OPTIMIZATION OF VEHICLE ASSIGNMENT FOR SHARING VEHICLES

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ABSTRACT

This paper optimizes the assignment of shared automated vehicles under users' uncertain departure times. Automated vehicles can drive themselves so no staff are needed to relocate vehicles in the one-way carsharing system. To formulate the one-way vehicle sharing system with departure time uncertainties, a two-phase stochastic optimization model is established. Phase 1 decides the strategic planning of vehicles distributed at stations by using a system optimization approach, followed by Phase 2, which tracks vehicle movements via an agent-based simulation model. The optimization solutions in Phase 1 serve as inputs for Phase 2. A case study in the six-county Austin, Texas area is conducted to verify the proposed model and corresponding solution approach. Under the base case setting, a fleet of 8,564 automated vehicles is deployed in the study region. Optimization results suggest that system profits are optimized when vehicle rental is priced at $0.8 per mile. The number of proactive vehicle relocations drops by 9.8% if the relocation operation cost increases from $0.06 to $0.2 per mile. The profit of serving each trip is $10.20 when using high-cost vehicles, and $11.60 if using low-cost vehicles. Three-hour simulation results show an average person-trip length of 15.6 miles with 29.6-minutes of average driving time. If a 24-hour day is simulated, the vehicle-occupied time and vehicle-miles traveled are 4 hours and 125 miles per vehicle, respectively. The low coefficient of variation of satisfied demand across 30 demand scenarios demonstrates the robustness of the proposed stochastic optimization model.

Keywords: Shared autonomous vehicles, Vehicle assignment, Fleet size optimization

INTRODUCTION

Vehicle sharing is becoming popular, allowing users to reserve unoccupied vehicles from traditional vehicle rental services by the minute or hour, instead of by the day. Vehicle users may have more flexibility when choosing a destination to return their vehicle with reduced cost, and fleet owners may profit due to higher utilization of the vehicles in their fleet. An existing study shows that vehicle serving time will rise by about 30% (Huang et al., 2020a). It indicates that vehicles need to run half of the time during daily operation, i.e., 6-hour driving time during a 12-hour vehicle sharing system service time. Owing to these benefits, many traditional vehicle rental companies are beginning to provide hourly vehicle sharing services to expand their market (Hertz 24/7, 2021). Popular vehicle sharing operators include companies like Car2Go (SHARE NEW, 2021) and Shanghai's EVCARD (EVCARD, 2021).

There are two main types of vehicle sharing services, depending on the vehicle return restrictions. Round-trip vehicle sharing requires users to return vehicles to the original departure location. Such a service is less popular among the public, especially for commuting trips. Therefore, one-way vehicle sharing has been popular in recent years, and is now becoming a major vehicle sharing service. Users can pick up and drop off rental vehicles at any allowed parking space (Lu et al., 2018). One-way vehicle sharing can be called station-based vehicle sharing if available parking stations are predetermined; otherwise, it is often called free-float vehicle sharing, and any prepaid or free parking spaces can be used (Balac et al., 2019).

Existing studies mainly focus on two kinds of problems when designing a vehicle sharing system in a new vehicle rental market: strategic planning and operational decisions (Huang et al., 2018). Strategic planning makes long-term decisions that include fleet size, vehicle assignment, and station location and capacity. Operational decisions focus on real-time vehicle movements, relocations, and pricing. Strategic planning often serves as the foundation of operational decisions, while the network performance can be leveraged to adjust planning strategies through feedback. Therefore, in a bi-level optimization model that considers both strategic planning and operational decisions, strategic planning is often formulated as an upper-level model while the operation decisions are the lower-level.

One challenge of designing the vehicle sharing system is the vehicle assignment (or "vehicle initialization") at the predetermined stations at the beginning of daily operations. The vehicle assignment procedure determines the fleet size and the number of vehicles allocated to each parking station. An excessive number of vehicles deployed (or initialized) in traffic zones can lead to an underutilized vehicle fleet. However, if inadequate vehicles are deployed across the network, vehicle rental demand cannot be satisfied, leading to a loss of profits. Regarding this, an optimized vehicle assignment at the beginning of day can properly balance the vehicle use and avoid the two kinds of profit loss mentioned above. Existing studies often build mathematical optimization models that take the whole day's demand into account (Huang et al., 2020a; Xu et al., 2018; Xu and Meng, 2019). However, it is difficult to track and capture the detailed vehicle movements on road with such a model. Simulation models can fill this gap by recording detailed vehicle route choices and decisions (Illgen and Höck,2020). Even so, the time-varying travel demand cannot be well reflected in simulation models.

