



1 congestion of the road network is crucial, and numerous studies have been conducted so far.

2 Congestion pricing can be a possible solution to reduce traffic congestion. If the road is priced for its  
3 congestion, fewer drivers will use the tolled road and this will lead to improvement of travel conditions.  
4 However, in what degree to toll the road and how the toll should be measured is the key in congestion  
5 pricing techniques. In this paper, a congestion pricing model based on traffic conditions is suggested,  
6 reactive over time and congestion level.

7 Another possible solution to lower the congestion is improvement of the vehicle itself. In the future,  
8 automated vehicles will be driving with better performance and information processing ability than  
9 traditional human drivers. These automated vehicles will be driving in faster speed and more effectively,  
10 leading to lower congestion than the human drivers. Thus, the adoption of automated vehicles is also applied  
11 in the network. Combining the congestion pricing technique with the advent of automated vehicles, the  
12 effect of congestion pricing and future travel conditions with automated vehicles can be analyzed.

13

## 14 LITERUATURE REVIEW

15 Road congestion can be monetized to evaluate how much cost is imposed by its users. The cost of  
16 congestion is dependent on the number of trips made by the users, meaning a higher cost is imposed when  
17 trip frequency increases. There are two types of costs associated with congestion pricing. Average cost  
18 refers to the normative cost imposed by users, and marginal cost refers to the cost of adding an extra vehicle  
19 to the traffic stream. Thus, average cost is the expense users would expect to occur when using a road, while  
20 marginal cost is the social cost imposed by the users of the road. Although marginal cost should be covered  
21 by the network's users, they are usually unaware of or unwilling to take extra burden that exceeds the  
22 average cost they should pay. Thus, the difference between average cost and marginal cost of any number  
23 of trips represents the price of congestion for that number of trips. (Button, 2010; Yang & Huang, 1998)

24 As opposed to the relation between cost and number of trips, demand decreases with respect to the number  
25 of trips. It is obvious that fewer drivers would be willing to use a road that is frequently used by others.  
26 When the demand meets the marginal cost in point O at number of trips  $N_O$ , shown in Fig. 1, it would be  
27 the optimal point where the demand meets the cost of users. However, as users are unaware of or are  
28 unwilling to consider the marginal cost, actual number of trips appear in point A at number of trips  $N_A$ ,  
29 where the demand meets the average cost. When the number of trips is larger than its optimal value ( $N_O$ ),  
30 users benefit ( $N_A G$ ) is smaller than the cost ( $N_A M$ ) imposed by the users. Thus, the shaded area in Fig. 1 is  
31 the welfare loss caused by congestion. A smaller number of trips than  $N_O$  is also suboptimal, since the  
32 consumers' surplus would not be utilized completely in that case.

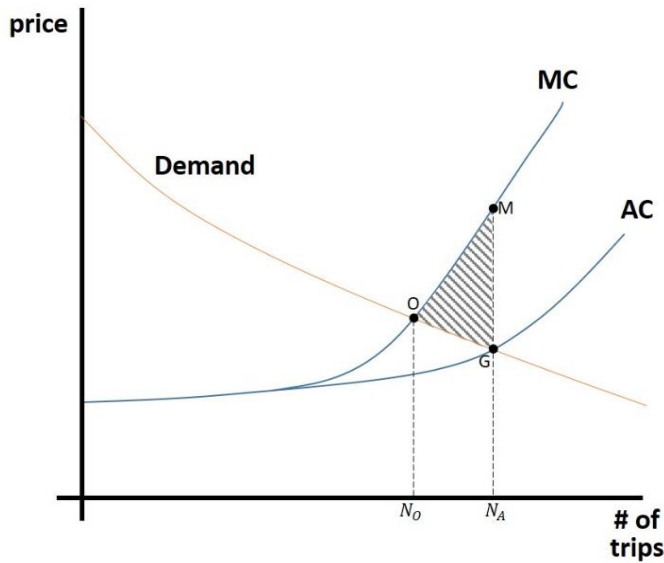
33 Congestion pricing and tolling strategies seek to find an optimal point between demand and supply, to  
34 determine how far a state or region is from reaching that point. Toll prices can be imposed on the users to  
35 fill in those gaps, such as the difference between marginal cost and average cost at a certain number of trips.  
36 A lot of research has been conducted so far, with each study providing distinct methods on how to determine  
37 the optimal point and fill in the gap between actual usage and optimal usage with the toll.

38 From an economic standpoint, a lot of congestion pricing research is based on determining the cost of  
39 congestion and how toll pricing can offset it. Cost functions, such as average cost and marginal cost, are  
40 generally used as objective variables and economic concepts, such as social welfare, revenue use, and  
41 monetizing externalities are discussed in this field. (Walters, 1961; Morrison, 1986; Newberry, 1990; Small,  
42 1992; Thompson, 1998)

43 An alternative approach to quantify the congestion is analyzing the utility to use the road (Kockelman &  
44 Lemp, 2011; Verhoef, 2005; Kunniyur et al., 2003; Liu et al., 2009). Users' utility is generally not  
45 observable but can be quantified by using values such as travel time. Congestion pricing strategies based

1 on utility concepts seek to optimize the utility of users who travel on the roads by deriving the toll price  
 2 that would maximize users' utility. The advantage of this method is that it can expand the discussion of  
 3 congestion into broader areas such as equity and the accountability of users since the unit to define utility  
 4 is not limited to monetary value. However, this can also be a disadvantage, since utility cannot be observed,  
 5 it cannot be directly applied in reality.

6



7

8 **FIGURE 1. Marginal Congestion Pricing at given Demand Level**

9

10 The last approach is more closely tied to traffic theories and focuses on optimizing traffic-oriented variables  
 11 such as throughput, high-occupancy toll (HOT) lane, and travel speed, by adopting tolling strategies. Thus,  
 12 the objective function is more microscopic as compared to other two approaches mentioned above. Traffic-  
 13 oriented characteristics can have advantages over other methods in optimizing the performance of tolling  
 14 from a traffic standpoint. However, other aspects that are not highly related to traffic, such as social  
 15 externalities, equity, and consumer surplus are usually difficult to analyze with this method. Examples of  
 16 this approach include controlling the travel speed and throughput to a certain level (Zhang et al., 2008), or  
 17 applying this technique to multi-lane situations (Yin et al., 2009).

18

19 **TRAFFIC ALGORITHM**

20 **Car following (Newell's Model)**

21 Car-following models determine the longitudinal acceleration of vehicles through the relationship between  
 22 the leader and follower vehicle. On a typical road, there are usually leaders driving in front of a follower,  
 23 and the acceleration/speed must be reactive to the leader's movement to prevent collision. Such a situation  
 24 is called 'car-following', and numerous researchers have developed their own car-following model to  
 25 describe this movement.

