WELFARE ANALYSIS USING LOGSUM DIFFERENCES VS. RULE OF HALF: A SERIES OF CASE STUDIES

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ABSTRACT

Logsum differences and rule-of-half (RoH) calculations are two different methods to estimate consumer surplus in transport economics. As a traditional and relatively straightforward (and potentially more robust) procedure, RoH has been widely used in project investment and policy analysis, and much of the literature seems to agree that logsums are somewhat superior to the RoH when valuing user benefits- at least when the true travel behaviors stem from random-utility maximization with Gumbel error terms.

This paper explores the differences in both methods, through a careful review of literature and many case study results. The comparison of RoH and logsum methods relies on three specifications, in order of increasing complexity: binary logit, multinomial logit, and nested logit models, under a variety of settings/scenarios. This work offers a closer look at three numerical examples, and concludes that the difference between RoH and logsum solutions rises with increases in travel times or costs, and changes in parameters. The monetized differences in logsums is usually smaller than RoH solution for welfare changes under most situations, and gives a more exact result for consumer surplus than RoH (which assumes a linear demand relationship with respect to cost); Larger coefficients on affected variables (like travel time and cost) in the random-utility expressions tend to increase differences between logsum- and RoH-based estimates. Such findings should be of interest to policy-makers and planners when developing transportation planning and land use models and interpreting their results, for more accurate and rigorous and behaviorally defensible project evaluations.

Key Words: logsum differences, rule-of-half, consumer surplus, travel demand modeling, user benefits analysis

INTRODUCTION

As a traditional procedure for calculating changes in user benefits (CS, consumer surplus), the Rule of Half (RoH) has been widely used in transportation project investment, policy analysis, and operations (e.g., tolling decisions) (see, e.g., De Raad 2004, Geurs et al. 2010, Brunton 2012). This method assumes a linear demand function, to create a trapezoid (including a rectangle and a triangle) for generalized cost savings or losses for consumers of a good (like transport) following changes in costs (with travel time effects monetized), as shown in Figure 1. The area of the trapezoid is the increment of consumer benefits and suitable for the RoH method (De Jong et al. 2005, Brunton 2012). In fact, CS is based on an uncompensated or Marshallian demand curve, while compensating variation (CV) and equivalent variation (EV) represent areas under compensated (Hicksian) demand curves (see, e.g., Varian [1992]).

McFadden's (1978, 1981) logsum differences are based on random utility maximization (RUM) assumptions (with Gumbel-type error terms), and used to estimate user benefits and losses, when their travel (or other) context changes. In this method, travel demand is estimated as a result of each individual's choice context (e.g., travel time and cost) changes, and the monetized differences in all individuals' logsum values characterize the change in consumer surplus. Binary logit (BL), multinomial logit (MNL), and nested logit (NL) models are generally used to determine the shares of modes, and/or other choice alternatives.

This paper investigates the differences in estimating user benefits based on RoH versus logsum measures, via a review of the literature and an examination of three progressively more complex applications, using BL, MNL and NL specifications (under a series of settings or scenarios). Existing literature helps illustrate how user benefits under both methods vary by circumstance, but does not explain when and why these differences occur, what parameters or variables impact these differences most, and whether the three specification contexts (using BL, MNL and NL models) exhibit similar differences in outcomes. This paper addresses each of these questions through examples and related discussions. The work begins with literature review, followed by a description of methods and model specifications, case studies, and key findings.

LITERATURE SYNTHESIS

Several studies have investigated the theoretical issues involved in logsum formulations. McFadden (1978) outlined the mathematical formulations of the RUM choice model and welfare functions. Ben-Akiva and Lerman (1979) noted that the value of maximum utility increases with choice set size and average utility of each alternative. McFadden (1996) found that "the expected utility change is bounded by the averages of these utility changes per alternative, weighted by the original (lower) and final choice probabilities (upper bound)," while Herriges and Kling (1999) used real data and three methods (a simulation procedure, an approximation based on a representative consumer approach, and some bounds on the true value of the surplus), to assess consumer surplus in preference settings that are nonlinear in income. Karlstrom (2000) and Daly

(2004) identified conditions for when logsums are appropriate, the foremost of which requires the constant marginal utility of money in the generalized extreme value (GEV) model[1].

Applications using logsum differences as an evaluation measure have been conducted in Europe, the U.S., and many other countries, for policy and investment decisions in the areas of land use, congestion pricing of roadways, housing location and traffic analysis. For example, the EXPEDITE Consortium (2002) studied the combined effects of an increase in car operating costs and reductions in train and bus/tram/metro costs to illustrate the effects of policy measures. Odeck et al. (2003) used logsums to estimate the relative magnitude of impacts across socio-economic groups under Oslo's cordon toll, based on changes in generalized costs. Castiglione et al. (2003) used San Francisco's activity-based model and logsum differences to estimate user benefits based on changes in travel costs and induced travel. Gullipali and Kockelman (2008), Gupta et al. (2006), and Kalmanje and Kockelman (2005) used logsum differences to evaluate the impact of credit-based congestion pricing in Texas.

The US DOT (2004) compared results of integrated travel demand-land use models to those using demand models only, and used Small and Rosen' approach (1981) to measure consumer benefit (also known as compensating variation, CV). The authors wondered whether consumer surplus measures for travel demand shifts are still valid when land use demands shift. Essentially, travelers can offset some negative system effects or exploit transport system improvements by moving their home origins, resulting in different (hopefully less negative) welfare implications, but land prices also can change to offset travel benefits, resulting in higher rents. Ma and Kockelman (2014) have a new investigation on such impacts.

Finally, Geurs et al.'s (2010) evaluations of Netherland's data (to anticipate climate change impacts and evaluate potential land-use strategies) suggest that logsum differences help value benefits from changes in trip production and destination utility, which may be quite large and are not measured using the RoH (since RoH assumes that all accessibility benefits accruing to economic agents are attributable to generalized cost changes within the transport system). In this case, logsum and RoH accessibility benefits from the additional road-investment package are quite different (e.g., \$148 versus \$247 million per year, across 1,000 persons), but on the same order-of-magnitude. In contrast, their differences across the different land-use scenarios were very far apart (\$27M versus \$697M per year), suggesting that more welfare-characterization research recognizing land use's welfare impacts may be needed. Most recently, Delle Site and Salucci (2013) proposed welfare calculation methods in the presence of before-after correlations (of the error terms in choice-related utilities), and their example delivered a close correspondence in logsum differences versus RoH values (i.e., 16.41 vs. 16.46 euros per month).