Furthermore, the case of different departure times across the day brings challenges to vehicle assignment, and has not been well explored. Most existing studies assume travel demand and departure time to be fixed based on historical or predicted data (Jorge et al., 2015). In terms of the uncertainty in travel demand, only a few papers explored the case when departure time of vehicles leaving one traffic zone follows a distribution function (Lu et al., 2018; He et al., 2017 &

2020).

This paper aims to address the research gaps in dealing with the vehicle assignment problem under departure time uncertainties. Shared autonomous vehicles (SAVs) are used such that no staff are hired to drive and relocate vehicles. A Bi-level Mixed-integer Linear Programming (MILP) model is proposed, with an objective of maximum profit from the vehicle operator's perspective. Decision variables include vehicle assignment patterns (i.e., number of vehicles initialized at each station) across traffic zones in the strategic planning level, and vehicle movements in the operational decision level with uncertain departure times. A two-phase stochastic optimization programming is established to solve this problem. Phase 1 is a mathematical optimization model that solves for the vehicle assignment pattern that maximizes profit, based on the base case setting. Leveraging the assignment pattern in Phase 1, the simulation model in Phase 2 is used to track exact vehicle movements. The scenarios with uncertain departure time are tested one-by-one. To demonstrate the proposed research methodology, this paper applies a case study of the 6-county Austin area with 2,210 traffic analysis zones (TAZs) and 3-hour morning peak travel demand.

The remainder of this paper is organized as follows. In Section 2, the related literature in system optimization and simulation studies are reviewed. The problem setting with a general model is proposed in Section 3. The two-level method, mathematical optimization, and simulation are presented in Section 4. Section 5 shows a case study in Austin's 6-county area. Conclusions and future research directions are given in the last section.

LITERATURE REVIEW

The literature of studies on fleet size optimization for shared vehicles is summarized in Table 1. This table differentiates studies using gasoline vehicles (GVs) and electric vehicles (EVs), as well as two solution methods, namely mathematical optimization, and system simulation.

Most papers adopt mathematical optimization models to plan vehicle assignment. With given travel demand, Kek et al. (2009), Fan (2014), Nourinejad and Roorda (2014), Repoux et al. (2015), Weikl and Bogenberger (2015), Vasconcelos et al. (2017), Lu et al. (2018), Xu et al. (2018), Li et al. (2019), Xu and Meng (2019) and Lu et al. (2020) optimized vehicle distribution among stations during daily operations. The vehicle purchase costs are often set in the objective function, and the known station capacity is set as a constraint. Fleet size, station location and station capacity are joined together in some other studies, such as Nair and Miller-Hooks (2011), Correia and Antunes (2012), Jorge et al. (2012), Jorge et al. (2014), Boyaci et al. (2015), Jorge et al. (2015), Nourinejad et al. (2015), Deng and Cardin (2018), Huang et al. (2018), Boyacı et al. (2019) and Huang et al. (2020a). Joint optimization can avoid parking space waste and maximize the operator's profit.

Apart from optimization models, a few studies use the simulation model (Huang et al., 2022; Yan et al., 2020). When deciding strategic planning, those existing simulation models use the given fleet size and vehicle assignment without considering the operations. Combining optimization and simulation can avoid the limitation. For example, Monteiro et al. (2021) built an agent-based simulation model to obtain the system performance of vehicle sharing, while the fleet size is simply optimized by a mathematical model.

An increasing number of studies have begun to explore the application of EVs (Huang et al., 2020a; Xu et al., 2018; Xu and Meng, 2019; Zhao et al., 2018). This brings many challenges for vehicle sharing organization. Due to limited battery capacity and long charging times, the charging station location problem is normally proposed when deciding the electric vehicle fleet size in an urban area.