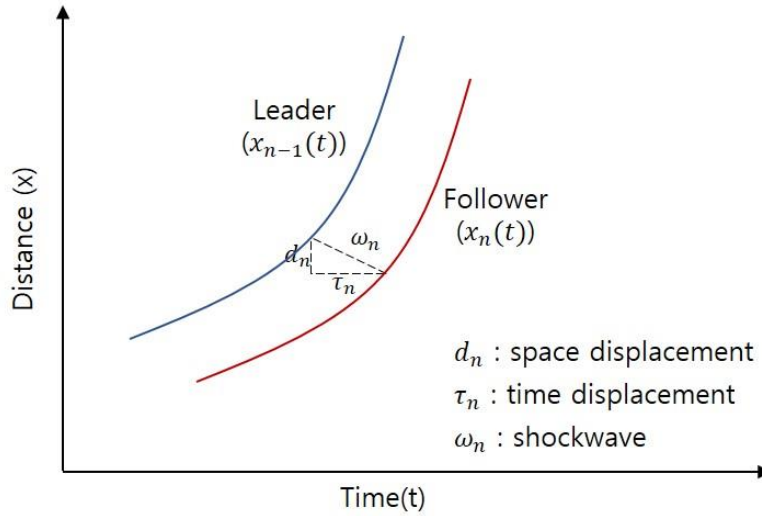
26 In this research, Newell's simplified car-following model (Newell, 2002) was used to describe the car-  
 27 following behavior of human drivers. Newell's model assumes that with a certain time displacement ( $\tau_n$ ),  
 28 a follower will maintain certain space displacement ( $d_n$ ) with the leader vehicle. These displacements of

1 time and space are different from the concept of headway and spacing. Rather, they represent that the  
 2 shockwave between the leader and the follower will be maintained constantly according to the follower's  
 3 characteristics. This can be modeled as Eq. (1).

4

$$5 \quad x_n(t + \tau_n) = x_{n-1}(t) - d_n \quad (1)$$

6



7

8

**FIGURE 2. Newell's Car-following Model**

9

10 This concept can be generalized using Eq. (2) and derive the speed of the follower at the next time step with  
 11 Eq. (3), where  $T_n$  is the time step. The time step  $T_n$  should be between 0 and  $\tau_n$ , but if the function is  
 12 smooth, it can be approximated to  $T_n = \tau_n/2$  according to mean value theorem (Newell, 2002). To  
 13 simplify the time step of the simulation,  $T_n$  is fixed to 1 so that  $\tau_n = 2$  is used. However, to apply  
 14 randomness to the simulation,  $d_n$  is selected randomly between 0 and 10m (32ft). Therefore, Newell's  
 15 model illustrates that every follower has its own distinctive following behavior to maintain a certain  
 16 shockwave ( $\omega_n$ ) with its leader.

17

$$18 \quad x_n(t + \tau_n) = x_n(t) + v_n(t + T_n)\tau_n = x_{n-1}(t) - d_n \quad (2)$$

$$19 \quad v_n(t + T_n) = \frac{x_{n-1}(t) - x_n(t) - d_n}{\tau_n} \quad (3)$$

20

## 21 Cooperative Adaptive Cruise Control (CACC)

22 Newell's car-following model is modeling the conventional drivers who are not supported with automation  
 23 or communication devices. However, drivers in the future will be assisted with automation during their  
 24 driving. In this paper, cooperative adaptive cruise control (CACC) is used to model these automated  
 25 vehicles. CACC means that the follower can maintain short spacing with the leader, since the CACC vehicle  
 26 is assisted with communication devices, information technologies, and advanced braking ability. A CACC  
 27 follower can measure the distance to the leader, while exchanging information with the leader, assuming it

1 is also a CACC vehicle. The CACC algorithm used in this paper is developed by Van Arem et al. (2006). It  
 2 determines the acceleration of the follower ( $a_{flw}$ ) by comparing the speed displacement with respect to  
 3 desired speed ( $a_{ref\_d}$ ) and the relationship between the leader and the follower ( $a_{ref\_r}$ ). Speed  
 4 displacement means the displacement of the follower's current speed and its desired speed. Thus, this can  
 5 be modeled as Eq. (4).

$$7 \quad a_{ref\_d} = \Psi(v_{des} - v_{flw}), \Psi = 0.3 \quad (4)$$

8 Where,  $v_{des}$ : desired speed (m/s) (33.3 m/s, 75mi/h)

9  $v_{flw}$ : current speed of the follower (m/s)

10  
 11 The relationship between the leader and the follower ( $a_{ref\_r}$ ) is more complicated since it must consider  
 12 the acceleration of the leader, as well as the relative speed and spacing between the leader and follower. It  
 13 is modeled as Eq. (5). The reference spacing ( $s_{ref}$ ) is the maximum value among safe following distance  
 14 ( $s_{safe}$ ), following distance according to system time setting ( $s_{system}$ ), and a minimum allowed distance  
 15 ( $s_{min}$ ), (Van Arem et al., 2006).

$$17 \quad a_{ref\_r} = \theta_1 a_{ld} + \theta_2(v_{ld} - v_{flw}) + \theta_3(s_{flw} - s_{ref}), \theta_1 = 1; \theta_2 = 0.58; \theta_3 = 0.1 \quad (5)$$

18 Where,  $a_{ld}$ : acceleration of the leader (m/s<sup>2</sup>)

19  $v_{ld}$ : speed of the leader (m/s)

$v_{flw}$ : speed of the follower (m/s)

20  $s_{flw}$ : spacing between the leader and the follower (m)

$s_{ref}$ : reference spacing (m)

21  
 22 The safe following distance ( $s_{safe}$ ) is computed with the speed of the follower, and the braking ability of  
 23 both the leader ( $d_{ld}$ ) and the follower ( $d_{flw}$ ). The following distance according to the system target time-  
 24 gap setting ( $s_{system}$ ) is the distance the follower has driven while maintaining a certain time gap with the  
 25 leader. This equation varies depending on whether the leader is a CACC vehicle, or a conventional vehicle.  
 26 If the leader is a CACC vehicle,  $t_{system}$  is set to 0.5 second, but otherwise,  $t_{system}$  is set to 1.4 second.  
 27 Thus, the reference spacing ( $s_{ref}$ ) required for Eq. (5) is derived from Eq. (6). Comparing the speed  
 28 displacement with respect to desired speed ( $a_{ref\_d}$ ) and the relationship between the leader and the follower  
 29 ( $a_{ref\_r}$ ), the acceleration of the follower ( $a_{flw}$ ) can be obtained with Eq. (7).

