits are observed, including a delay in the onset of congestion. Dynamic prices vary substantially over the analysis period, reflecting changes in congestion. Reasons for any inconsistencies between dynamic and static results are discussed, along with important enhancements to future implementation.

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INTRODUCTION AND BACKGROUND

It is well known in the literature that two broad methods to combat traffic congestions are supply improvement and demand management. The supply-sided solutions include capacity expansion, managed lanes and ramp metering, to name a few. According to the Pigou-Knight-Downs paradox (see Arnott and Small, 1994), increasing capacity actually allows latent demand to consume much of the travel time savings. Clearly, demand management is key, emphasizing behavioral modification in order to shift and shorten car trips and reduce their frequency and peaking over times of day. (See, e.g., Kockelman 2004) One rather obvious option is the tolling of congested roads, or congestion pricing (CP).

The concept of road space rationing is not new and dates back to early 20th century (Pigou, 1920 and Knight, 1924). Early work in CP includes Vickrey (1963), who observed that an effort was underway to differentiate between peak and off-peak demand in several markets, and something similar for transportation would be useful. There are numerous works on static traffic assignment (STA)-based CP, which essentially assumes steady-state traffic conditions (e.g. Zhao and Kockelman, 2006; and Dial, 1999) Given the time varying nature of traffic flow on a transportation network, to truly assess the traffic and economic impacts of CP, one should seek a dynamic traffic assignment based CP. The advantages of a dynamic CP over the static CP are as follows: i) more realistic marginal costs that are calculated at various time slices and likely to be equitable, and ii) the better representation of traffic flows as traffic dynamics and spatial and temporal vehicular interactions can be captured. Moreover, the recent new commitments by municipal, state and federal governments to construct and operate roadways with dynamic toll pricing, which is a toll pricing method that changes based on traffic conditions to maximize the performance of the tolled facility, have crucially motivated the need for dynamic CP models (Friesz et al., 2007).

The literature on the dynamic congestion pricing is very limited. Joksimovic et al. (2005) formulated the second-best toll design problem in the dynamic traffic network as a bi-level optimization problem, considering elastic demand. They only showed a small hypothetical network and solved it by complete enumeration, and their DTA scheme is an extension of the static traffic assignment. Wie and Tobin (1998) proposed two dynamic congestion pricing models based on the first-best marginal cost pricing. Their DTA scheme employs a link performance function to estimate link travel time. Friesz et al. (2007) introduced the dynamic optimal toll problem with user equilibrium constraints, and formulated two formulations based on differential variational inequalities and equilibrium network design, respectively. The DTA models in these papers lack realistic traffic conditions, and they cannot capture traffic interactions across adjacent links. Friesz et al. (2004) and Yang et al. (2007) consider day-to-day dynamic congestion pricing, which forces the traffic condition to system optimum. They consider steady-state traffic condition within each day, and employ a typical link performance function. Mahmassani et al. (2005) proposed an efficient approximation algorithm for finding bi-criterion time-dependent efficient paths in large-scale traffic networks. Lu et al. (2006) and Lu and Mahmassani (2007) proposed a bi-criterion dynamic user equilibrium model and solution algorithm to support the planning, operation and evaluation of various dynamic congestion pricing schemes, but do not consider calculating the dynamic optimal tolls.

This paper presents a simulation-based heuristic algorithm to calculate dynamic tolls under the dynamic marginal cost pricing (MCP) scheme, which is a direct extension of static

MCP (see, e.g., Kockelman 2004). Our approach involves the incorporation of the proposed MCP computation to the Inner Approximation Dynamic User Equilibrium (IADUE) algorithm, which is implemented in the DTA module of the Visual Interactive System for Transport Algorithm (VISTA) (Ziliaskopoulos and Waller, 2000). The DTA module employs RouteSim (Ziliaskopoulos and Lee, 1996), which is a mesoscopic simulator based on an extension of Daganzo's (1994) cell transmission model, to propagate traffic while accounting for traffic realisms such as link capacity, queue spillbacks and shockwaves. The static model requires the strict guarantee of first-in-first-out (FIFO) condition, which can be relaxed in the dynamic model. For example, vehicles approaching an intersection may not satisfy the FIFO condition. Thus, it overcomes the weakness of using link performance functions as typical in the literature.

This paper is organized as follows. The next section describes the simulation-based approximation algorithm for dynamic MCP. Our computational experience is then presented, and the limitations in our approach are discussed. Finally, the major conclusions are summarized and possible directions for future research are given.

SIMULATION-BASED APPROXIMATION ALGORITHM FOR DYNAMIC MARGINAL COST PRICING

Our approach does not attempt to solve for the truly optimal marginal-cost dynamic pricing policy because it is a highly complex problem. Instead, an approximate method is used, employing each link's marginal cost estimate to compute time-dependent tolls, updating these every ten minutes. This heuristic method assumes that a vehicle entering a tolled link imposes marginal costs only on vehicles that use the same link, rather than impacting travel times (and thus costs) on other links. We first show the approximation algorithm for computing dynamic MCP tolls. Then, we present the modified IADUE algorithm that incorporates the proposed dynamic MCP computation.

Approximation of Dynamic MCP Toll

When a vehicle enters a transportation network, it imposes two types of costs: the average cost experienced by the vehicle and a marginal cost (experienced essentially by those following the new vehicle, under very slightly reduced speeds) (Liu and McDonald, 1999). In this study, we consider an approximation for calculating the marginal costs on tolled links. The assumption is that a vehicle entering a tolled link imposes marginal costs only on all following vehicles using this same link. Thus, we assume that it does not impact vehicles using other, upstream (or downstream) links. The time-dependent link toll is the product of the time-dependent link marginal cost (in seconds) and the value of travel time (VOTT, in \$ per second).