Table 1 Literature of vehicle sharing fleet size study

Literature	Vehicle type		Solution method		Uncertain demand
	GV	EV	Optimization	Simulation	
Barrios and Godier (2014)	√			√	
Boyaci et al. (2015)	√		√		
Boyacı et al. (2019)	√		√		
Correia and Antunes (2012)	√		√		
Correia et al. (2014)	√		√		
Deng and Cardin (2018)	√		√		√
Fan (2014)	√		√		√
Huang et al. (2018)	√		√		
Huang et al. (2020a)		√	√		
Jorge et al. (2012)	√		√	√	
Jorge et al. (2014)	√		√		
Jorge et al. (2015)	√		√		
Kek et al. (2009)	√		√		
Lu et al. (2018)	√		√		√
Lu et al. (2020)	√		√		
Li et al. (2019)	√		√		√
Monteiro et al. (2021)	√			√	
Nair and Miller-Hooks (2011)	√		√		
Nourinejad and Roorda (2014)	√		√		
Nourinejad et al. (2015)	√		√		
Repoux et al. (2015)	√		√		
Xu et al. (2018)		√	√		
Xu and Meng (2019)		√	√		
Vasconcelos et al. (2017)	√		√		
Weikl and Bogenberger (2015)	√		√		
Zhao et al. (2018)		√	√		

When taking the uncertain departure times into account, vehicle sharing demand is usually assumed to follow a distribution, such as a Poisson or normal distribution (Lu et al., 2018; Li et al., 2019). Two main methods used to solve such a problem are stochastic optimization and robust optimization. Table 1 also shows limited studies focusing shared vehicles with departure time uncertainty, but there is no mature methodology in this field.

PROBLEM SETTING

This paper formulates the upper level of the optimization model as the strategic planning of shared vehicles under departure time uncertainty. The objective is to decide the vehicle assignment among traffic zones by maximizing the total profits of the vehicle sharing operator. The selected study region includes $|I|$ traffic zones with a certain zone denoted as $i \in I$. The daily operation time periods are equally divided into $|T|$ time steps (or intervals) and one time period is defined as $t \in T$. In the strategic level, number of vehicles $\overline{V_i}$ in each traffic zones i are the decision variables. The optimal solution of vehicle assignments is used as an input to decide the vehicle sharing services Q_{ijt} and relocations N_{ijt} at the operational level.

Travelers' departure time is uncertain and may change from day to day. Let Ξ be the set of daily departure time scenarios (can be seen as demand scenarios) and the probability of scenario $\omega \in \Xi$ is $\mathbf{E}_\omega > 0$ with $\sum_{\omega\in\Xi} \mathbf{E}_\omega = 1$. The daily operational decisions are only decided based on the scenario $\omega \in \Xi$.