Logsum differences come from RUM behaviors, and BL, MNL and NL behavioral specifications are used to anticipate demand changes in most of the literature surveyed here. However, other choice behaviors may dominate. To investigate this idea, Chorus (2010) evaluated route choices under variable travel time, congestion levels, crash exposure, and travel costs, using both RRM (Random Regret Minimization) and RUM (Random Utility Maximization) bases for the MNL specification. He relied on stated preference survey data to compute the logsums which were

[1] McFadden (1978) noted that, "A random-utility model in which the utilities of the alternatives have independent extreme value distributions yields the Luce (MNL) model. Considering non-independent extreme value distributions leads to the generalized extreme value (GEV) models".

then compared to survey responses (regarding willingness to pay), with only weak correlations found.

Kockelman and Lemp (2011) illustrated four-level NL logsum methods (two destinations, three modes, three times of day, and two routes) to equilibrate a toy network's travel times and choices, and then estimate class-specific user benefits across eight scenarios. They found that road pricing can reduce congestion levels while producing significant and largely positive consumer surplus benefits, though no direct comparisons with RoH valuations were conducted. Brunton (2012) proposed a BL-based example to estimate user benefits in a logsum setting by improving bus transit through decreased travel times, comparing outcomes with RoH estimates, while noting that logsum give a more exact result for consumer surplus than the RoH (which assumes a linear demand relationship).

Koopmans and Kroes (2004) and De Raad (2004) compared logsum-based estimates of CS with traditional vehicle-hours-lost (VHL) values, and found logsum method give a higher benefits and increase less rapidly with increasing congestion level than the traditional VHL method. De Jong[2] et al.'s (2005) comprehensive survey of logsum and RoH comparison results concluded that traditional RoH evaluations should be replaced by logsum differences, to account for non-linear demand assumptions. Brunton's (2012) example drew a similar conclusion. However, each of these conclusions were based on singular specific scenario examples (e.g., combined project impacts from bus improvements, enhanced capacity, and road toll policy were not evaluated simultaneously in combination in any of the scenarios).

Although several works suggest that logsum differences are better than the RoH when valuing user benefits (De Jong et al. 2005; Kockelman and Lemp 2011; Brunton 2012), they also recommend further testing, for more confidence in the details of such results. The literature appears somewhat mixed regarding when the RoH method should closely track logsum differences, and when it should give substantially different results. The following sections describe such comparisons.

METHODOLOGY: Using Rule of Half to Estimate User Benefits

RoH is one traditional measurement for calculating consumer surplus in transport economics. This method assumes that the consumer demand (in this case, transport demand) curve is linear with respect to generalized costs, at least within the changing context between original and new scenarios. As shown in Figure 1, when generalized cost changes from GC^0 to GC^1, travel demand in the form of person-trips is assumed to respond accordingly by changing from T^0 to T^1. Therefore, the change in consumer surplus (ΔCS) is denoted by the shaded area of the trapezoid.

In accordance with Figure 1's illustration, ΔCS can be computed as follows:

$$\Delta CS = \frac{1}{2}(T^1 + T^0)(GC^1 - GC^0) \tag{1}$$

where CS denotes consumer surplus, T^0 and T^1 are, respectively, the transportation demand before and after a change in scenario context (e.g., tolling changes and/or capacity additions), and GC^0 and GC^1 are the generalized costs before and after the change, respectively.

[2] This is a survey article, describing much of the research and many applications using logsum methods before the year 2004.

Using logsum to estimate user benefits (consumer surplus)

The purpose of measuring consumer surplus change is usually to evaluate the social welfare implications resulting from a particular policy or project (De Jong et al. 2005). Since consumer surplus is usually associated with a set of alternatives, when using a logit model with RUM assumption, the change in consumer surplus is calculated as the difference between the expected consumer surplus E(CS_n) before and after the change in context (or across scenarios). This procedure relies on the indirect utility of choice alternatives, and is formulated as follows:

$$\Delta E(CS_n) = (1/\alpha_n)[\ln(\sum_i e^{V^1{}_{ni}}) - \ln(\sum_i e^{V^0{}_{ni}})],\ \forall\ n,\ i \qquad (2)$$

where superscript 0 and 1 refer to before and after the change, α_n represents the marginal utility of income for person *n*, and can also be expressed dU_n/dY_n (assumed to be constant in subsequent case studies investigated here), where Y_n is the income of person *n*, $U_n$is the overall utility for person n, V_n is the indirect utility for person *n*, and *i* denotes the choice alternatives available to person *n* . Therefore, U_{ni} is the overall utility for person *n* choosing alternative *i*, and V_{ni} denotes the systematic or representative utility for person *n* choosing alternative *i.*

This procedure also determines the probabilities that a given person will choose each of the alternatives by using a logit model. These probabilities are estimated by evaluating alternative characteristics in order to assess an indirect utility associated with that alternative. In a MNL model, it is expressed using the following formula:

$$P_i = e^{V_i} / \sum_{i=1}^{K} e^{V_i} \qquad (3)$$

where P_i is the probability of a traveler choosing alternative *i* from alternative choice set K; and V_i is the indirect utility of alternative *i*, which is usually a linear function of the attributions of mode *i* that describe its attractiveness. When using a BL model, the only difference is that the choice set contains just two alternatives. NL models are more complicated than BL and MNL model specifications due to the nested structure and typically greater number of alternatives. However, Equation 3 is the basis of NL within-nest choice decisions, and therefore this equation still governs much of the NL's behavior.

CASE STUDIES

In order to fully appreciate the differences between consumer surplus impacts when using logsum and RoH methodologies to estimate user benefits, three broad model categories (BL, MNL and NL models) were investigated here, with several settings or scenarios explored for each model.

RoH and Logsum Valuation Comparisons using a Binary Logit Model

Brunton (2012) developed a case study assuming 1,000 people traveling from point A to point B. The journey was assumed to take 18 minutes by bus and 10 minutes by car, with assumed VOTTs at $12/hour. Under this scenario, user benefits were calculated from potential decreases in bus travel time using both RoH and logsum methodologies, assuming a BL model. In order to comprehensively assess the difference when using logsum and RoH methodologies, improvements in bus travel times (from 1 to 17 minutes) were investigated here, with outcomes shown in Table 1**.**

The BL model's basic specification and user benefits changes (using logsum methodologies) are expressed as follows:

$$P_i = \exp(\lambda V_i) \Big/ \sum_i \exp(\lambda V_i) = \exp(\lambda GC_i) \Big/ \sum_i \exp(\lambda GC_i) \quad (4)$$

$$\Delta E(CS_n) = (1/\lambda)[\ln(\sum_i e^{\lambda \cdot GC^1{}_i}) - \ln(\sum_i e^{\lambda \cdot GC^0{}_i})] \quad (5)$$

where P_i, V_i, $\Delta E(CS_n)$ are the same meanings as above, superscript 0 and 1 refer to before and after the bus travel time improvements, *GC* is generalized cost (in cents), and λ is scaled parameter (assumed here to be -0.03, as in Brunton's example).