Peeta and Mahmassani (1995) proposed the following computation of a link's approximate time-dependent marginal cost, $t^{ta} \quad \forall t, a$, in terms of travel time (for all time periods t and links a):

$$t^{ta} = T^{ta}(x) + x^{ta} \frac{\partial T^{ta}(x)}{\partial x^{ta}}$$

where $T^{ta}(x)$ represents the travel time experienced by another vehicle entering link *a* at time *t, x* is the vector of time-dependent vehicle counts on all links (x^{ta}, $\forall t, a$), and *dT/dx* equal the link's marginal cost (in seconds).

The spatial interactions and n^{th} order temporal interactions (global marginals) are ignored in this computation. These effects may not be significant compared to the direct effect on link *a* at time *t*, $\frac{\partial T^{ta}(x)}{\partial x^{ta}}$ (i.e. local marginals). Under such conditions, the solutions obtained using the global marginals and the local marginals will be relatively close. However, if the interactions are significant, the solution obtained using the local marginals may deviate from that obtained using the global marginals.

$T^{ta}(x)$ and x^{ta} are obtained directly from simulation. Figure 1 illustrates the approach used for the computation of the derivative $\frac{\partial T^{ta}(x)}{\partial x^{ta}}$. The approach used here assumes that the time-dependence of the derivative is due to time-varying link performance functions. This means the performance curve in Figure 1a for link *a* at time *t* depends on the traffic flow conditions on the link at that time. This time-dependence is very significant; a link's travel time can differ significantly the same number of vehicles at two different times depending on the fraction of vehicles that are queued. A link's link performance curve changes somewhat gradually over time. If the time interval between successive evaluations of *dT/dx* derivatives (marginals) is small, it appears reasonable to assume that three consecutive points in time are on the same link performance curve, as illustrated in Figure 1a. A quadratic fit using the three points results in the time-varying link performance curve at time *t* and the slope of this curve at time *t* gives $\frac{\partial T^{ta}(x)}{\partial x^{ta}}$, as shown in Figure 1b. For example, at times *t*-1, *t* and *t*+1, the corresponding link inflows are 100, 250 and 400 vehicles, and the corresponding link travel times 1500, 2500 and 600 seconds. Following the procedure in Figure 1b, the link marginal cost at time *t* is 14.973 seconds/vehicle. Assuming a VOTT of $10.00 / vehicle-hour, the MCP toll at time *t* is 4.16 cents plus an existing flat toll (if any).

Peeta and Mahmassani (1995) suggested that the consideration of small time intervals (on the order of a few seconds) may cause some instability in the curves because the VOTTs and the number of vehicles in successive intervals may exhibit "jumps" at certain times. Hence, the length of the time interval between successive data points involves trade-offs between the accuracy and robustness of the curves. To achieve stability in the curves, the simulation of 6-second time intervals may be too small for updating OD paths , since no appreciable change takes place in the system in such a short duration. In the implementation of the solution algorithm, paths are updated every assignment interval (i.e., every 10 minutes for the DFW network). The marginal values may be computed for assignment intervals only and not for simulation intervals, thereby reducing the computational burden of the path-processing step. In the next section, this dynamic MCP computation is incorporated into the IADUE algorithm.

Modified IADUE Algorithm for Dynamic MCP

The IADUE algorithm (Chang, 2004) is a solution algorithm for the variational inequality (VI) formulation of the single-mode automobile dynamic user equilibrium problem. The algorithm estimates the equilibrium path assignment using inner approximation methods. Conventionally, vehicle assignment has been performed using the method of successive averages, which assumes that all previous solutions contribute equally to the final equilibrium, so it assigns vehicles equally among the set of past solutions. In contrast, the IADUE algorithm searches the feasible set of path assignments for the assignment that minimizes an equilibrium gap function. It searches within a subspace defined by a set or subset of the extreme points of the feasible space. The IADUE algorithm is similar to simplicial decomposition (e.g. Von Hohenbalken, 1977; Hearn et al., 1984), but differs in the descent direction used for each iteration.

The IADUE algorithm iteratively employs the TDSP algorithm (Ziliaskopoulos and Mahmassani, 1994) to generate vehicle paths with the implicit assumption that drivers have perfect information and can divert to alternate paths if it reduces travel time. In this study, a minor modification in TDSP is made in order to account for tolls, under an assumption of homogeneous users (i.e., a single value of VOTT). The VOTT is used to convert tolls to a time penalty, characterized as additional time perceived by each driver. For example, with the VOTT of \$10, \$0.33 toll charge is equivalent to 2 minutes of additional travel time. This added time or "delay" obviously does not impact the actual time spent within the network, but it is used within the TDSP algorithm. The modified algorithm is called the time-dependent least cost path (TDLCP) algorithm, and it routes each vehicle such that it chooses a path with the least generalized cost (toll-based time penalty plus actual travel time).

The notations are first described. Let's denote by V the set of nodes, A the set of arcs, and $[0, T]$ an assignment period. d_{rs}^{t} is the number of automobile trips departing from node r to node s ($r,s \in V$) at time $t \in [0,T]$. P is the set of all spatiotemporal paths from all origins to all destinations. $P(r,s,t) = \{p^1, p^2, \dots, p^\pi\}$ is the set of paths departing at time t from nodes r to s. ξ^{p^k} is the number of automobiles choosing to follow path p^k. Ξ is the vector of path flows; i.e., $\Xi = (\xi^{p^k})$. $\psi^{p^k}(\Xi)$ is the generalized cost on path p^k. $\Psi(\Xi)$ is the vector of path generalized costs; i.e., $\Psi(\Xi) = (\psi^{p^k}(\Xi))$.