$$31 \quad s_{ref} = \max(s_{safe}, s_{system}, s_{min}) \quad (6)$$

$$32 \quad \text{Where, } s_{safe} = \frac{v_{flw}^2}{2} \left( \frac{1}{d_{ld}} - \frac{1}{d_{flw}} \right) \text{ (m)}$$

$$33 \quad s_{system} = t_{system} v_{flw}, t_{system} = 0.5s \text{ if leader is CACC, and } 1.4s \text{ otherwise (m)}$$

$$34 \quad s_{min} = 2 \text{ (m)}$$

$$35 \quad a_{flw} = \min(a_{ref\_d}, a_{ref\_r}) \text{ (m/s}^2\text{)} \quad (7)$$

1

## 2 **Free-flow Model**

3 The common feature of Newell's model and CACC model is that there is always a leader in front of the  
4 follower. Newell's model expects that the follower will maintain a certain wave speed with respect to the  
5 leader, while the CACC model defines that the follower will maintain safe distance through communication.  
6 However, vehicles can drive without having any leader in front of them, or at least have a distant leader that  
7 does not affect the movement of the follower at all. Both Newell's model and the CACC model cannot  
8 define the movement in such situations.

9 However, it is obvious that if there is no hindrance (leader vehicle) at all in front of them, drivers will  
10 choose the maximum acceleration and speed that are allowed. Thus, in such free flow situation, the drivers  
11 will choose maximum acceleration ( $a_{max}$ ) and they will desire to reach their maximum speed ( $v_{max}$ ). In  
12 this paper,  $3\text{m/s}^2$  ( $9.84\text{ft/s}^2$ ) is assumed for  $a_{max}$  and  $33.3\text{m/s}$  ( $75\text{mi/h}$ ) is assumed for  $v_{max}$ .

13 The determination of whether a vehicle should be driving in car-following mode or free-flow mode is  
14 achieved by comparing the spacing between the leader and follower, and the minimum stopping sight  
15 distance (MSSD) of the follower. If the spacing is larger than MSSD, the follower will be driving in free-  
16 flow mode with its maximum driving capacity. If the spacing is smaller than MSSD, the follower will be  
17 driving in either Newell's model or the CACC model depending on its vehicle type. The MSSD of a vehicle  
18 is derived with Eq. (8).

19

$$20 \text{ MSSD} = v_{flw} \text{ tr} + \frac{v_{flw}^2}{2g(\mu+\theta)} \quad (8)$$

21 Where, tr: driver's reaction time (2s assumed)

22 g: gravitational acceleration ( $9.8\text{m/s}^2$ )

23  $\mu$ : coefficient of friction (0.4 assumed)

24  $\theta$ : grade of the road (0% flat road assumed)

25

## 26 **CONGESTION PRICING**

### 27 **Value of Travel Time (VOTT)**

28 While drivers choose their travel mode, travel route, and partners to travel with, they have different values  
29 of travel time (VOTT). For example, an individual will have a much higher VOTT if they needed to catch  
30 a plane than if they planned to drive to an appointment with a friend. Thus, VOTT defines how the driver  
31 is willing to pay for a travel and it affects his/her overall travel choices including mode and route choice.

32 In this paper, two types of VOTT are introduced to adopt heterogeneity among drivers, \$15/hr and \$7/hr.  
33 The former represents the drivers with higher VOTT, while the latter represents drivers with lower VOTT.  
34 The drivers with higher VOTT might be willing to cover the cost of congestion pricing for access to the  
35 shorter path. Their high VOTT is an indication that they are less willing to withstand congestion and traffic  
36 delays than an average driver. However, the drivers with lower VOTT might be more willing to bypass  
37 through non-tolled, longer roads to avoid the congestion pricing.

38 For drivers in Newell's model, the percentage of each VOTT is set to be equal, 50% to 50%. For CACC  
39 drivers, the percentage of higher VOTT drivers will be larger than that of lower VOTT drivers. This is

1 because of the assumption that CACC vehicles are state-of-the-art, and drivers who would pursue such  
 2 advanced technology are looking to further reduce their travel time. According to Quarles et al., (2018), 64%  
 3 of survey respondents showed interest of owning an autonomous vehicle if they are affordable in the US.  
 4 Thus, 64% is used to represent the percentage of higher VOTT CACC drivers in the model. The rest of the  
 5 CACC drivers (36%) are lower VOTT CACC drivers.

## 7 Traffic Stream Model

8 Before defining the congestion pricing scheme, the traffic stream model (flow-density relationship) should  
 9 be defined first. Traffic stream models are used to interpret the relationship between density, flow, and speed.  
 10 In this paper, Greenshield's model as Eq. (9) is used in estimating the flow-density relationship of the road.  
 11 Greenshield's model was chosen because of its simplicity in calculation. This traffic stream model will be  
 12 used to compute the relevant congestion pricing required for the road conditions.

$$14 \quad v = v_f \left(1 - \frac{k}{k_j}\right) \quad (9)$$

$$15 \quad q = v_f \left(1 - \frac{k}{k_j}\right) k$$

16 Where, q: flow rate (veh/h)      v: speed (km/h)      k: density (veh/km)

17       $k_j$ : jam density (veh/km)       $v_f$ : free flow speed (speed limit, km/h)

## 19 Travel Time Estimation

20 The estimation of travel time is crucial for both route choice and toll pricing. Drivers will choose the route  
 21 with minimum travel time. Likewise, the toll price should increase if travel time is long, and it should  
 22 decrease, if travel time is short. The time required to drive a certain link of road is estimated with BPR  
 23 equation as in Eq. (10).

$$25 \quad T_i = t_{ff} \left(1 + \alpha \left(\frac{q}{c}\right)^\beta\right), \quad \alpha = 0.84; \quad \beta = 5.5 \quad (10)$$

26 Where,  $T_i$  = travel time of link  $i$  (min)

27       $t_{ff}$  = free flow travel time of link  $i$  (min)

28      q = throughput (veh/h)

29      c = capacity (veh/h)

31 In the original BPR equation, q variable should be the demand. However, in this microscopic simulation,  
 32 only throughput is observable, and demand is not. The travel time derived with throughput means only the  
 33 running time and waiting time or delay in intersections are not included. In this simulation, waiting time  
 34 means the time that drivers waited to enter the node of origin because of congestion. When the origin is  
 35 occupied by another driver, the driver scheduled to enter the road is relocated to a waiting zone and waits  
 36 until the node of origin is empty. The delay in intersection means the delayed time caused by signals or stop