Notation table

Table 2 List of notations

Sets	
$I:\{i\},\{j\}$	Set of traffic zones
$T:\{t\}$	Set of time steps (or intervals)
$\Xi:\{\omega\}$	Set of demand scenarios with varying departure time from day to day
Parameters	
c_f	Fixed costs per vehicle per day including depreciation, maintenance and insurance costs
c_e	Costs of power consumption of a vehicle running for one mile
c_r	Costs of relocating a vehicle per mile
p_{pu}	Pick-up payment per trip
g_{ij}	Travel distance from traffic zone $i \in I$ to traffic zone $j \in I$
q_{ijt}	Travel demand upper bound from traffic zone $i \in I$ to traffic zone $j \in I$ where $i \neq j$ at time step $t \in T$
s	Average speed of shared vehicles
$\mathbf{E}_\omega$	Probability of demand scenario $\omega \in \Xi$
Decision variables	
P	Shared automated vehicle payment per mile
$\overline{Q}_{ijt}$	Number of served travel requests from traffic zone $i \in I$ to traffic zone $j \in I$ where $i \neq j$ at the beginning of time step $t \in T$
$Q_{ijt}(\omega)$	Number of served travel requests from traffic zone $i \in I$ to traffic zone $j \in I$ where $i \neq j$ at the beginning of time step $t \in T$ in scenario $\omega \in \Xi$
$\overline{N}_{ijt}$	Number of proactive vehicle relocations from traffic zone $i \in I$ to traffic zone $j \in I_i$ where $i \neq j$ at the beginning of time step $t \in T$
$N_{ijt}(\omega)$	Number of proactive vehicle relocations from traffic zone $i \in I$ to traffic zone $j \in I$ where $i \neq j$ at the beginning of time step $t \in T$ in scenario $\omega \in \Xi$
$\overline{V}_i$	Number of vehicles in traffic zone $i \in I$ at the beginning of daily operation
$\overline{V}_{it}$	Number of vehicles parked in traffic zone $i \in I$ at the beginning of time step $t \in T$
$V_{it}(\omega)$	Number of vehicles parked in traffic zone $i \in I$ at the beginning of time step $t \in T$ in scenario $\omega \in \Xi$
Auxiliary variables	
$W_{it}(\omega)$	Number of vehicles idling in traffic zone i at time instant t in scenario $\omega \in \Xi$
$\overline{W}_{it}$	Number of vehicles idling in traffic zone i at time instant t
$\overline{U}_{ijt}$	Number of vehicles leaving from traffic zone i to traffic zone j at time instant t
$U_{ijt}(\omega)$	Number of vehicles leaving from traffic zone i to traffic zone j at time instant t

in scenario $\omega \in \Xi$

Price based elastic demand function

The elastic demand of vehicle sharing is affected by the payment of users. Hence, elastic demand function Equation (1) is introduced. The potential demand will be lower when increasing the payment because users will be less inclined to use shared vehicles. On the contrary, a lower price will encourage more users to choose vehicle sharing.

$$\sum_{t\in T}\sum_{j\in J}\sum_{i\in J} Q_{ijt}(\omega) \le \sum_{t\in T}\sum_{j\in J}\sum_{i\in J} q_{ijt}(\omega)\left(1+\alpha P\right) \qquad \forall \omega \in \Xi \qquad (1)$$

In this paper, we set α as -0.2. When the price rises, the demand will drop. Figure 1 shows the elastic demand with varying price. A linear relationship is assumed and no users are expected at a very high cost of \$5 per mile. Such an elastic demand function can be extended to other forms, like exponential or log-linear functions.

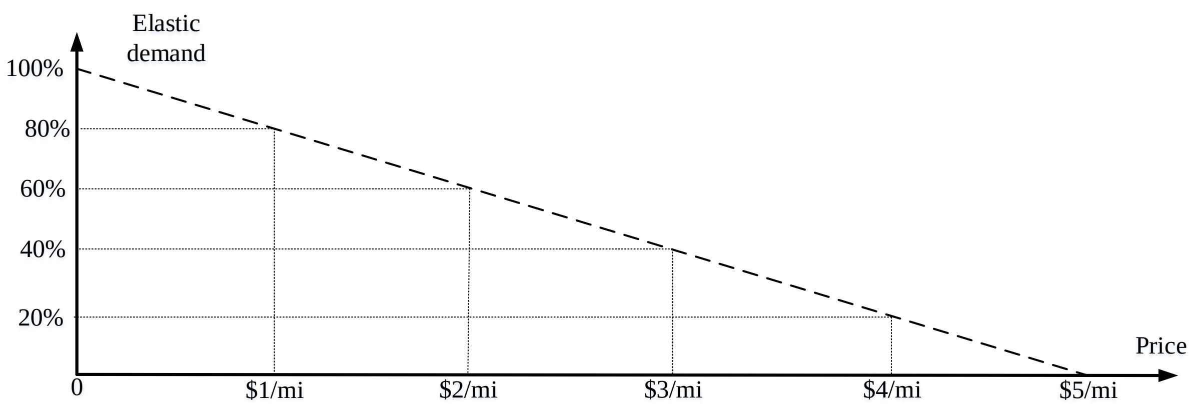

Figure 1 The elastic demand with different price

Integrated model

$$\mathbf{P0} \max_{U,V,N,Q} \phi = -\sum_{i\in I} c_f \overline{V}_i + \mathbf{E}_{\omega\in\Xi}\left[\sum_{t\in T}\sum_{i\in I}\sum_{j\in I}\left[\left(P-c_e\right)g_{ij}+p_{pu}\right]Q_{ijt}(\omega) - \sum_{t\in T}\sum_{i\in I}\sum_{j\in I} c_r g_{ij} N_{ijt}(\omega)\right] \qquad (2)$$