Since the demand relationships $\sum_{p^k \in P(r,s,t)} \xi^{p^k} = d_{rs}^{t}$ for all *(r,s,t)* form a closed, bounded and convex space $D \subset R^\pi$, any assignment Ξ in D is feasible, given that the traffic flow propagation law adopted prevents gridlock and allows all vehicles to complete their trips within time T. It is assumed that a driver's selection of an alternative path is a unilateral decision based on the current traffic conditions. Chang (2004) showed that a Wardrop equilibrium solution, Ξ^*, exists where

$$\Psi(\Xi^*)^T(\Xi - \Xi^*) \ge 0 \;\; \forall \Xi \in D$$

Since the equilibrium conditions defined in this VI formulation are difficult to solve directly, Chang (2004) formulated a gap function based on Smith (1983) such that the optimal solutions of the gap function coincide with points that satisfy the equilibrium conditions. Let Ω be the set of equilibrium solutions that satisfy the VI conditions, then *Gap* is a gap function for these conditions if $Gap(\Xi) = 0 \;\; \forall \Xi \in \Omega$ and $Gap(\Xi) \ge 0 \;\; \forall \Xi \in D$. After translating the equilibrium conditions into a gap function, numerical search approaches can be applied. The proposed IADUE algorithm iteratively selects a descent direction (Δ) in the assignment space D and a step length that minimizes the gap function. The algorithm terminates when $Gap(\Xi) = 0$. We refer to Chang (2004) for the detailed descriptions of *Gap* and Δ.

For dynamic MCP, we add a new module, which computes dynamic MCP, to the original IADUE algorithm. The modified IADUE algorithm is shown below.

Step 0: Initialization
- Set $n = 0$
- Set link travel times to free flow travel times
- Compute the link generalized costs (using the existing flat tolls)
- Run TDLCP to obtain least cost paths for each *(r,s,t)*
- Set Ξ_0 to all-or-nothing assignment of d_{rs}^{t} to shortest path for *(r,s,t)*
- Simulate traffic conditions with the assignment Ξ_0
- Compute dynamic MCP tolls for tolled roads
- Update link generalized costs (using the MCP tolls plus the existing flat tolls)

Step 1: Run TDLCP and add new paths to the path set P

Step 2: Choose new solution Ξ_{n+1}
- Determine the descent direction Δ_n
- Select step length λ_n that minimizes the gap function
  $(\lambda_n = \arg\min_{\lambda}(Gap(\Xi_n + \lambda\Delta_n)))$
- Assign demand to $\Xi_{n+1} = \Xi_n + \lambda_n\Delta_n$

Step 3: Update generalized costs
- Simulate traffic conditions with Ξ_{n+1}
- Compute MCP tolls for tolled roads
- Update link and path generalized costs
  (using the MCP tolls plus the existing flat tolls)

Step 4: Check for convergence
  If $Gap(\Xi_{n+1}) \geq 0$, set $n = n+1$ and return to Step 1. Otherwise, terminate.

The computational bottleneck is the TDLCP algorithm. After a number of iterations of the modified IADUE, we assumed that there are a sufficiently large number of paths generated for each OD pair. Then, the module UPDATE-COST-DTA in VISTA is executed that runs Steps 2 to 4 of the modified IADUE. That is, the algorithm keeps updating time-dependent path generalized costs of all paths in the generated path set P (without generating new paths) until convergence. As such, an iteration of UPDATE-COST-DTA is much faster than an iteration of the modified IADUE, since the TDLCP algorithm is not performed. Next, we show the computational experience on a real large-size network.

COMPUTATIONAL EXPERIENCE

This work was originally developed for the application of credit-based congestion pricing (CBCP) in Texas, which is briefly described as follows. CBCP (Kockelman and Kalmanje, 2004) is a novel strategy which seeks to overcome the negative equity impacts of congestion pricing CP by allocating monthly budgets to eligible travelers to spend on congestion tolls. The first step in predicting CP's impacts involves traveler behavior modeling. Kockelman et al. (2005) estimated joint destination-mode choice models for the Dallas-Fort Worth (DFW) metroplex from the region's 1996 household and on-board transit survey datasets for different trip purposes. These models were applied for short-term (employment locations held fixed) and long-term (employment locations flexible) static cases with full feedback (using the method of successive averages) to the DFW region. In addition to the status quo (which has some tolling), two MCP scenarios were simulated for the 1999 modeling year: MCP just on the region's freeways and MCP applied on all roads (see Gulipalli, 2005). Full model feedback of travel times and costs was implemented, and the method of successive averages (MSA) was used for route, destination, and mode choice equilibration for each of five daily time periods.

Since the MCP-on-all-roads scenario has very high associated initial and recurring costs, and it is not likely to be practically feasible in the near future (Kockelman et al., 2005). Due to these facts and especially the excessive computational time of a DTA run for large-scale networks, the MCP-on-all-roads scenario is not considered here. Specifically, we employ the proposed simulation-based heuristic to evaluate the MCP scenario when only freeways are priced, and the simulation-based DTA to evaluate the status quo for the DFW region. Due to highly intensive computational requirements for large-scale DTA applications, the status quo and MCP scenarios are simulated without feedback (of travel times and costs, for destination and other choices). Only route choices are permitted to vary.

The DTA analysis focuses on the AM peak (6 to 9 AM), and an assignment interval is ten minutes, resulting in 18 assignment intervals over three hours of simulation time. A simulation time step is six seconds, yielding 1800 time steps over three hours. A single VOTT of $10.00 per hour (per vehicle) is assumed, so that the results can be comparable to Kockelman et al.'s (2005) static analysis. In order to build a set of competitive path choices for each time step and every OD pair, five IADUE iterations are run (approximate CPU time = 1 month). Then, we run the UPDATE-COST-DTA until achieving convergence for both scenarios (status quo and MCP-on-freeways) (approximate CPU time = 1 week). The proposed simulation-based algorithms run on a Dell PowerEdge 6600 Server with dual 2.3 GHz Xeon processors and 4 GB of RAM, running under Redhat Fedora Core 3. The modified IADUE algorithm takes approximately 5 weeks to evaluate the results of dynamic MCP-on-freeways without feedback.