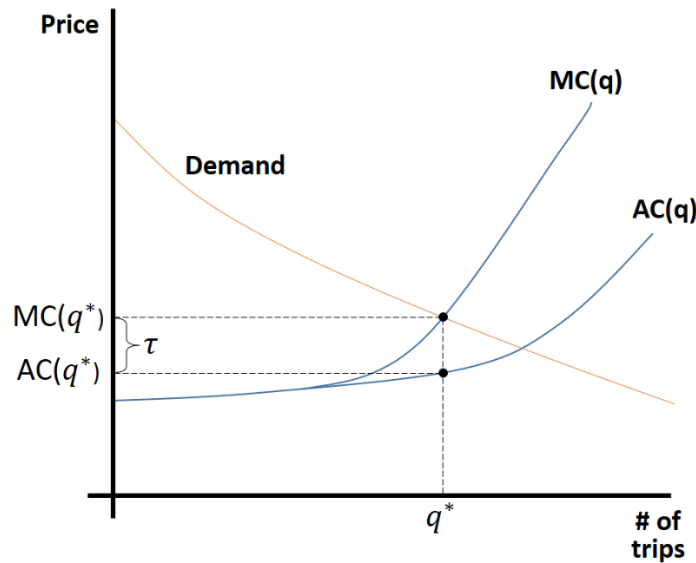
1 signs. This delay time and waiting time is added to the running time derived by the BPR equation to form  
 2 the travel time of a road.

3

4 **Congestion Pricing based on Traffic Conditions**

5 The developed congestion pricing scheme relies on traffic conditions that are derived from traffic stream  
 6 models. The traffic condition is converted to congestion pricing through the following equations developed  
 7 by Li (2002). It is a first-best tolling method that is derived from the point where demand is equal to  
 8 marginal cost. Here, the difference between marginal cost ( $MC(q^*)$ ) and the average cost ( $AC(q^*)$ ) is the  
 9 congestion pricing that should be imposed to the drivers. This can be shown as Fig. 3.

10



11

12 **FIGURE 3. First-best Tolling Method**

13

14 According to Li (2002), the average cost in each number of trips ( $AC(q)$ ) can be calculated with the  
 15 multiplication of VOTT and travel time, where travel time will be derived with distance and speed. The  
 16 total cost of the road ( $TC(q)$ ) imposed by every driver is calculated by multiplying the total flow by the  
 17 average cost. This can be calculated with Eq. (11). Through the traffic stream model section, it is well  
 18 known that flow rate and speed used in Eq. (11) has a special relationship. This paper uses Greenshield's  
 19 model, which can be used to calculate the marginal cost ( $MC(q)$ ), since it is developed through the  
 20 relationship between flow rate and speed in this paper.

21

22 
$$AC(q) = VOTT * \frac{d}{v} = \frac{VOTT}{v} \text{ (when } d=1\text{km, } 0.62\text{mi is assumed)}$$
 (11)

23 
$$TC(q) = q \frac{VOTT}{v}$$

24 Where,  $q$ : flow rate of the road in certain period

25  $v$ : average speed of vehicles in certain period



1 d: travel distance

2  
3 The marginal cost (MC(q)) is the derivative of the total cost with respect to the flow rate, which can be  
4 derived with Eq. (12). Through the calculation of marginal cost, the relationship between flow rate and  
5 speed should be used, where this paper uses Greenshield's model. Finally, the congestion pricing ( $\tau$ ) can be  
6 calculated by substituting marginal cost with average cost as seen in Eq. (13). Through Eq.(13), the  
7 congestion pricing of a given road can be derived based on traffic conditions.

8  
9 
$$MC(q) = \frac{dTC(q)}{dq} = \frac{VOTT}{v} - \frac{qVOTT}{v^2} \frac{dv}{dq} \quad (12)$$
  
10 
$$= \frac{v}{v^2} \frac{VOTT - q}{v} \frac{VOTT}{dq} = AC(q) - \frac{q}{v^2} \frac{VOTT}{dq} \frac{dv}{dq}$$

11 
$$\tau = MC(q) - AC(q) = -\frac{q}{v^2} \frac{VOTT}{dq} \frac{dv}{dq} = -\frac{VOTT}{v} \left( \frac{q}{v} \frac{dv}{dq} \right) \geq 0 \quad (13)$$

12 Where,  $\frac{q}{v} \frac{dv}{dq}$  represents the elasticity of the flow rate with respect to speed

13  
14 In Eq. (13), the elasticity of flow rate with respect to speed is the key to calculating the congestion pricing.  
15 According to Gang et al. (2005), it is possible that this elasticity can be derived with the use of traffic stream  
16 models. In this paper, Greenshield's model is used and Eq.(13) can be transformed to Eq.(14). The  
17 congestion pricing can be easily derived through Eq. (14), which is based on traffic conditions.

18  
19 If congested (when  $v \leq v_f/2$ )

20 
$$\tau = -\frac{VOTT}{v} \left( \frac{q}{v} \frac{dv}{dq} \right) = -\frac{VOTT}{v} \left( \frac{v_f - v}{v_f - 2v} \right) \quad (14)$$

21  
22 Gang et al. (2005) used this methodology to calculate the congestion pricing of Gangbyeon Expressway,  
23 Seoul, South Korea. However, their method was applied to a given traffic data set during a certain period,  
24 where the congestion pricing did not change according to time. This paper is willing to model reactive  
25 congestion pricing according to traffic condition and time interval. Thus, when the traffic conditions change  
26 by time, the congestion pricing scheme should change also.

27 This can be achieved by aggregating 0-5 minute travel time data with the BPR equation in Eq. (14), and  
28 estimate the average velocity required to calculate the toll from that 0-5 minute span. Based on the 0-5  
29 minute travel results, impose the congestion pricing scheme for the 5-10 minute interval. During this  
30 process, the 5-10 minute traffic data is aggregated, the average velocity for the 5-10 minute time interval is  
31 estimated, and the congestion pricing scheme required for the 10-15 minute interval is calculated. By  
32 repeating this process, reactive congestion pricing is decided based upon traffic conditions, which can be  
33 modeled with any time interval.

34 One trivial issue is when there is no traffic at all, since the estimation of velocity can be tricky. However,  
35 since there is no traffic, the travel time will be equal to free flow travel time, so travel speed will be equal  
36 to free flow speed. This results in zero congestion pricing, since there is no traffic at all and it is not  
37 congested. If at least one single vehicle is on the road, congestion can be measured, and a congestion pricing

1 scheme can be imposed that best represents the current traffic condition.