Constraints (1), plus:

$$V_{i1}\left(\omega\right) = \overline{V}_i \qquad \forall \omega \in \Xi \qquad (3)$$

$$Q_{ijt}\left(\omega\right) \le q_{ijt}\left(\omega\right) \qquad \forall i \in I, j \in I, t \in T, \omega \in \Xi \qquad (4)$$

$$\sum_{j\in I} Q_{ijt}(\omega) + \sum_{j\in I} N_{ijt}(\omega) \le V_{it}\left(\omega\right) \qquad \forall i \in I, t \in T, \omega \in \Xi \qquad (5)$$

$$U_{ijt}\left(\omega\right) = Q_{ijt}(\omega) + N_{ijt}(\omega) \qquad \forall i \in I, j \in I, t \in T, \omega \in \Xi \qquad (6)$$

$$W_{it}\left(\omega\right) = V_{it}\left(\omega\right) - \sum_{j\in I} U_{ijt}(\omega) \qquad \forall i \in I, t \in T, \omega \in \Xi \qquad (7)$$

$$V_{it+1}\left(\omega\right) = W_{it}\left(\omega\right) + \sum_{j\in I} U_{jim}(\omega) \qquad \forall i \in I, t \in \left|T\right|-1, m = max\left\{0, t+1-\left\lceil g_{ji}/s \right\rceil\right\}, \omega \in \Xi \qquad (8)$$

$$N_{ijt}(\omega), Q_{ijt}(\omega), \overline{V}_i, V_{it}(\omega), U_{ijt}(\omega), W_{ijt}(\omega) \in Z^0 \qquad \forall i \in I, j \in I, t \in T, \omega \in \Xi \qquad (9)$$

The objective function (2) is to maximize profit for the vehicle sharing operator. It is equal to the revenue minus the vehicle fixed cost, power consumption cost caused by the vehicle sharing service and proactive relocation. The revenue and cost of vehicle movements are obtained in random demand scenarios, so that the probability $\mathbf{E}_{\omega\in\Xi}[\ ]$ is used. In the following explanation, we omit (ω) in all the decision variables for notation simplicity.

Constraints (3) require the number of vehicles at the beginning of daily operation to follow a static vehicle assignment planning. In Constraints (4), the number of served vehicle sharing requests Q_{ijt} cannot be larger than total demand q_{ijt}. Constraints (5) require that the number of served requests cannot go beyond the number of available vehicles (either at the station or relocated from other zones) in traffic zone i at the beginning of time step t. Constraints (6) calculate U_{ijt}, the total number of vehicles moved from traffic zone i to traffic zone j at time instant t. Constraints (7) calculate the total number of vehicles W_{it} idling in traffic zone i at time step t. Constraints (8) calculate the total number of available vehicles in traffic zone i at next time instant $t+1$. W_{it} indicates the number of idling vehicles. $\sum_{j\in I}U_{jim}$ indicates the number of vehicles arriving i between time step t and $t+1$. Constraints (9) specify the domain of the decision variables.

Non-linear challenge

The established model **P0** is not positive definite (PSD), because the term $\left[\left(P-c_e\right)g_{ij}+p_{pu}\right]Q_{ijt}(\omega)$ in the objective function is a square equation. Hence, model **P0** cannot be solved by using a commercial solver. One of the solution methods is to use a dynamic iteration that sets a fixed value of price and then solve the optimization model.