The network and demand matrix assembly is described next, followed by the comparison of the status quo and MCP-on-freeways from static and dynamic traffic assignment. Possible reasons of considerable difference between STA and DTA results are discussed. Then, we show the exploration of density on certain arbitrary freeway links from DTA analysis. Although it is unclear if STA and DTA models are comparable as they are based on different modeling assumptions, our comparison provides a systematic approach to compare the differences between these two approaches for CP. This comparison should facilitate future research to compare the obtained results with observed data.

Network and Demand Matrix Assembly

As provided by the North Central Texas Council of Governments (NCTCOG), the 1999 DFW roadway network has 26,748 lane miles and 22,187 links. For a static approach to network assignment (implemented in TransCAD), the DFW network has 13,694 nodes, 4,874 centroids, 22,187 links, and 9,805 centroid connectors. TransCAD's links and centroid connectors can be either unidirectional or bidirectional. Since VISTA has its own data format requirements, the TransCAD network data are converted into VISTA file formats. All links in the VISTA database must be unidirectional. Thus, the number of links used by VISTA is greater than that in

TransCAD, but the number of nodes remains the same. A major concern in running DTA is the amount of memory consumed – especially when generating the competitive least-cost paths for every 6-second time step and every OD pair. Since the number of zones (centroids) impacts these memory requirements directly, a relatively aggregate zonal system is used. In VISTA, the DFW roadway network still has 13,694 nodes, but just 919 centroids. It has 35,732 links, and 3,642 centroid connectors (or 4 connectors per centroid). The zonal structure is that previously used by NCTCOG.

The time-dependent OD demands are derived as follows. Gulipalli's (2005) short-term static travel demand model application for the status quo yielded trip origin-destination (OD) information for the region's five traffic assignment periods and four modes. The static ODs for the three modes (drive alone, shared ride and truck trips) in the five times of day then were combined by first converting each truck trip to two passenger car equivalents (HCM 2000) and summing up the drive-alone trips, shared-ride trips and converted truck trips for each OD and time of day. With these 24-hour static OD trips, the demand profiling method by Karoonsoontawong et al. (2008) is used to generate the smoothed time-dependent OD trips every six seconds. Only the demands during the three-hour AM peak (6:00 AM – 9:00 AM) are used for this analysis.

Comparison of Traffic Impacts for the Status Quo and MCP-on-freeways from Static and Dynamic Traffic Assignment

The DTA results are compared both to one another (status quo vs. MCP-on-freeways scenarios) and to the static analysis with its full behavior feedbacks. It is noted that the value of MCP toll for static analysis is based on the standard Bureau of Public Roads (BPR) formulation:

$$t_i = t_{i,f}\left[1+\alpha\left(\frac{v_i}{C_i}\right)^{\beta}\right]$$

where t_i = travel time of link i; $t_{i,f}$ = free-flow travel time of link i; v_i = volume of link i; C_i = capacity of link i; and α and β = calibration parameters. The static MCP toll is determined from differentiating t_i with respect to v_i; thus, the value of link toll for static MCP scenario is (Gulipalli, 2005):

$$Toll_i = k_i + VOTT \cdot t_{i,f} \cdot \alpha \cdot \beta\left(\frac{v_i}{C_i}\right)^{\beta}$$

where k_i = any existing toll on link i.

Traffic impacts for the status quo and MCP-on-freeways scenarios are compared in terms of VMT, VHT and average speed during the AM peak period. For the comparison with the static models, the long-term STA traffic impacts come from all five time periods, so they are not perfectly comparable to the 3-hour AM peak period results found using DTA. However, the differences are so striking that this distinction is not of major consequence. All STA results are taken from Kockelman et al. (2005)[1].

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[1] The BPR parameters used in Kockelman et al. (2005) (α=0.15; β=4) were based on effective capacity (maximum service flow under level of service (LOS) C, rather than true capacity, under LOS E) and thus were biased low. In

System Level Comparison: The predicted changes in system-level (i.e., total) VMT, VHT and average speed during the AM peak when freeways are priced, versus status quo, are shown in Table 1. The DTA results appear insignificant, both in isolation and when compared to the behavioral changes evident under the STA approach. Part of this insensitivity is due to the smaller share of VMT on freeways in DTA model (see Figure 2) when compared with STA model. The directions of changes for STA and DTA results are not in agreement for system VHT and average speed. The STA results seem to be consistent with expectations. The DTA results seem less so. Of course, the DTA model runs do not permit the behavioral feedbacks that the STA approach allows, so the travelers are far more constrained and changes are fully expected to be much less dramatic. Although it is well accepted by researchers (e.g. Peeta and Ziliaskopoulos, 2001; Mahmassani, 2001) that DTA captures traffic realities better, it should be noted that a more valid comparison should involve the observed traffic data to conclusively test the validity of the proposed approaches. Our research in this work is a step towards that direction.

Comparison from DTA Analysis: Estimates of %VMT, %VHT and average speed categorized by different roadway facility types for DTA analysis are shown in Figures 2-4. Freeway and ramp VMTs for the MCP-on-freeways scenario are predicted to rise very slightly (again by less than 1%), while freeway and ramp VHTs are predicted to fall by 1.53% and <1%, respectively. Freeway and ramp average speeds for the MCP-on-freeways are predicted to rise by 1.89% and <1%. This may imply that more short-trip travelers using freeways in the status quo tend to choose non-freeway routes to avoid MCP tolls, while more long-trip travelers switch to stick with freeways, thanks to the travel time savings offsetting MCP tolls.