Figure 2 (-a1, -b1, and –c1) illustrates the differences between RoH and logsum calculations, along with estimated bus travel shares as bus travel times fall (-a2, -b2, and –c2), all else equal.

Figure 2-a2, shows how the share of bus users increases non-linearly as bus travel times fall, with its graph effectively representing a demand curve (and rotated 90 degrees). The point at which logsum and RoH valuations become equal as bus travel times fall is highlighted by the green dotted line that crosses the curve. This point is critical when comparing RoH- or logsum-based benefits (Figure 2-a1). Before this point, the differences between RoH and logsum methodologies present a trend of small-large-small (with a maximum difference of $178.20 per day, or 43.8% in the two valuations [for the 1,000 travelers]) - when bus and car travel times are equal), until reaching Figure 2-a's green dotted line (the point at which bus travel time is 8 minutes less than car travel time, the reverse of the initial scenario). In addition, the logsum benefits curve is lower than the RoH benefits curve, meaning the benefits calculated using logsum differences are lower than those calculated using the RoH. After the inflection point the opposite is true, and differences between the two methodologies become larger again, with the logsum benefits curve higher than the RoH benefits curve. Readers should note that under these circumstances bus travel time is less than 20% of car travel time, an unlikely scenario. However, the binary logit model is structured such that the same results would be obtained for identical bus travel time reductions explored here, even if the initial travel times were 28 and 20 minutes, respectively, for bus and car travel, given that the scale parameter was unchanged.

Various λ are investigated here, to determine parameters' effects on the RoH and logsum values, and their differences (Figure 2). When $\lambda = -0.01$ (Figure 2b), the share of bus users almost linear with respect to change in the travel time, and user benefits calculated by RoH and logsum differences are almost identical (with maximum variations between the two of just 4.9% or $32 per day, total across 1000 travelers). When $\lambda = -0.05$ (Figure 2c), the situation is similar to $\lambda=-0.03$, except that the differences between RoH and logsum valuations is even larger (with maximum variations growing to 66.8% or $126, over the 1000 affected travelers). Scale parameters (λs) with values of -0.001, -0.005, -0.1 are also investigated here, as shown in Table 1.

From these results, we can draw the following conclusions:

1. If bus percentage grows approximately linearly with decreasing travel times, the benefits calculated by RoH and logsum differences will be very close.

2. Under most circumstances, RoH-calculated benefits are larger than those calculated using logsums, though in extreme cases (like in the very low bus travel times BL scenario), RoH methods may result in smaller user benefits than logsum differences. This should generally hold true when persons shift from one high-use alternative to a lower-use alternative, as the costs of the second alternative fall.
3. Figures 2a, 2b, and 2c show the same trend of the differences between RoH and logsum valuations, and the differences grow as λ increases in magnitude. When λ lies near zero (e.g., λ = -0.001 to -0.002), the ratios of logsum/RoH approach 1.0, meaning that the benefits calculated by RoH and logsums are almost the same. When λ grows in magnitude (e.g., λ = -0.1), the differences between RoH and logsums become much more substantial.

RoH and Logsum Valuation Comparisons using a Multinomial Logit Model

Equation 3 noted previously shows the formula used to estimate the probability that a traveler would select a given mode when applying a MNL model. Among this equation, indirect utility, V_i, is generally estimated as a linear function. Here, Equation 6 shows one common expression for indirect utility (from NCHRP Report 365 [Martin and McGuckin, 1998]):.

$$V_i = a_i + b_i \times IVVT_i + c_i \times OVVT_i + d_i \times COST_i \quad (6)$$

where $IVTT_i$ represents the in-vehicle travel time of mode i (in minutes), $OVTT_i$ represents the out-of-vehicle travel time of mode i (include walk, wait and transfer times, in minutes), $COST_i$ denotes the out-of-pocket cost of mode i (in dollars), a_i, b_i, c_i, and d_i are all constant coefficients.

Assume that there are 3 modes (Car, Bus, Metro) travelers can choose when they travel from origin O to destination D, where the distance between O and D is 15 miles. Bus and Metro speeds are assumed to be the same as the Car speed (50 mph), however, flat 20 minute and 15 minute penalties are added to Bus and Metro times respectively, to represent their added wait, access, and egress times. Further, bus fare is set at \$0.50 per trip, metro fare is set at \$2 per trip, and a fixed \$0.20/mile Car operating cost is assumed[3], with a \$1.0 parking fee per trip. Therefore, the total Bus travel time is 38 minutes (IVTT 18 min and OVTT 20 min), with \$0.50 out of pocket costs; the total Metro travel time is 33 minutes (IVTT 18 min and OVTT 15 min), with \$2.00 out of pocket costs; and total Car travel time is 18 minutes (only the IVTT), with \$4.00 out of pocket costs. Alternative specific constants (a_i) are assumed to be 0.0 for Car, -1.8 for Bus, and -2.0 for Metro, with b_i = -0.025, c_i = -0.050, and d_i = -0.004 (Martin and McGuckin, 1998). An average income of \$35,000/year, 2080 working-hours/year, and a value of time equal to 25% of income were also assumed, resulting in a $VOTT$ = \$16.80/hour.

10,000 travelers are assumed here, with no appreciable congestion or bus capacity limitations. That is, the available roadway capacity is large enough such that travel speeds are not impacted, and travelers who shift to the Bus or Metro modes can always find a space. In this scenario, the probabilities of a traveler selecting each mode are calculated, with the Car mode share (0.72) being the largest, due to its relatively high utility. Additionally, four other scenarios are investigated to illustrate the differences in estimated user benefits compared to the base case scenario: Scenario 1 decreases Bus wait times, from the current 18 minutes to just 2-minute waits; Scenario 2 simultaneously decreases Bus and Metro wait times, by 2 minutes each across 6

[3] Kockelman and Lemp (2011, p. 828) assumed \$0.20/mile operating costs, noting that it is "less than the American Automobile Association (AAA 2006) recognizes for full-cost accounting of vehicle ownership and use but about 35% more than current gas costs, assuming a 20 mi/gallon vehicle".

progressive reductions; Scenario 3 increases Car operating costs from $1 to $8, and Scenario 4 simultaneously decreases Bus and Metro out-of-pocket costs from the present fares to free rides. As previously noted, logsum-estimated user benefits are calculated using Equation 5.Table 2 shows how decreasing Bus OVTTs reduce overall travel times and increase bus shares. There are slight differences between the benefits evaluated using logsum and RoH methodologies, and the RoH is a little larger than the Logsums (Scenario 1). The other three scenarios present similar situations, with just slight differences between logsum and RoH methodology outcomes. This being noted, the magnitude of these differences grows larger with greater changes from the base-case scenario.

b_i, c_i and d_i parameter values were also changed from -0.025, -0.050 and -0.004, to -0.05, -0.1 and -0.008, respectively. The results show similar trends (larger parameter values result in greater differences between logsum and RoH valuations), which is largely due the impacts of travel cost growing.