2

### 3 **Driver Heterogeneity with respect to VOTT**

4 The congestion pricing model of Gang et al. (2005) uses a single VOTT, so that drivers with different  
5 VOTTs cannot be modeled. Thus, every driver has equal VOTT and the dynamics related to variance in  
6 VOTTs are not modeled. In this paper, this limitation will be modified to apply the driver heterogeneity  
7 with respect to VOTT.

8

$$9 \quad q = q_1 + q_2 \quad (15)$$

10 Where,  $q$ : total flow rate regardless of VOTT

11  $q_1$ : sum of flow rate that has VOTT<sub>1</sub>

12  $q_2$ : sum of flow rate that has VOTT<sub>2</sub>

13

14 Heterogeneity is based on the conservation law requiring that the sum of flow rates with different VOTT is  
15 equal to the total flow rate regardless of VOTT. This conservation law can be written as Eq. (15). Thus,  
16 Eq.(11) through Eq.(14) can be modified to Eq.(16) through Eq.(19), and heterogeneity of drivers will be  
17 assumed in this paper.

18

$$19 \quad AC(q) = \frac{VOTT_1}{v_1} + \frac{VOTT_2}{v_2} \quad (16)$$

$$20 \quad TC(q) = q_1 \frac{VOTT_1}{v_1} + q_2 \frac{VOTT_2}{v_2}$$

21 Where,  $q_1$  &  $q_2$ : flow rate of the road w.r.t. VOTT <sub>$i$</sub>  ( $i=1$  or  $2$ )

22  $v_1$  &  $v_2$ : average speed of vehicles w.r.t. VOTT <sub>$i$</sub>  ( $i=1$  or  $2$ )

$$23 \quad MC(q) = \frac{dTC(q_1)}{dq_1} + \frac{dTC(q_2)}{dq_2} = \frac{VOTT_1}{v_1} - \frac{q_1 VOTT_1}{v_1^2} \frac{dv_1}{dq_1} + \frac{VOTT_2}{v_2} - \frac{q_2 VOTT_2}{v_2^2} \frac{dv_2}{dq_2} \quad (17)$$

$$24 \quad = AC(q) - \frac{q_1 VOTT_1}{v_1^2} \frac{dv_1}{dq_1} - \frac{q_2 VOTT_2}{v_2^2} \frac{dv_2}{dq_2}$$

$$25 \quad \tau = MC(q) - AC(q) = \sum_{i=1}^2 \left\{ -\frac{VOTT_i}{v_i} \left( \frac{q_i}{v_i} \frac{dv_i}{dq_i} \right) \right\} \geq 0 \quad (18)$$

$$26 \quad \tau = \sum_{i=1}^2 \left\{ -\frac{VOTT_i}{v_i} \left( \frac{v_f - v_i}{v_f - 2v_i} \right) \right\} \quad (19)$$

27

### 28 **Route Choice**

29 Drivers will select their route according to their VOTT, travel time, and the toll. From the beginning of his  
30 travel, the driver will compare these values and select his/her route that is most suitable to them. While  
31 driving along the route, he will not change his initial decision. Thus, the driver's route decision will be  
32 made by Eq. (20). The travel time of a given route will be calculated by dividing the distance by the average

1 speed of the last time interval.

2

3 Driver with  $VOTT_i$  will choose  $Route_x = \underset{x}{\operatorname{argmin}} (VOTT_i * travel\ time_x + toll_x)$  (20)

4

## 5 Summary of the Congestion Pricing Model

6 The drivers in this model will be either a conventional human driver or a CACC driver. The drivers will  
7 have two types of VOTT, one is higher and the other is lower. The likelihood of a high or low VOTT will  
8 vary depending on a driver's vehicle type. Conventional drivers will have equal probability of having higher  
9 or lower VOTT, while CACC drivers will have increased probability of having higher VOTT. The traffic  
10 conditions of the road will be estimated by the BPR equation considering waiting time and delay. The  
11 average speed derived from travel time will then be used to calculate the congestion pricing based on traffic  
12 conditions. Varying VOTTs will be applied during the estimation of congestion pricing.

13

## 14 SIMULATION ANALYSIS

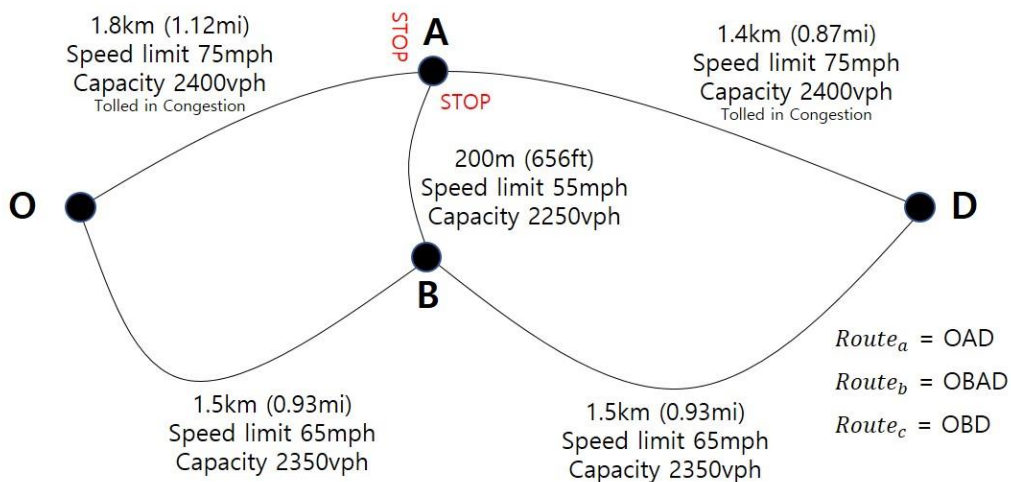
### 15 Network Design

16 The sample road network used in analysis is shown in Fig. 4. It has three routes that connects Origin (O)  
17 and Destination (D), which are  $Route_a$ ,  $Route_b$ , and  $Route_c$ . Only OA and AD segment will be tolled,  
18 thus  $Route_a$  will be fully tolled and  $Route_b$  will be partially tolled when the road is congested.

19 The drivers will select the route that has the lowest cost (VOTT\*travel time + toll). VOTT will vary for  
20 each driver, with travel time dependent upon the current traffic conditions, and the toll will be determined  
21 according to both VOTT and traffic conditions.

22 At the beginning when the road is not congested, most vehicles will be driving on  $Route_a$  since it is the  
23 shortest path. However, when  $Route_a$  gets congested, there will be congestion tolls on OA and AD, so  
24 that some drivers might select  $Route_b$  or  $Route_c$ . Thus, it is expected that higher VOTT drivers will  
25 choose  $Route_a$ , while lower VOTT drivers will choose  $Route_b$ , and  $Route_c$ .