Principal-arterial, minor-arterial and frontage-road VHTs for the MCP-on-freeways scenario are predicted to rise very slightly (by less than 1%), while their speeds are predicted to fall by <1%, <1% and 3.32%, respectively. This means the principal arterial, minor arterial and frontage roads are predicted to become somewhat more congested when MCP tolls are applied on freeways because more short-trip travelers that use freeways in the status quo leave the freeways to use these facility types, when tolls are applied.

Principal-arterial, minor-arterial and frontage-road VMTs for the MCP-on-freeways are predicted to fall by <1%. This may imply more longer-trip travelers that use arterials in the status quo are attracted to freeways in the MCP-on-freeways. All these results are expected to be amplified considerably when behavioral feedbacks for destination and mode choice shifts are permitted.

Comparison between DTA and STA Results: The %VMT, %VHT and average speed by roadway types for STA analysis are also shown in Figures 2-4. In the DTA model, minor-arterial VMT is predicted to be highest, followed by principal-arterial VMT, and freeway VMT (around 10 to 15% of total VMT). In dramatic contrast, under the STA model freeway VMT is predicted to be highest (around 45-55% of total VMT), followed by minor-arterial VMT, and principal-arterial VMT. A similar trend is witnessed for VHT's distribution across the network.

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this paper, we employ the static results based on more appropriate BPR parameters (α = 0.85; β =5.5), which imply higher static tolls, and somewhat higher VMT reductions and speed increases.

Of course, the STA model is a 24-hour model, so freeways may attract more travel during the off-peak hours, but the contrast is still striking if one considers just the 3-hour peak period for the STA analysis. The STA results match our expectations better than the DTA results, since one expects a greater share of VMT on freeways than on other facility types (due to higher speeds on freeways). However, the percentage of freeway VMT under the STA model may well be too high, simply because many local network links are not coded. Thus, true freeway %VMT should lie somewhere between STA and DTA results.

Predicted freeway speeds average between 45 and 65 mph, under the 24-hour STA analysis, while under a DTA approach for the morning peak period predicts just 30-35 mph. The range of estimated average speeds for all facility types under the STA approach is 25-65 mph, while that for DTA in the 3-hour peak is just 15-35 mph. These indicate a drastic difference between STA and DTA results, and suggest very different behavioral assumptions regarding traffic performance.

Possible Reasons of Considerable Differences between STA and DTA Results

As also discussed in the limitations section, the dramatic differences between DTA and STA results may stem from the following modeling distinctions. First and foremost, the static analysis allows feedback of travel time and cost information to destination and mode choices. The dynamic model is without feedback: travelers are only allowed to change route. Thus, travelers in the dynamic model are substantially more restricted, resulting in negligible estimates of VMT, VHT and speed changes when MCP is applied. Secondly, the centroid structure employed in the DTA model is much more limited than in the STA case. It consists of 919 zones (or centroids) and 3,642 centroid connectors, while the STA model uses 4,874 zones and 9,805 centroid connectors. Thus, the DTA's centroid structure cannot load and unload the network as uniformly or rapidly as the STA's structure. This may result in more congestion around entry nodes in the DTA network. Third, there is a difficulty in DTA peak period analysis: a small number of travelers' departures cannot complete their trips by the end of the 3-hour AM peak-period analysis. The ideal analysis is a 24-hour period; however, this is effectively impossible at present, due to computer memory limitations. The three-hour study period (AM peak) may not be sufficient for "warming up" and "cooling down" the network; substantial traffic shoulders may exist, making congestion more severe at the start and end of the peak period. Note that in the beginning of DTA, the network is empty, so the warming-up period is employed to populate the network; then, the true analysis period starts. In the same way, the cooling down period is employed after the analysis period to clear all vehicles from the network. All DTA results are determined only from the analysis period.

Exploration of Density on Freeway Links and Toll Rates

Next, we explore the number of vehicles or traffic density along four arbitrary links over the analysis period, using DTA methods. Figures 5a-5c show the density of freeway links that presently operate without tolls. It can be seen that time shifts in traffic demands take place on these freeways, as a result of MCP tolls. The MCP toll estimates under the DTA-based heuristic method range from $0.10 to $29.2 per mile on the region's freeway links during the AM peak (6 to 9 am). Obviously, anything over $10 or $20 per mile is probably unrealistic, at any time of day, even for very short sections (such as narrow bridge crossings). The DTA model seems to be overestimating MCP tolls. In the STA approach, the toll rates went only as high as $0.79 per mile.

LIMITATIONS

This DTA analysis assumes inelastic demand, in both destination and mode choice, substantially limiting behavioral changes. Due to computational limitations in VISTA, the numbers of vehicle trips for the status quo and MCP-on-freeway scenarios between all OD pairs come from the static analysis (the applications of joint DM choice model in Gulipalli, 2005), and these are considered fixed. Departure times are also considered fixed. Although VISTA's path-based simulation model can assume different VOTTs across OD pairs (as explained in Ziliaskopoulos et al., 2004), a single VOTT of $10.00 per vehicle-hour is used here, since it is more comparable to the STA approach and no obvious means of ascertaining VOTT variations by OD pair is available. The centroid structure is aggregated to enable DTA analysis. The static demand is smoothed across times of day by the selected model. Lastly, the dynamic MCP toll calculation is an approximation method, unlike the analytical method in the STA.

CONCLUSIONS

An approximation algorithm for computing dynamic marginal cost pricing (MCP) tolls was developed, employing the link marginal costs to calculate the time-dependent toll. This approximation was incorporated into the inner approximation dynamic user equilibrium (IADUE) algorithm in VISTA to evaluate the dynamic MCP. This represents a wholly new application environment for VISTA and a major step forward for this kind of DTA model.