RoH and Logsum Valuation Comparisons Using a Nested Logit Model

To illuminate the user benefit differences estimated by RoH and logsum methodologies using a NL model, an example of multiple alternatives for travel between a single origin and two destinations is proposed here. The alternatives include the choice of destination (A versus B), mode (Auto, Bus, or Walk), and route(1, 2). Figure 3 shows the overall nesting structure of mode-choice NL model.

This scenario reflects a configuration similar to that used in Kockelman and Lemp (2011), with two destination options (A and B) available to each user. Destination A is a location close to the origin (1 mile) while destination B is much farther away (8 miles), though the "attractiveness" (e.g., the natural log of jobs) of Destination B is much more than that of Destination A (i.e., 200 versus 10). Also, Destination A may be reached using motorized modes at just 10 mph in contrast to average speeds of 60 mph to reach Destination B. In the base-case scenario, both routes to Destination B are identical, non-tolled, and free of congestion. Bus and Auto speeds are assumed to be the same; however, a flat 15-minute penalty is added to the Bus time to represent added wait, access, and egress times. Walk is only available to Destination A, with an assumed speed of 4.47 mph. Furthermore, a fixed $0.50 per-trip Bus fare is assumed, along with a $0.20/mile Auto operating cost. Alternative specific constants (ASC_m) are assumed to be 0.0 for Auto, -1.1 for Bus, and -1.3 for Walk (as discussed in Kockelman and Lemp [2011]). 10,000 trip-makers with $12/hour VOTTs were assumed to be traveling, and able to choose either destination, any of the modes, and either of the two routes (when traveling to the further destination).

It is important to discuss the scale parameters (which are the inverse of the inclusive value coefficients, and reflect the degree of substitution that occurs between nested alternatives versus alternatives outside the nest) in each level of the nested model. As shown in Figure 3, scale parameters of 1.6 in the lowest nest (μ_1 for driving to Destination B via Route 1 or Route 2), 1.4 in the next lowest nest (μ_2 for Walk versus Bus versus Auto mode), and 1.2 in the upper level nest (μ_3 for Destination A versus Destination B) were assumed[4]. The greater the scale parameter,

[4] Kockelman and Lemp (2011,p. 830) developed a 4-layer (destination-mode-TOD-route) NL model, scale parameters (μ_1, μ_2, μ_3, μ_4) from the lowest level nest to the highest level nest were assumed as 1.8, 1.6, 1.4 and 1.2, with their order consistent with random utility maximization (Ben-Akiva and Lerman 1985).

the greater the substitutability among nested alternatives, versus other alternatives. Then, the associated equations, for generalized trip costs, systematic utilities, inclusive values of the nested choices and choice probabilities are as follows:

$$\mathrm{GC}_{\mathrm{dmr}} = VOTT \cdot OVTT_{\mathrm{dmr}} + VOTT \cdot \mathrm{IVTT}_{\mathrm{dmr}} + COST_{\mathrm{dmr}} \tag{7}$$

$$\mathrm{V}_{\mathrm{dmr}} = \left[\ln(attr_{d}) - \ln(attr_{B})\right] + \mathrm{A}SC_{\mathrm{m}} - \mathrm{GC}_{\mathrm{dmr}} \tag{8}$$

$$\Gamma_{\mathrm{dm}} = \frac{1}{\mu_1} \ln[\exp(\mu_1 \cdot V_{\mathrm{dm,\ route1}}) + \exp(\mu_1 \cdot V_{\mathrm{dm,\ route2}})] \tag{9}$$

$$\Gamma_{\mathrm{d}} = \frac{1}{\mu_2} \ln[\exp(\mu_2 \cdot V_{\mathrm{d,Auto}}) + \exp(\mu_2 \cdot V_{\mathrm{d,Bus}}) + \exp(\mu_3 \cdot V_{\mathrm{d,Walk}})] \tag{10}$$

$$\mathrm{Pr}_{\mathrm{d}} = \frac{\exp(\mu_3 \cdot \Gamma_{\mathrm{d}})}{\sum_{j \in \mathrm{D}} \exp(\mu_3 \cdot \Gamma_{j})} \tag{11}$$

$$\mathrm{Pr}_{\mathrm{dm}} = \mathrm{Pr}_{\mathrm{d}} \cdot \frac{\exp(\mu_2 \cdot \Gamma_{\mathrm{dm}})}{\sum_{\mathrm{j} \in \mathrm{M}} \exp(\mu_2 \cdot \Gamma_{\mathrm{dj}})} \tag{12}$$

$$\mathrm{Pr}_{\mathrm{dmr}} = \mathrm{Pr}_{\mathrm{dm}} \cdot \frac{\exp(\mu_1 \cdot \mathrm{V}_{\mathrm{dmr}})}{\sum_{\mathrm{j} \in \mathrm{R}} \exp(\mu_1 \cdot \mathrm{V}_{\mathrm{dmj}})} \tag{13}$$

Here, *GC* is the generalized cost, *V* stands for systematic utility of the alternative (as measured in dollars), Γ denotes the inclusive value or expected maximum utility for an upper level alternative, $\Pr(\cdot)$ represents the probability of a particular choice, *d*, *m* and *r* denote Destination (A, B), Mode (Auto, Bus, Walk) and Route (Route1, Route2). *VOTT* denotes the value of travel time, μ_1, μ_2, and μ_3 are scale parameters for the Route, Mode, and Destination, respectively, *COST* represents the out-of-pocket travel costs (include fare, toll and operating cost) and has no coefficient (so that utilities are in dollars), *IVTT* and *OVTT* denote the travel time spent in and out of the vehicle, *attr* characterizes the "attractiveness" of each destinations (and $attr_B$ is the "attractiveness" of destination B), and ASC_m represents the mode-specific (alternative-specific) constants.

Consumer surplus change estimates (ΔCS) for each scenario were also computed. The ΔCS computation using normalized logsums of systematic utilities are estimated as follows:

$$\Delta \mathrm{C}S = \frac{1}{\mu_3} \{\ln[\sum_{\mathrm{d} \in \mathrm{D}} \exp(\mu_3 \Gamma^1{}_{d}) - \ln[\sum_{\mathrm{d} \in \mathrm{D}} \exp(\mu_3 \Gamma^0{}_{\mathrm{d}})]\} \tag{14}$$

While ΔCS can be measured between any two scenarios, this investigation examines changes in consumer surplus relative to the base-case scenario.