26



27

1 **FIGURE 4. Road Network**

2

3 For the demand of the network, fixed, high values of demand will be imposed to make the network  
 4 congested. For 5-minute interval, the demand will change from 2200, 1400, 1300, 1000, 2000, 2000, 1600,  
 5 1800, 1300, and 1200 veh/h. The simulation will run for 1 hour and will be iterated 100 times to derive the  
 6 average result of the simulation.

7

8 **Simulation Results**

9 *Travel Speed, Travel Time and Toll with Respect to CACC Rate*

10 The travel speed of each route is calculated with the actual time individuals spent driving and the delay  
 11 time. Delay time includes the time drivers were waiting because of high congestion to enter the network,  
 12 and the time consumed while crossing a stop-signed road. According to Table 1, the travel speed of every  
 13 route increases with the increase of CACC rate, where CACC rate refers to the percentage of CACC vehicle  
 14 drivers in the simulation. This is reasonable since CACC vehicles have better performance than  
 15 conventional vehicles, meaning they can travel faster than conventional vehicles. Moreover, the increased  
 16 travel speed means that there is less congestion with the increase of the CACC rate. Thus, the average  
 17 congestion pricing imposed to the Route<sub>a</sub> decreases with respect to the CACC rate.

18 When comparing the tolled scenario and non-tolled scenario, travel speed of Route<sub>a</sub> is higher in the tolled  
 19 scenario, while the other two routes show lower speed in tolled scenarios. When tolling is present, fewer  
 20 drivers select Route<sub>a</sub> (fully tolled route), so that travel conditions of Route<sub>a</sub> is improved. However,  
 21 other routes experience more congestion from the tolling scenario, since drivers who could not afford the  
 22 congestion pricing will be using Route<sub>b</sub> and Route<sub>c</sub> more frequently than the non-tolled scenario.

23

24 **TABLE 1. Travel speed, Travel time and Toll with respect to CACC rate**

| Average value from 100 Iterations |                      | CACC 0% |         |         | CACC 50% |         |         | CACC 100% |         |         |
|-----------------------------------|----------------------|---------|---------|---------|----------|---------|---------|-----------|---------|---------|
|                                   |                      | Route a | Route b | Route c | Route a  | Route b | Route c | Route a   | Route b | Route c |
| Tolled Scenario                   | Average Toll (\$/mi) | 5.95    |         |         | 4.04     |         |         | 1.55      |         |         |
|                                   | Travel Speed (mi/h)  | 33.02   | 23.46   | 20.66   | 37.11    | 26.92   | 23.96   | 47.12     | 36.89   | 32.12   |
|                                   | Travel time (min)    | 3.61    | 4.92    | 5.41    | 3.22     | 4.29    | 4.67    | 2.53      | 3.13    | 3.48    |
| Not-tolled Scenario               | Travel Speed (mi/h)  | 31.09   | 28.21   | 27.01   | 34.70    | 31.77   | 31.30   | 45.24     | 42.45   | 38.54   |
|                                   | Travel time (min)    | 3.84    | 4.09    | 4.14    | 3.44     | 3.64    | 3.57    | 2.64      | 2.72    | 2.90    |

25

26 *Route Choice with respect to VOTT and CACC Rate*

27 In the beginning, Route<sub>a</sub> will be used mostly, since it is the shortest path with no congestion. However,

1 as the road becomes congested, the congestion toll will be high, and some drivers might select Route<sub>b</sub> or  
 2 Route<sub>c</sub>. This assumption has been justified in the simulation results.

3 Table 2 shows the result of each driver's route choice and the toll rate on each time interval (300s) of a  
 4 sample simulation run. The toll developed in this paper is reactive to the congestion and updates every 300  
 5 seconds. Thus, drivers will have varying route choice depending on the toll rate and the traffic conditions.  
 6 It shows a tendency of selecting Route<sub>b</sub> or Route<sub>c</sub> when there is high toll, and drivers select Route<sub>a</sub>  
 7 when the toll is low or there is no toll.

8 The CACC rate affects both the toll rate and the percentage of drivers selecting Route<sub>a</sub> (tolled road).  
 9 When the CACC rate increases, the toll rate decreases. Also, the proportion of drivers selecting Route<sub>a</sub>  
 10 increases with the CACC rate. This is expected to represent the effect of CACC's improved driving  
 11 performance. The congestion caused by CACC vehicles are generally low, meaning the toll rate is also low.  
 12 Since the toll rate is inexpensive, drivers are less reluctant to select a tolled route with the use of CACC  
 13 vehicles. However, CACC drivers have a higher likelihood to have increased VOTT (64%), so that they  
 14 will naturally select tolled roads more. Thus, the route choice among lower VOTT drivers should be  
 15 analyzed further to understand the relationship between route choice and CACC rate.

16

17 **TABLE 2. Route Choice with respect to CACC Rate**

| Time (s)    | CACC 0%          |                  |                  |              | CACC 50%         |                  |                  |              | CACC 100%        |                  |                  |              |
|-------------|------------------|------------------|------------------|--------------|------------------|------------------|------------------|--------------|------------------|------------------|------------------|--------------|
|             | Route a veh. (#) | Route b veh. (#) | Route c veh. (#) | Toll (\$/mi) | Route a veh. (#) | Route b veh. (#) | Route c veh. (#) | Toll (\$/mi) | Route a veh. (#) | Route b veh. (#) | Route c veh. (#) | Toll (\$/mi) |
| 300         | 125              | 0                | 1                | 0.00         | 132              | 0                | 0                | 0.00         | 150              | 0                | 1                | 0.00         |
| 600         | 2                | 3                | 124              | 13.21        | 1                | 14               | 117              | 15.31        | 6                | 28               | 120              | 6.99         |
| 900         | 44               | 10               | 102              | 3.65         | 27               | 45               | 68               | 3.07         | 63               | 11               | 34               | 0.00         |
| 1200        | 1                | 62               | 23               | 15.66        | 0                | 37               | 44               | 8.06         | 44               | 0                | 29               | 0.00         |
| 1500        | 49               | 7                | 93               | 0.00         | 70               | 3                | 82               | 0.00         | 83               | 1                | 77               | 0.00         |
| 1800        | 15               | 38               | 87               | 6.68         | 13               | 41               | 89               | 3.38         | 1                | 69               | 78               | 3.69         |
| 2100        | 67               | 2                | 96               | 0.00         | 78               | 3                | 86               | 0.00         | 79               | 10               | 70               | 0.00         |
| 2400        | 0                | 35               | 90               | 22.15        | 0                | 46               | 89               | 9.14         | 0                | 70               | 72               | 3.44         |
| 2700        | 49               | 9                | 97               | 1.03         | 45               | 11               | 65               | 0.00         | 56               | 1                | 44               | 0.00         |
| 3000        | 0                | 70               | 35               | 14.24        | 0                | 62               | 33               | 4.10         | 63               | 0                | 48               | 0.00         |
| 3300        | 0                | 0                | 0                | 0.00         | 0                | 0                | 0                | 0.00         | 0                | 0                | 0                | 0.00         |
| 3600        | 0                | 0                | 0                | 0.00         | 0                | 0                | 0                | 0.00         | 0                | 0                | 0                | 0.00         |
| Average (#) | 29.33            | 19.67            | 62.33            | 6.39         | 30.50            | 21.83            | 56.08            | 3.59         | 45.42            | 15.83            | 47.75            | 1.17         |
| Percent (%) | 26.3             | 17.7             | 56.0             |              | 27.4             | 19.6             | 50.4             |              | 40.8             | 14.2             | 42.9             |              |