SOLUTION METHOD

Optimization-based method

In the optimization-based method, the demand in the base case setting is used to optimize the long-term vehicle assignment. All randomness $\omega\in\Xi$ is removed. New variables of $\overline{Q}_{ijt}$, $\overline{N}_{ijt}$, $\overline{V}_{it}$ and $\overline{Q}_{it}$ are introduced in this model. The following shows the mathematical model.

$$\mathbf{P1}\ \max_{U,V,N,Q}\phi=-\sum_{i\in I}c_f\overline{V}_i+\sum_{t\in T}\sum_{i\in I}\sum_{j\in I}\left[\left(p-c_e\right)g_{ijt}+p_{pu}\right]\overline{Q}_{ijt}-\sum_{t\in T}\sum_{i\in I}\sum_{j\in I}c_r g_{ijt}\overline{N}_{ijt} \qquad (10)$$

Subject to:

Constraints (3)-(8), plus:

$$\overline{N}_{ijt},\overline{Q}_{ijt},\overline{V}_i,\overline{V}_{it},\overline{U}_{ijt},\overline{W}_{ijt}\in Z^0 \qquad \forall i\in I,\ j\in I,t\in T \qquad (11)$$

The objective function (10) maximizes profit for the vehicle sharing operator with the expected demand. Constraints (11) specify the domain of the decision variables.

Simulation-based method

SUMO simulation setup

Based on the values of $\overline{V}_{i1}$ from the optimization model, the simulation model further offers a realistic representation of traffic flow. The simulation exercise is conducted using simulation of urban mObility (SUMO), which is an agent-based simulation tool that enables detailed real-time vehicle and passenger tracking. The mesoscopic version of SUMO is used, to offer faster computation speeds for the vehicle-sharing demand, while still capturing the key system performance. The link transmission model is leveraged instead of the car-following model and lane-changing model in the mesoscopic version for the traffic flow model.

The vehicles are initialized based on the pattern suggested in the optimization model. A person is assumed to make a request at the parking lot, so the vehicle can pull out immediately. The vehicle then heads towards the destination parking lot using the shortest travel time path. After the vehicle arrives at the destination station, the person parks and leaves the vehicle. The vehicle is then available for others to use. When there is no vehicle available in the parking lot, the request is considered a failed match, and the revenue is lost.

The parking lots are initialized at an existing edge of the responding TAZ, and this edge is considered as the only destination of all the trips entering/leaving this TAZ. The parking lots are assumed to have unlimited capacity but when all the available vehicles have left a certain parking lot, this parking lot cannot offer vehicles to serve more people. The vehicle sharing clients in one TAZ are free to use the available vehicles at the edge of the current TAZ.

Simulation framework

The simulation starts with the optimization model to obtain the profit-maximizing vehicle assignment pattern, which is fed into the simulation model. These vehicles will be initialized at the parking lots and serve riders for the morning peak period. At each timestep, the desired departures and arrivals are examined. For the desired departures, the list of the people who would like to use the vehicles at this timestep are checked. If there are available vehicles at the parking lot, a random vehicle is assigned to serve this person. In terms of arrivals, the list of all vehicle arrivals at each time step is examined so that destination parking lots are identified, and the parking lot information is updated accordingly. Finally, the vehicle and system performance are collected after the simulation ends.

CASE STUDY

Austin traffic network

The proposed methodology is adopted in the traffic network of Austin, Texas. All 6 counties of the Austin area are involved, including 2,210 traffic analysis zones (TAZs) and 23,576 links. Figure 1 illustrates the traffic network. As the purpose of this study is to explore the vehicle sharing system optimization, the shortest routes among TAZs are selected.