The base-case scenario assumes two identical, congestion-free, non-tolled routes to Destination B, with scenario summary results (including destination, mode and route choice probabilities) shown in Table 3. Then, In order to compare the differences between user benefits using RoH and logsum methodologies when relying on a NL model, six distinctive alternative scenarios are investigated. Scenario 1 assesses flat tolls on one of the routes to Destination B at rates varying between $0.10 and $0.50 per mile. Scenario 2 explores the impacts of VOTT based on a fix toll-rate ($0.20 per mile). Scenario 3 evaluates the impacts of varying operating speeds (from 20 mph to 80 mph, Route 2 to Destination B), reflecting potential roadway facility upgrades with higher speeds or worsening overall congestion with lower speeds. Scenario 4 changes bus wait times reflecting policies that increase or decrease the level-of-service for public transit, while Scenario 5 alters bus fares. Scenario 6 varies auto operating costs, reflecting changing gasoline prices and parking fees.

Each scenario assumes 10,000 persons who want to travel (to Destination A or B), and with user benefits compared across scenarios (relative to base-case scenario), using RoH and logsum methodologies, with results shown in Table 4.

Results show that the benefits calculated when using logsum versus RoH methodologies differ more substantially with travel time changes, and the differences become more significant in Scenario 1 when a route is tolled. When Autos are tolled on Route 2 to Destination B in Scenario 1, the difference between the two become larger with increased toll-rates, with logsums valuations typically smaller than RoH valuations. In Scenario 2, when changing the VOTT from $1/hr to $12/hr, the logsum/RoH ratio rises from 0.761 to 0.822, suggesting that higher values of time may lead to more consistent results in logsum and RoH valuations. In Scenario 3,when altering the speed from 20 mph to 80 mph on Route 2 to Destination B, the logsum/ RoH ratios rise, and they are greater than/less than 1.0 when the base-case speed is greater than/less than 60 mph. The user benefits evaluated using logsum and RoH methodologies are closer when travel speeds change modestly, from 50 mph and 70 mph, rather than more dramatically. In Scenario 4, the logsum/RoH ratios fall as the Bus OVTT change increases. In Scenarios 5 and 6 (which vary Bus fares and Auto operating costs), there are only slight differences between the two methods for calculating user benefits.

Analysis of the 3 Cases

In analyzing the results of these 3 cases and their associated scenarios, the following conclusions can be drawn:

1. As the magnitude of parameters and variables in utility expression grow, the percentile and absolute differences between logsum and RoH valuations become larger.
2. With slight changes in travel time, travel cost and other variables, the percentile and absolute differences between logsum and RoH valuations are very small; but these differences grow as the alternative scenario increasingly diverges from the base-case scenario. Also, when travel demand is a near-linear function of travel time and other variables, the user benefits calculated using RoH and logsum methodologies are close.
3. Under most circumstances, the magnitude of the impacts (including negative user benefit valuations) calculated using RoH are larger than using logsum differences. However, in some instances RoH may be smaller than Logsum differences (e.g., when the changes in travel cost are very large, compared to base scenarios).

These differences between estimated logsum and RoH valuations may be further illustrated by simultaneously comparing all scenarios and modeling results, providing a useful and quick-reference framework for transport planners, managers and decision-makers. Table 5 shows how travel time, travel cost, tolls and other parameters influence the differences between RoH and logsum valuations, as input variables change in sign and magnitude. These input changes reflect potential transport policies, projects, and/or management decisions, and may serve as a reference for future planning and decision-making efforts.

CONCLUSIONS

Much work currently exists on examining logsum differences to estimate potential user benefits from various policies or projects, but existing comparisons between RoH and logsum benefits is largely lacking, particularly in the context of a comprehensive comparison evaluating the impacts of changing travel times, travel costs, added tolls and other important parameters aspects. This paper uses case studies to analyze and summarize the differences between RoH and logsum valuations as a measure of user benefits. As shown using model types, the ratio between logsum and RoH valuations varies on scenario context and the degree to which input parameters change. The tollway scenario illustrates this phenomenon, as the difference in estimated benefits when using these two methodologies becomes larger as toll rates grow. This implies the RoH method may sometimes over-estimate the effects of a given policy, especially when the change is significant compared to the base-case scenario. In these three cases, the differences between RoH and logsum valuations when using the MNL model appear to be smaller than when using either BL or NL models. This is mainly due to the changes in probabilities of each alternative are almost linear or near-linear in MNL model compared in BL and NL models. While this last conclusion is indicated by assessing results from these three specific cases, it is possible that it may not hold under all circumstances, so further research may be needed.

Results also indicate that when the transport demand is a linear function, the ratio between logsum and RoH valuations is close to 1, though transport demand usually exhibits nonlinear trends. As such, it is recommended that policy-makers estimate user benefits using logsum valuations when travel time or travel cost impacts are anticipated to be large, since logsums are more accurate than traveler welfare valuations estimated using the RoH methodology.

Of course, the analysis provided here illustrates only a limited number of idealized scenarios under three governing model formulations. Many other potential explorations and scenario extensions exist, which could further highlight key issues involved in these differences. For example, other evaluations could examine if multiple inputs simultaneously change (for example, travel times under varying congestion levels and toll prices). In addition, when a given route is tolled, the entire transportation network may be impacted, potentially further influencing differences in logsum and RoH valuations. One could also explore other underlying model structures to generate the demand functions, and investigate which method is more robust to other behavioral assumptions. Linear demand functions are likely to favor the RoH, which was at a disadvantage here (thanks to starting off with a random-utility logit-based model for all choice behaviors).

In summary, the comparison evaluating the differences between logsum and RoH valuations should help transportation planners and policy-makers understand how the choice of evaluation methodology will influence and potentially bias the expected benefits from a given project or policy. When relatively minimal impacts are expected to the overall underlying generalized cost of a given choice alternative, the two methods are roughly equivalent. However, when changes in such costs are expected to be substantial, there is a strong chance that using the RoH methodology may produce a substantially mis-estimated result, potentially overestimating or underestimating benefits by up to half.