18

19 According to Table 3, higher VOTT drivers generally choose Route<sub>a</sub> more than lower VOTT drivers. This  
 20 is because higher VOTT drivers have an increased willingness to pay for their travel, and thus are more  
 21 likely to select tolled road to avoid congestion and lower their travel time.

22 When the CACC rate increases, the drivers are more likely to select the tolled road. This nuance is not only  
 23 restricted to the higher VOTT drivers, but also results in lower VOTT drivers selecting Route<sub>a</sub> more.

1 Drivers' with lower VOTT would choose Route<sub>a</sub> more with respect to the increase of CACC rate. For  
 2 example, the likelihood of selecting Route<sub>a</sub> among the drivers with lower VOTT is 23.73% when there  
 3 are 0% CACC drivers in the simulation, while it increases to 40.17% when 100% of the drivers in the  
 4 simulation are CACC drivers. This is considered to be the effect of a low toll rate and low congestion with  
 5 an increase in the CACC rate. Thus, the lower VOTT drivers will benefit from the increases in CACC rate  
 6 by selecting Route<sub>a</sub> more, while higher VOTT drivers will also benefit from the reduced congestion and  
 7 travel time. Thus, it is reasonable to conclude that the increase in CACC rate will benefit most people  
 8 regardless of VOTT.

9 In the non-tolled scenario, drivers' selection of Route<sub>a</sub> is always higher than that of the tolled scenario.  
 10 This is because the fastest route is provided for free, so that drivers will choose that route more. On the  
 11 other hand, fewer drivers choose Route<sub>b</sub> and Route<sub>c</sub> in the non-tolled scenario compared to the tolled  
 12 scenario. This difference in route choice affected the travel speed and travel time result shown above. In  
 13 the tolled scenario, there are fewer drivers in Route<sub>a</sub> than the non-tolled scenario, so that the travel speed  
 14 is higher and travel time is lower compared to the non-tolled scenario. However, there are more drivers in  
 15 Route<sub>b</sub> and Route<sub>c</sub>, so that the travel speed is lower and travel time is higher compared to the non-tolled  
 16 scenario. Thus, tolling is affecting the drivers' route choice and travel conditions of the network.

17 The non-tolled scenario also shows a higher selection of Route<sub>a</sub> with the increase of CACC rate. This  
 18 means that CACC is lowering the congestion of the network, and drivers can use Route<sub>a</sub> more. For both  
 19 the tolled and non-tolled scenario, an increase of the CACC rate improves the traffic conditions of the  
 20 network.

21  
 22 **TABLE 3. VOTT and Route Choice with respect to CACC Rate**

| Average value from 100 Iterations |                             | CACC 0%       |               |               | CACC 50%      |               |               | CACC 100%     |               |               |
|-----------------------------------|-----------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|                                   |                             | Route a (%)   | Route b (%)   | Route c (%)   | Route a (%)   | Route b (%)   | Route c (%)   | Route a (%)   | Route b (%)   | Route c (%)   |
| Tolled Scenario                   | VOTT <sub>1</sub> (\$15/hr) | 13.21 (26.88) | 10.19 (20.74) | 25.74 (52.38) | 16.59 (29.25) | 11.13 (19.62) | 29 (51.13)    | 22.92 (36.11) | 13.62 (21.46) | 26.93 (42.43) |
|                                   | VOTT <sub>2</sub> (\$7/hr)  | 12.07 (23.73) | 10.56 (20.76) | 28.23 (55.51) | 12.71 (29.36) | 8.45 (19.52)  | 22.12 (51.12) | 14.67 (40.17) | 6.66 (18.22)  | 15.2 (41.61)  |
| Non-tolled Scenario               | VOTT <sub>1</sub> (\$15/hr) | 18.99 (38.93) | 1.97 (4.04)   | 27.82 (57.03) | 24.00 (44.94) | 3.63 (6.79)   | 25.77 (48.27) | 35.83 (56.51) | 1.95 (3.07)   | 25.62 (40.42) |
|                                   | VOTT <sub>2</sub> (\$7/hr)  | 19.38 (37.84) | 1.42 (2.77)   | 30.42 (59.39) | 20.22 (43.38) | 3.39 (7.28)   | 22.99 (49.34) | 19.39 (52.98) | 2.1 (5.74)    | 15.11 (41.28) |

23 \* ( ): percentage within same VOTT level

24  
 25 *Cost of Travel Time and Revenue with respect to CACC Rate*

26 In the above sections, tolling improves the travel conditions of Route<sub>a</sub>, but the travel conditions of the  
 27 other two routes have worsened. Thus, it can be ambiguous to judge whether the tolling strategy has  
 28 improved the overall travel condition of the network. If the benefits from tolling are greater than the  
 29 disadvantages experienced by the other two routes, it would be reasonable to conclude that toll collection  
 30 is advantageous to the network. This analysis is achieved by comparing the monetary cost of change in  
 31 travel time and considering the revenue collected from toll.

32 The cost of travel time for a single driver is derived by multiplying the VOTT and travel time of that driver.  
 33 This is the cost that the driver spent while traveling a certain route. For the tolled route, the price of the toll  
 34 is not included to compare the pure travel time change between the tolled and non-tolled scenario. The total  
 35 cost of travel time is the sum of every driver's cost of travel time. Revenue is derived from multiplying the

1 price of toll with the number of drivers who paid that toll.