The 1999 DFW roadway network was converted from TransCAD format into VISTA format. Due to computer memory limitations, DFW's past, more aggregated zonal system was used, composed of 919 zones/centroids and 3,642 centroid connectors. The network structure used in the dynamic analysis was the same as the static analysis, but the analysis focused on the AM peak (6:00 AM – 9:00 AM). The time-dependent OD demands for the status quo and MCP-on-freeways scenarios were the same, as derived from the status quo of the static analysis. The DTA parameters are the following: the assignment interval of 10 minutes, the simulation time step of 6 seconds and the single value of travel time (VOTT) of $10 per hour. For both scenarios, the DTA module was run for 5 iterations, followed by running the UPDATE-COST-DTA module until convergence.

The results of static and dynamic traffic assignments were remarkably inconsistent. The reasons for this are felt to be as follows: (1) The STA approach allowed behavioral feedbacks, whereas the DTA did not. (2) The time periods and traffic demand profiles were distinct (all times of day constant-demand STA results were compared to AM-only DTA results for a smoothed demand profile). (3) The STA employs link performance functions; in contrast, the DTA model employs the cell transmission model, a traffic flow theoretical model, to propagate traffic. (4) The DTA's MCP method is an approximation, whereas that in STA is analytical. (5) The DTA zone and centroid connection structure was relatively coarse, so the DTA network could not load (and unload) as smoothly as the STA network.

Minor changes in DTA-predicted freeway use following implementation of MCP-on-freeways suggest that short-trip travelers may avoid the priced freeways while longer-trip travelers are more willing to pay the congestion tolls. Using the DTA-based approximation method, MCP tolls on freeways during the AM peak period were predicted to range between \$0.1 and \$29.2 per mile. Predicted freeway link densities and associated MCP tolls indicated observable shifts in traffic flows due to pricing. Some minor system benefits were observed, including a delay in the onset of congestion. Of course, traffic impact predictions would have been much more dramatic had behavioral feedbacks been incorporated. Future implementations should allow such feedback, to incorporate destination, departure-time, and mode-choice decisions as well as heterogeneous users. This paper is essentially the first step toward the more realistic, deployable model for dynamic MCP. In essence, this paper identifies the possibilities and the challenges that should be addressed while implementing DTA for large networks. The insights from computational implementation obtained from this work should allow other researchers and practitioners to draw lessons while solving large scale DTA problems.

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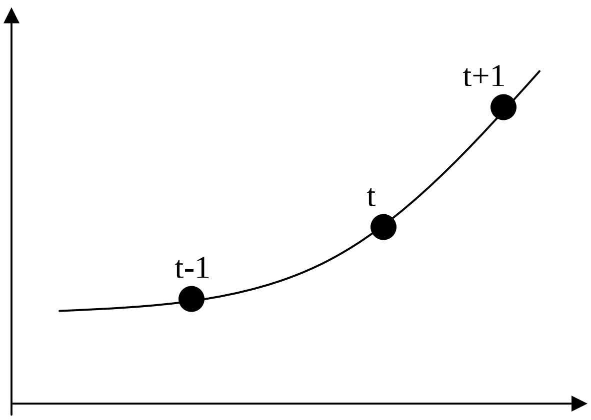
Table 1. Traffic Impacts of MCP-on-Freeways, as Compared to the Status Quo

	DTA (AM Peak)	STA (Long-Term; All TODs)
System VMT	Fall by <1%	Fall by 9.4%
System VHT	Rise by <1%	Fall by 16.7%
System Average Speed	Fall by <1%	Rise by 8.7%

Figure 1. Computation of Derivative for Link Marginal Cost and Time-Dependent MCP Toll

a) Graph of Link Travel Time and Cumulative Link Inflows

Link Travel Time of Link *a* at Time τ ($T^{\tau a}$)



Cumulative Link Inflows (vehicles) of Link *a* at Time τ, ($x^{\tau a}$)

b) Procedure to C

Input: (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) associated with times t-1, t and t+1

Output: the gradient at point (X_2, Y_2)

Step 0: Form the system of quadratic equations:

$$AX_1^2 + BX_1 + C = Y_1$$
$$AX_2^2 + BX_2 + C = Y_2$$
$$AX_3^2 + BX_3 + C = Y_3$$

where A, B and C are variables.

Step 1: The solution of the system in Step 0 is:

$$L_1 = \frac{X_1 + X_2}{X_1 X_2}, L_2 = \frac{X_1 + X_3}{X_1 X_3}, R_1 = \frac{Y_1 X_2^2 - Y_2 X_1^2}{X_1 X_2 (X_2 - X_1)},$$

$$R_2 = \frac{Y_1 X_3^2 - Y_3 X_1^2}{X_1 X_3 (X_3 - X_1)},\ C = \frac{R_1 - R_2}{L_1 - L_2},\ B = R_1 - CL_1,\ A = \frac{Y_1}{X_1^2} - \frac{B}{X_1} - \frac{C}{X_1^2}.$$

Step 2: Determine the gradient at point (X_2, Y_2): $\frac{dY}{dX}(X_2, Y_2) = 2AX_2 + B$.

Step 3: MCP toll at time t (associated with point (X_2, Y_2)) is $2AX_2 + B$ + existing flat toll.