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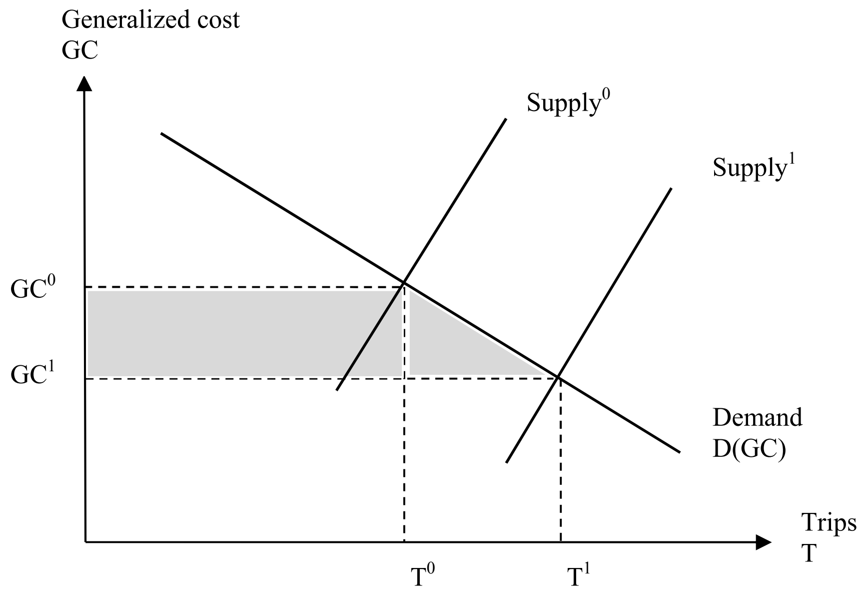
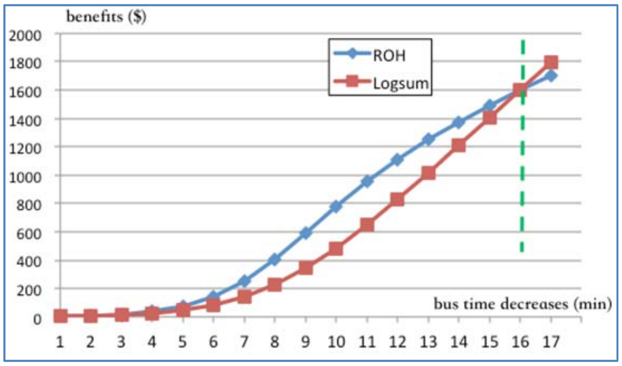


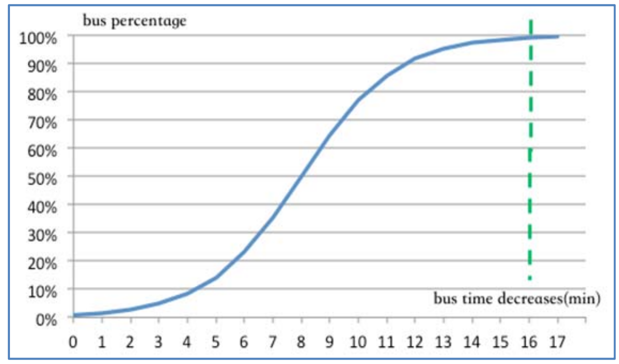
Figure1. Using rule-of-half (RoH) to estimate user benefits

Table 1 User benefits calculated using RoH and logsum methods, with bus travel time falling (BL model specification)

λ = -0.03																			
Modes	Base scenario		Bus travel time decreases (min)																
	car	bus	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Travel time (min)	10	18	17	16	15	14	13	12	11	**10**	9	8	7	6	5	4	3	**2**	1
Cost (cents)	200	360	340	320	300	280	260	240	220	**200**	180	160	140	120	100	80	60	**40**	20
Probability	0.992	0.008	0.008	0.015	0.027	0.047	0.083	0.141	0.231	**0.354**	0.500	0.646	0.769	0.858	0.917	0.953	0.973	**0.985**	0.992
Benefits ($)	RoH		2.3	7.0	16.7	36.5	75.0	143.8	253.8	**406.5**	588.4	776.7	952.9	1110.0	1249.0	1374.2	1490.1	**1600.0**	1706.2
	Logsum		2.2	6.3	13.5	26.2	48.3	85.0	143.1	**228.3**	343.1	485.0	648.3	826.2	1013.5	1206.3	1402.2	**1600.0**	1798.8
Logsum/RoH ratio			0.972	0.900	0.807	0.718	0.643	0.591	0.564	**0.562**	0.583	0.624	0.680	0.744	0.811	0.878	0.941	**1.000**	1.054
λ = -0.01																			
RoH			36.6	79.9	131.1	191.2	261.2	341.6	432.7	534.4	646.0	766.7	895.0	1029.5	1168.8	1311.1	1455.2	1600.0	1744.4
Logsum			36.5	79.4	129.4	187.2	253.6	329.1	414.2	509.2	614.2	729.1	853.6	987.2	1129.4	1279.4	1436.5	1600.0	1769.1
Logsum/RoH ratio			0.998	0.994	0.987	0.979	0.971	0.964	0.957	0.953	0.951	0.951	0.954	0.959	0.966	0.976	0.987	1.000	1.014
λ = -0.05																			
RoH			0.1	0.6	2.1	7.3	23.9	71.7	188.5	400.3	658.3	881.1	1048.2	1178.8	1291.7	1397.0	1499.1	1600.0	1700.4
Logsum			0.1	0.4	1.3	3.6	9.7	25.3	62.6	138.6	262.6	425.3	609.7	803.6	1001.3	1200.4	1400.1	1600.0	1800.0
Logsum/RoH ratio			0.924	0.762	0.605	0.486	0.404	0.353	0.332	0.346	0.399	0.483	0.582	0.682	0.775	0.859	0.934	1.000	1.059
λ = -0.001, -0.002, -0.005, -0.1																			
logsum/RoH ratio	λ = **-0.001**		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	λ = **-0.002**		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	λ = **-0.005**		1.000	0.999	0.998	0.998	0.997	0.996	0.995	0.994	0.993	0.993	0.993	0.994	0.995	0.996	0.998	1.000	1.003
	λ = **-0.1**		0.762	0.482	0.332	0.250	0.200	0.168	0.152	0.173	0.268	0.409	0.547	0.667	0.769	0.857	0.933	1.000	1.059

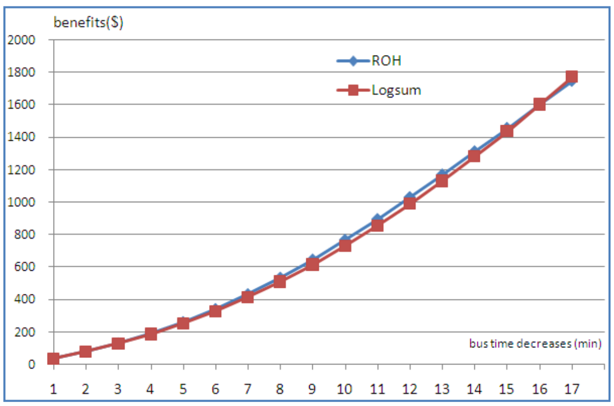
Note: These benefits are calculated for 1,000 people traveling from point A to point B for one trip. Benefits in Figures 2a, 2b, and 2c share this same basis.