2 According to Table 4, regardless of CACC rate, Route<sub>a</sub> experiences a decrease in travel cost when it is  
 3 tolled. This is highly related to the fact that the travel speed and travel time is improved when it is tolled  
 4 when compared to the non-tolled scenario. However, the other two routes have increases in travel cost,  
 5 since they result in increased travel time when tolled. The increase in travel cost of two routes is larger than  
 6 the reduction in travel cost of Route<sub>a</sub>, so that drivers might experience negative surplus from the tolling  
 7 strategy. However, the revenue earned from the tolling is larger than this negative surplus, so that the tolling  
 8 strategy produces a positive surplus in the network. If this surplus can be invested to improve the travel  
 9 conditions of Route<sub>b</sub> and Route<sub>c</sub>, or sometimes to Route<sub>a</sub>, most drivers in the network can benefit from  
 10 the tolling strategy.

11 On the other hand, an increase of the CACC rate reduces this surplus, such that CACC 0% has a \$675.6  
 12 surplus while CACC 100% has only a \$5.53 surplus. This is because CACC vehicles have better driving  
 13 performance, so that the congestion is low enough. In CACC 0% scenario, the travel cost gain of Route<sub>a</sub>  
 14 in a tolled scenario is \$29.79, while this increases to \$74.28 in the CACC 50% scenario. However, in the  
 15 CACC 100% scenario, there is only a minor difference when compared to CACC 50%. Since the existing  
 16 congestion is low enough, the travel time decrease in tolled roads is limited. However, in CACC 100%,  
 17 other two non-tolled routes experience the lowest negative impact. In this sense, drivers under CACC 100%  
 18 also experience a benefit from the tolling strategy, but the degree of benefit is the lowest since the congestion  
 19 can be managed from both CACC vehicles' performance and tolling strategy.

20 In summary, most drivers in the network can benefit from the tolling strategy developed in this paper, and  
 21 the increase of CACC vehicle improves the travel conditions of the network.

22

23 **TABLE 4. Cost of Travel Time and Revenue with respect to CACC Rate**

| CACC 0%   |        |   |        |  |
|---|--------|---|--------|--|
| Toll<br>Total cost of travel time (\$, w/o toll)<br>(A) |        | Not-tolled<br>Total cost of travel time (\$)<br>(B) |        | Total change in cost<br>of travel time (\$)<br>(C=B-A) |
| Route a   | 233.60 | Route a   | 263.39 | 29.79  |
| Route b   | 104.86 | Route b   | 22.83  | -82.03   |
| Route c   | 381.36 | Route c   | 312.80 | -68.56   |
| Revenue earned from toll (\$)<br>(D)                    |        |   |        | 796.40   |
| Revenue + Change in cost of travel time (\$)<br>(C+D)   |        |   |        | <b>675.6</b>   |
| CACC 50%  |        |   |        |  |
| Toll<br>Total cost of travel time (\$, w/o toll)<br>(A) |        | Not-tolled<br>Total cost of travel time (\$)<br>(B) |        | Total change in cost<br>of travel time (\$)<br>(C=B-A) |
| Route a   | 191.64 | Route a   | 265.92 | 74.28  |
| Route b   | 152.28 | Route b   | 7.41   | -144.87  |

|   |        |                                       |        |                             |
|---|--------|---------------------------------------|--------|-----------------------------|
| Route c   | 327.33 | Route c                               | 49.67  | -277.66                     |
| Revenue earned from toll (\$)                   |        |                                       |        | 615.38                      |
| (D)   |        |                                       |        |                             |
| Revenue + Change in cost of travel time (\$)    |        |                                       |        | 267.13                      |
| (C+D)   |        |                                       |        |                             |
| <b>CACC 100%</b>                                |        |                                       |        |                             |
| <b>Toll</b>                                     |        | <b>Not-tolled</b>                     |        | <b>Total change in cost</b> |
| <b>Total cost of travel time (\$, w/o toll)</b> |        | <b>Total cost of travel time (\$)</b> |        | <b>of travel time (\$)</b>  |
| <b>(A)</b>                                      |        | <b>(B)</b>                            |        | <b>(C=B-A)</b>              |
| Route a   | 195.90 | Route a                               | 273.67 | 77.77                       |
| Route b   | 106.57 | Route b                               | 27.56  | -79.01                      |
| Route c   | 240.24 | Route c                               | 208.72 | -31.52                      |
| Revenue earned from toll (\$)                   |        |                                       |        | 38.29                       |
| (D)   |        |                                       |        |                             |
| Revenue + Change in cost of travel time (\$)    |        |                                       |        | 5.53                        |
| (C+D)   |        |                                       |        |                             |

\* Results from average of 100 iterations

1

2

### 3 CONCLUSION

4 In this paper, a traffic-based congestion pricing model was developed. This model is reactive to the traffic  
5 condition and can be applied to real traffic conditions since it is a microscopic agent-based model. Also,  
6 this paper attempted to apply the developed model to automated vehicles (CACC) to predict future traffic  
7 flow. It is shown that with the increase of CACC vehicles, the travel speed increases and the toll rate  
8 decreases. Since toll rate represents the degree of congestion, the increase of CACC vehicles will improve  
9 the traffic conditions.

10 It is shown that the value of travel (VOTT) time plays an important role in route choice. Higher VOTT  
11 drivers tend to choose tolled roads more, compared to lower VOTT drivers. However, with the increase of  
12 CACC vehicles, drivers with lower VOTT also select tolled roads more than before so that the introduction  
13 of CACC will benefit drivers on both ends of the spectrum.

14 By comparing the cost and revenue of congestion pricing, it is evident that most drivers can benefit from  
15 the tolling strategy. Although the drivers in tolled routes enjoy the improved travel conditions, the drivers  
16 in the non-tolled route experiences greater congestion because of the tolling. However, the revenue earned  
17 from tolling is higher than the negative impact, so that the negative surplus can be compensated, resulting  
18 in a positive surplus. This strategy also holds when all vehicles are automated, but the degree of positive  
19 surplus decreases with respect to the increase of automated vehicles. Thus, this paper showed that tolling  
20 and automated vehicles both contribute to the improvement of traffic conditions.

### 21 AUTHOR CONTRIBUTION STATEMENT

22 The authors confirm contribution to the paper as follows: study conception and design: Jooyong Lee; data  
23 collection: Jooyong Lee; analysis and interpretation of results: Kara Kockelman, Jooyong Lee; draft  
24 manuscript preparation: Kara Kockelman, Jooyong Lee. All authors reviewed the results and approved the  
25 final version of the manuscript.



1    **ACKNOWLEDGEMENTS**

2    The authors are grateful for the funding support of the Texas Department of Transportation (under 18  
3    research project 0-6975) and for the valuable suggestions of Soon beom Kwon during the development of  
4    this research work, along with the editing and administrative support of Scott Schauer-West.

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