Figure 2. VMT by Roadway Facility Type for Status Quo and MCP-on-freeways from DTA (AM Peak) and STA (Long Run, All TODs)

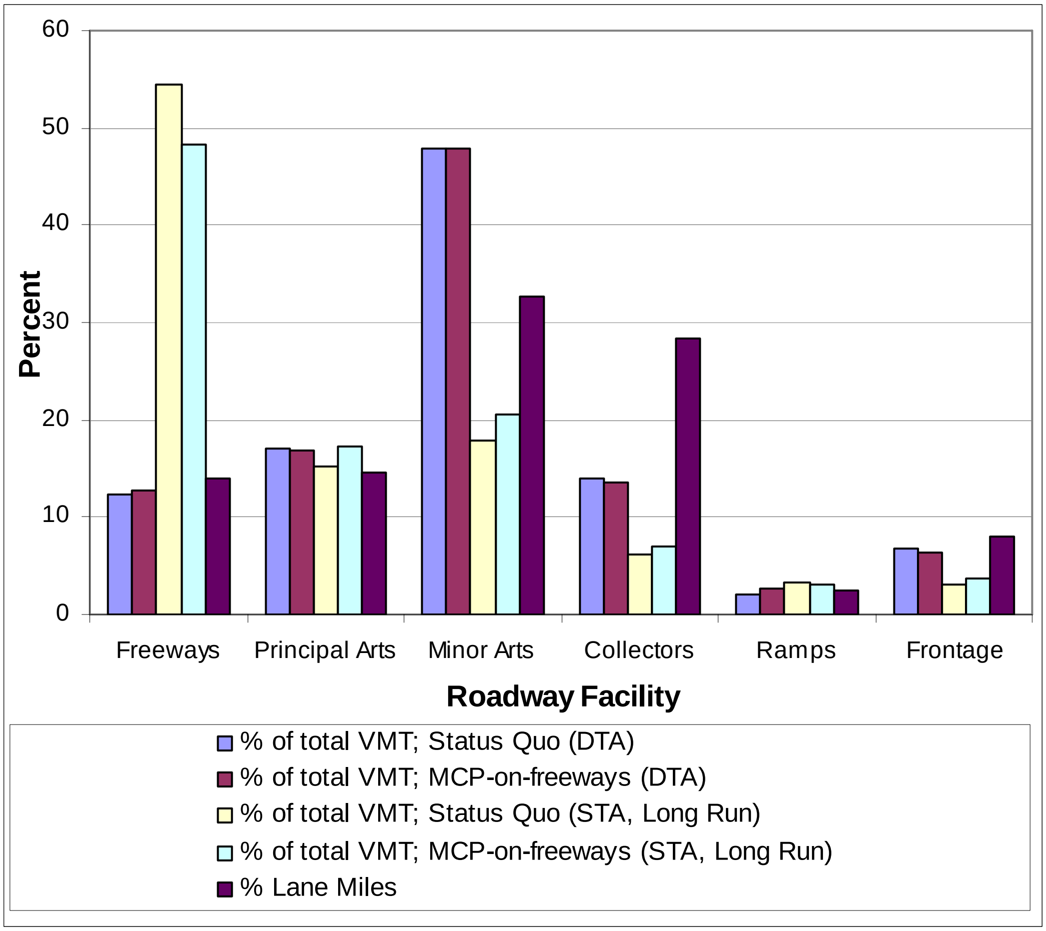


Figure 3. VHT by Roadway Facility Type for Status Quo and MCP-on-freeways from DTA (AM Peak) and STA (Long Run, All TODs)

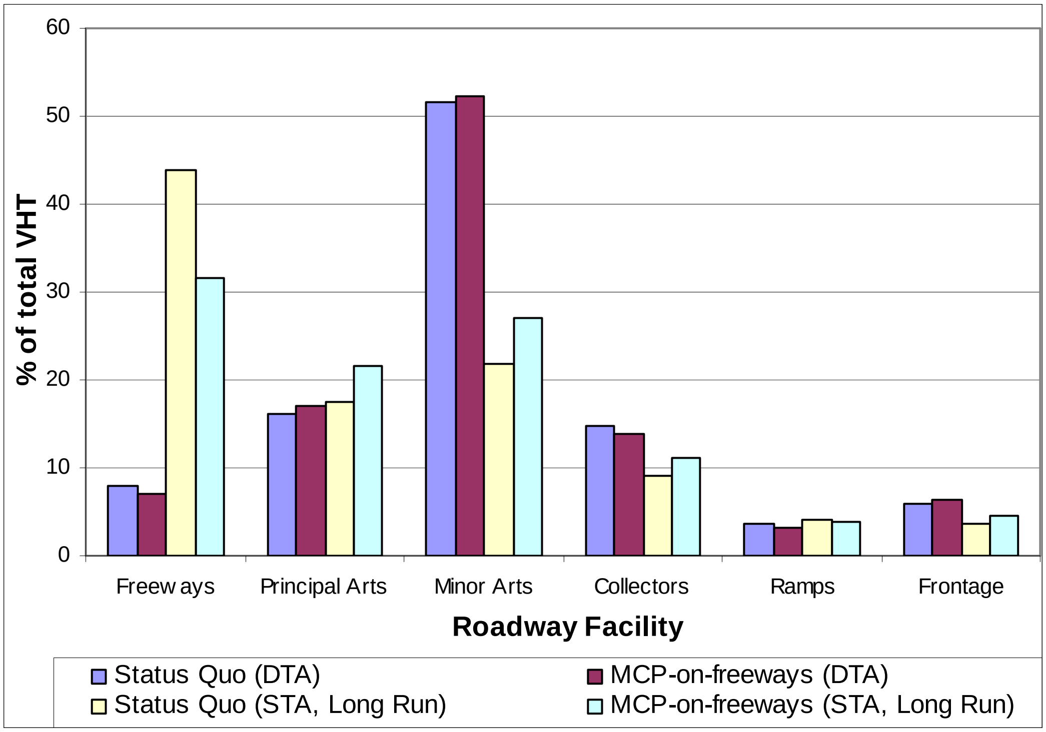


Figure 4. Average Speed by Roadway Facility Type for Status Quo and MCP-on-Freeways from DTA (AM Peak) and STA (Long Run, All TODs)