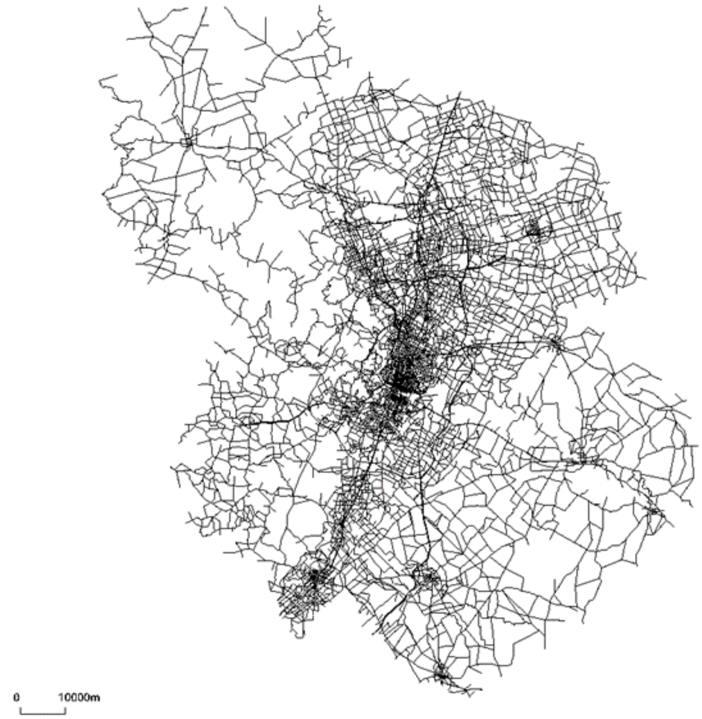

Figure 1. SUMO simulation traffic network in Austin, Texas

K-means clustering

The optimization-based method faces a challenge due to the large-scale network and discrete operation time. For the decision variables of satisfied demand $\overline{Q}_{ijt}$ or relocation operations $\overline{N}_{ijt}$, the optimization-based method will have 4,884,100 (2,210*2,210) variables for just one time step. To reduce the computation burden, TAZ clustering is desired. In this paper, a k-means clustering method is proposed such that a total of 2,210 TAZs are aggregated into 100 traffic zones (shown in Figure 2). K-means is a classic method in data clustering analysis. The local optimal group size is based on the Euclidian distance between the cluster center and each TAZ center. In the Austin network, after 52 iterations, the optimal clusters are obtained.

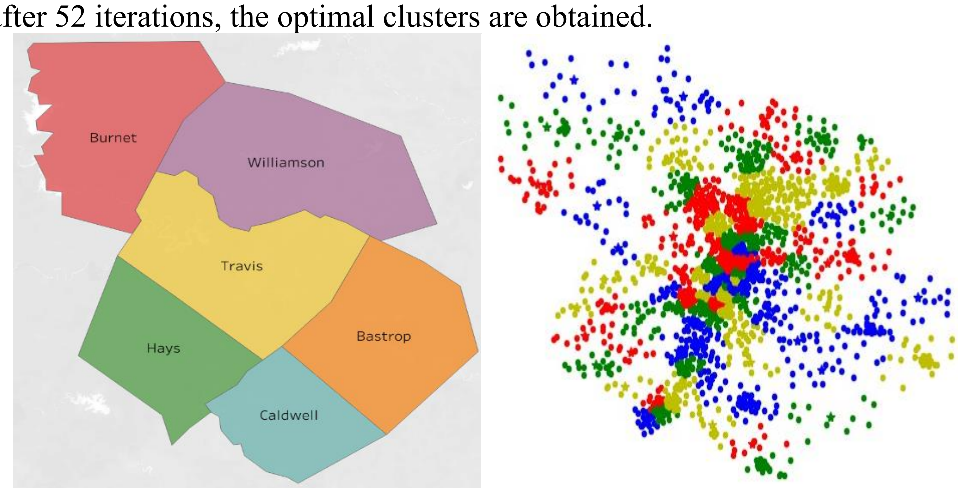

Figure 2. K-mean clustering for the 6-county Austin network

Demand generation with dynamic travel time

The travel demand from 7 am to 10 am is used in this case study. The initial optimization time step size tested 1 second as a unit, which led to a long computation time. Therefore, we divide the 3-hour operation period into 36 time steps, with 5 minutes per time step. Vehicle movements are allowed at the beginning of each time period.

Optimization results

Base case

In this paper, the parameter settings were adopted from existing studies (Huang et al., 2018 & 2020a & 2020b; Loeb and Kockelman, 2019; Segal and Kockelman, 2016; Gurumurthy and Kockelman, 2020). The optimization model is solved on a laptop with Gurobi 9.1.1 solver on an i7 processor @3.60GHz, 32GB RAM in a Windows 10 64 bit operating system. Since the problem does not have non-linear constraints, Gurobi is a preferred choice of solver.