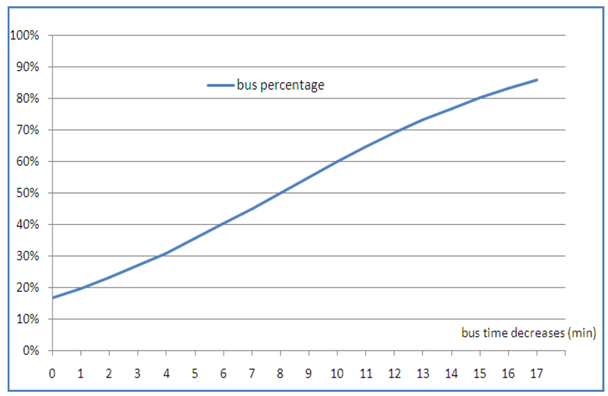




a1 **a2**

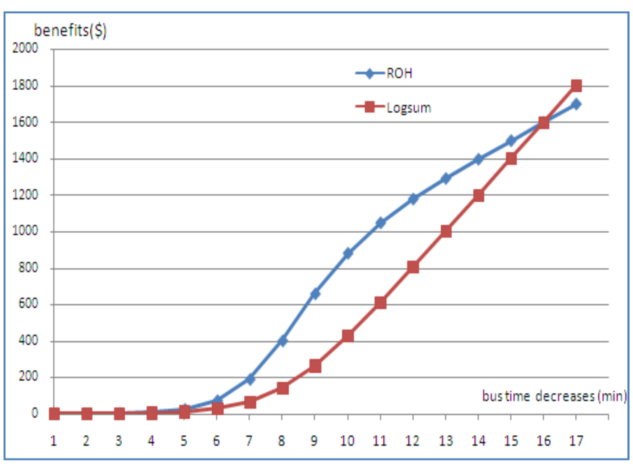
(a) λ = -0.03

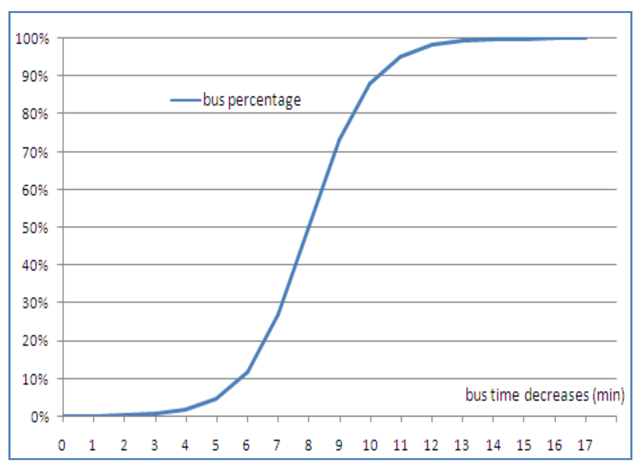




b1 **b2**

(b) λ = -0.01





c1 **c2**

(c) λ = -0.05

Figure 2. Total user benefits (per day, across 1000 travelers) calculated using RoH and logsum methods and bus share changes following reductions in bus travel times

Table 2. User benefits calculated using RoH and logsum methods, assuming MNL model behavior

Scenario 1: OVTT of Bus decreases										
OVTT (minutes)		18	16	14	12	10	8	6	4	2
Probability	Car	0.707	0.693	0.678	0.662	0.646	0.628	0.61	0.591	0.572
	Bus	0.193	0.209	0.226	0.244	0.263	0.282	0.303	0.325	0.347
	Metro	0.101	0.099	0.096	0.094	0.092	0.089	0.087	0.084	0.081
Logsum ($)		46.3	96.4	150.7	209.3	272.5	340.6	413.8	492.2	576.1
RoH ($)		46.3	96.6	151.2	210.6	275.0	344.9	420.5	502.1	589.9
Logsum/RoH ratio		1.000	0.998	0.996	0.994	0.991	0.988	0.984	0.980	0.977

Scenario 2: OVTTs of Bus and Metro both fall								
Bus OVTT (min)		18	16	14	12	10	8	6
Metro OVTT (min)		13	11	9	7	5	3	1
Probability	Car	0.699	0.678	0.656	0.633	0.609	0.585	0.561
	Bus	0.191	0.204	0.218	0.233	0.248	0.263	0.279
	Metro	0.11	0.118	0.126	0.134	0.143	0.152	0.161
Logsum ($)		72.6	150.4	233.6	322.5	417.3	517.9	624.6
RoH ($)		72.6	150.5	234.1	323.6	419.2	521.0	629.3
Logsum/RoH ratio		1.000	0.999	0.998	0.997	0.995	0.994	0.993

Scenario 3: Auto cost increases									
Dollar increase		+$1	+$2	+$3	+$4	+$5	+$6	+$7	+$8
Probability	Car	0.633	0.536	0.437	0.342	0.258	0.189	0.135	0.095
	Bus	0.233	0.294	0.357	0.417	0.47	0.514	0.548	0.574
	Metro	0.134	0.17	0.206	0.241	0.271	0.297	0.316	0.331
logsum		-677.5	-1262.5	-1748.7	-2137.1	-2435.9	-2658.3	-2819.3	-2933.3
RoH		-676.4	-1256.1	-1734.7	-2123.5	-2445.5	-2727.5	-2993.3	-3259.5
Logsum/RoH ratio		1.002	1.005	1.008	1.006	0.996	0.975	0.942	0.900

Scenario 4: Bus and Metro fares both fall						
Fare Reduction (%)		20%	40%	60%	80%	100%
Probability	Car	0.702	0.684	0.663	0.641	0.618
	Bus	0.18	0.183	0.184	0.186	0.186
	Metro	0.117	0.134	0.153	0.173	0.196
RoH		61.8	130.1	205.7	289.2	381.5
Logsum		61.8	130.6	207.2	293.0	389.1
Logsum/RoH ratio		0.999	0.996	0.992	0.987	0.981