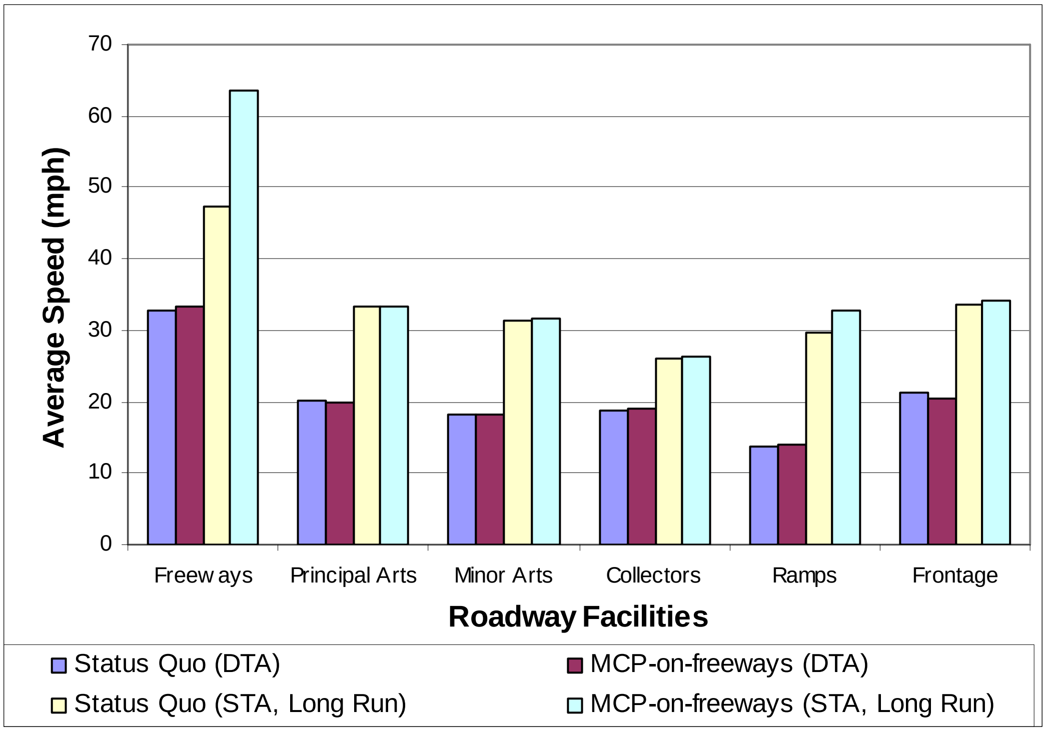
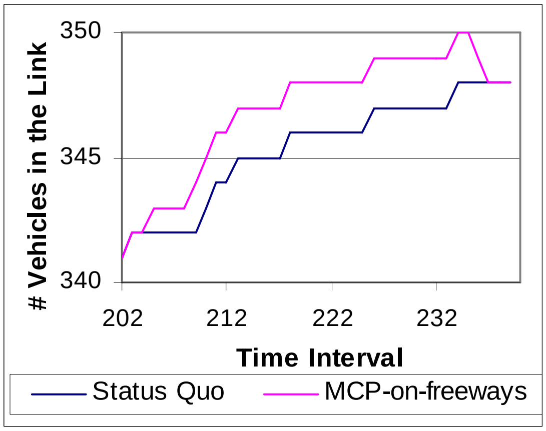
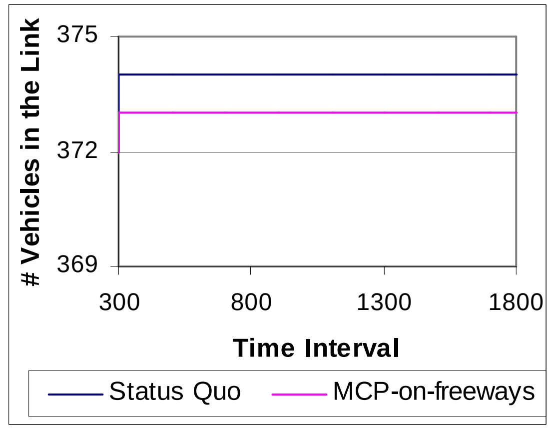


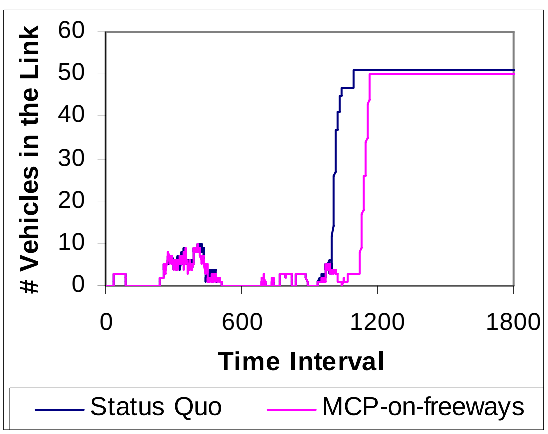
Figure 5. Time-Varying Traffic Density on Freeway Links

a) SH114 NB, Between Macarthur and W SH114 (Length = 1267.2 ft, MCP toll Rate Max = $0.27/mile) (Total Vehicles = 626114 for Status Quo, 624675 for MCP-on-freeways, Capacity = 6450 vph)





b) US287 SB, Near Wise CO LIN (Length = 3273.6 ft, MCP Toll Rate Max = $0.39/mile) (Total Vehicles = 41861 for Status Quo, 35109 for MCP-on-freeways, Capacity = 4600 vph)



c) IH30 WB, Near GALLOWAY (Length = 2112 ft, MCP Toll Rate Max = $0.04/mile) (Total Vehicles = 815561 for Status Quo, 798301 for MCP-on-freeways; Capacity = 4300 vph)