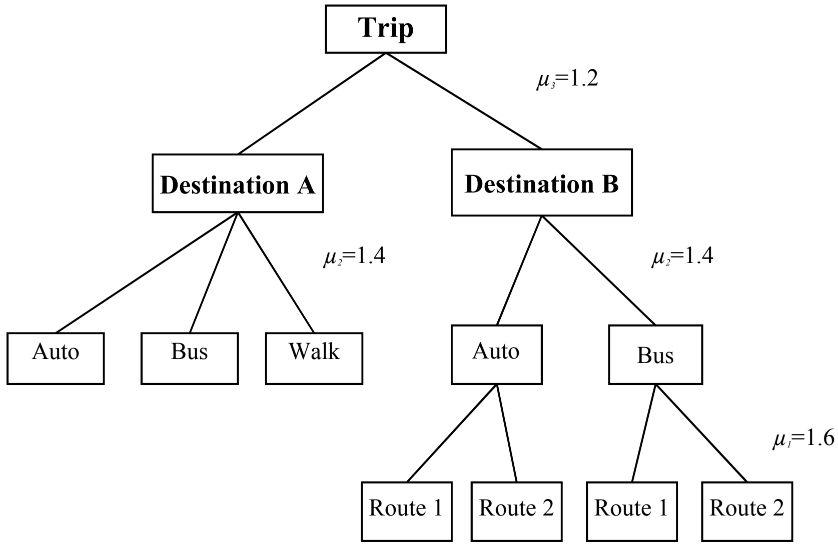


Figure 3. Mode-choice NL model structure

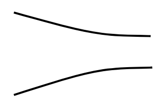
Table 3. Original settings and calculated probabilities for the base-case scenario (NL model)

Destination	Mode	Route	Travel time (minutes)		Travel cost ($)			Choice probability		
			IVTT	OVTT	Fare	Opera. cost	Toll	Pr_d	Pr_{dm}	Pr_{dmr}
A	Bus		6	15	0.5	0	0	0.125	0.0003	0.0003
	Auto		6	0	0	0.2	0		0.122	0.122
	Walk		0	13.4	0	0	0		0.0033	0.0033
B	Bus	Route 1	8	15	0.5	0	0	0.875	0.0148	0.0074
		Route 2	8	15	0.5	0	0			0.0074
	Auto	Route 1	8	0	0	1.6	0		0.860	0.430
		Route 2	8	0	0	1.6	0			0.430

Table 4. User benefits calculated using RoH and logsum methodologies using an NL model ($)

Scenario 1	**Route 2 to Destination B, where Auto is tolled**										
	Toll (cent/mile)	10 ct/mi.	20	30	40	50	10	20	30	40	50
	VOTT	$12/hour					$6/hour				
	Logsum	-2344.5	-3202.2	-3464.7	-3539.8	-3560.8	-2123.5	-2881	-3110.4	-3175.8	-3194.1
	RoH	-2427.9	-3895.2	-5358.6	-6954.8	-8626.3	-2212.6	-3564.3	-4921.3	-6395.9	-7936.4
	Logsum/RoH ratio	**0.965**	**0.822**	**0.646**	**0.509**	**0.413**	**0.959**	**0.808**	**0.632**	**0.496**	**0.402**
Scenario 2	**Route 2 to Destination B, where Auto is tolled at a fixed rate (20¢/mile) while considering various VOTTs**										
	VOTT($/hr)	1	2	3	4	5	7	8	9	10	11
	Logsum	-1741.6	-2030.4	-2298.6	-2533.3	-2727.8	-2996.4	-3079.3	-3136.1	-3172.4	-3193.1
	RoH	-2288.8	-2626.9	-2931.6	-3191.2	-3401.5	-3685	-3770.8	-3828.9	-3865.7	-3886.5
	Logsum/RoH ratio	0.761	0.773	0.784	0.794	0.802	0.813	0.817	0.819	0.821	0.822
Scenario 3	**Route 2 to Destination B, with speed variations (Base scenario is 60 mph)**										
	Route2 Speed	20	30	40	50	*70*	*80*	*90*			
	Logsum	-3619.9	-3272.8	-2392.7	-1209.2	*1100.6*	*2057.9*	is not realistic			
	RoH	-7075.6	-3967.1	-2474.8	-1212.7	*1100.9*	*2056.4*				
	Logsum/RoH ratio	**0.512**	**0.824**	**0.966**	**0.997**	**1.00**	**1.001**				
Scenario 4	**Changed Bus OVTT times (on wait, access, and egress times)**										
	Added time	-60% (6 min)	-40% (9 min)	-20% (12 min)	0	+20% (18 min)	+40% (21 min)	+60% (24 min)			
	Logsum	1138.1	456.6	140.8	---	-61.7	-88.5	-100.1			
	RoH	1544.9	536.5	145.5	---	-63.7	-105.8	-144.4			
	Logsum/RoH ratio	**0.737**	**0.851**	**0.968**	**---**	**0.965**	**0.836**	**0.693**			
Scenario 5	**Changed Bus fare (decrease and increase)**										
	Decrease/Increase	-100%	-80%	-60%	-40%	-20%	+20%	+40%	+60%	+80%	+100%
	Logsum	108.7	80.7	56.2	34.8	16.2	-14.1	-26.4	-37.2	-46.5	-54.6
	RoH	110.6	81.0	55.8	34.4	15.9	-13.9	-26.1	-36.9	-46.7	-55.6
	Logsum/ RoH ratio	**0.983**	**0.996**	**1.006**	**1.014**	**1.018**	**1.018**	**1.014**	**1.006**	**0.995**	**0.981**
Scenario 6	**Change Auto (car) operating costs (decrease and increase)**										
	Decrease/Increase	-20%	-40%	-60%	-80%	+20%	+40%	+60%	+80%	+100%	
	Logsum	2856.2	5804.9	8822.8	11891.7	-2734.9	-5315.4	-7705.9	-9873.0	-11790	
	RoH	2847.6	5765.6	8726.5	11711.4	-2732.1	-5292.2	-7627.1	-9692.8	-11468	
	Logsum/RoH ratio	**1.003**	**1.007**	**1.011**	**1.015**	**1.001**	**1.004**	**1.010**	**1.019**	**1.028**	

Table 5. Summary of the comparation of RoH and logsum methodologies

Logsum/RoH	Parameter	Travel Time	Travel Cost	Toll	Curve of Logsum/RoH with Change Increase	Relevant Policy
BL model	λ = -0.001	1.0				
	λ = -0.005	0.993-1.003				
	λ = -0.03	**0.562-1.054**				public transport priority
	λ = -0.05	0.332-1.059				
	λ = -0.1	0.173-1.059				
MNL model	b_i = -0.05, c_i = -0.01, d_i = -0.008	0.980-0.999	0.506-1.003			speed control
	b_i = -0.025, c_i = -0.05, d_i = -0.004	0.977-1.0	0.900-1.008			toll or road pricing
	b_i = -0.0125, c_i =-0.025, d_i = -0.002	0.991-1.0	1.0-1.011			
NL model	$\mu_1 = 1.6$			0.402-0.965		travel mode cost change
	$\mu_2 = 1.4$	0.512-1.001				
	$\mu_3 = 1.2$		0.983-1.028			