The basic setting assumes a fare of $1 per (traveler-occupied) mile, fuel cost of $0.06 per mile, vehicle relocation cost of $0.12 per mile, and vehicle depreciation, maintenance & insurance cost of $0.40 per mile. To reflect the uncertainty and variations (across settings) in the real world, sensitivity analyses are conducted on the cost assumptions, with 3 levels of price, relocation costs, and depreciation, maintenance & insurance costs. The optimization results are shown in Table 3.

Table 3 Shared automated vehicle fleet performance during 3-hour peak hours (with various price and cost settings)

Key price & cost values ($ per mile)		$ Profits per SAV	No. of SAVs	VMT per SAV (mile)	VHT per SAV (min)	Satisfied demand (%)	Proactive relocations
Price	$0.8/mi	$14.04 /SAV	11,963 SAVs	19.88 mi/SAV	39.17 min/SAV	71.77 (%)	69 SAVs
	1	10.86	11,000	21.62	41.52	67.12	58
	1.2	7.91	9,330	23.49	45.10	58.78	53
Vehicle relocation cost	0.06	10.87	10,999	21.63	41.52	67.11	61
	0.12	10.86	11,000	21.62	41.52	67.12	58
	0.20	10.86	10,997	21.63	41.52	67.10	55
Vehicle depreciation, maintenance & insurance cost	0.30	11.60	12,664	20.44	39.25	75.12	72
	0.40	10.86	11,000	21.62	41.52	67.12	58
	0.50	10.23	9,358	23.46	45.03	58.98	60

Note: bolded values are base case settings on these three variables; price is in a unit of $/mile paid by vehicle users; vehicle relocation cost is in a unit of $/mi paid by the fleet operator; vehicle fixed cost includes depreciation, maintenance, and insurance cost, in a unit of $/mi paid by the fleet operator.

The base case scenario demonstrates a profit of $10.86 per vehicle over the 3-hour morning peak period, with a total fleet size of 11,000 vehicles satisfying 67.12% of the vehicle-sharing travel demand. When increasing the vehicle rental price from $0.80 to $1.20 per mile, the average profits per vehicle decrease to $7.91 during the 3-hour morning peak period. If extending to 24-hour operation, the average profit per vehicle is $63.30 and the overall fleet size decreases by

22.0%. The percentage of served/satisfied requests drops significantly with the increase of fare, thanks to the smaller fleet size. Results indicate that arranging 11,963 vehicles with a rental price of $0.80 per mile in the Austin area is a stable strategic planning that will not cut off too many vehicle sharing requirements. For the vehicle relocation cost, $0.06 per mile is the best choice to achieve the highest profits. With the increase of vehicle relocation cost, the number of relocation operations drops lightly. Another finding is that the vehicle relocation cost does not affect the fleet size decision. With the increase of vehicle fixed cost, the profits and fleet size drop by about 11.8% and 26.1%, respectively. In this way, the satisfied demand falls from 75.1% to 59.0%.

Vehicle allocations

Vehicle allocation results in this section are based on the base case settings of price, fuel cost, vehicle relocation cost and vehicle maintenance cost in above section. The total profits are $119,489 by serving 67.12% vehicle sharing demand. A total of 11,000 vehicles are assigned to Austin in the morning peak hours, and 58 proactive vehicle relocations are conducted to move vehicles to high-demand areas. Vehicle in-service rate is 23%, as indicated by 41.52 minutes driving time (21.63 miles) during the 3-hour morning peak hours. Table 4 shows the vehicle distributions at the beginning of the daily operation. The average number of vehicles in traffic zones is 110, with the maximum value being 770, although some zones are not initialized with any vehicles. It shows that traffic zones #26, #37, #57, #80, #83 and #88 are high-demand zones that might be residential zones.

Table 4 Vehicle distributions in 100 traffic zones

Traffic zone	No. of vehicles	Traffic zone	No. of vehicles	Traffic zone	No. of vehicles	Traffic zone	No. of vehicles
#1	46 SAVs	#26	564 SAVs	#51	55 SAVs	#76	47 SAVs
2	41	27	1	52	59	77	94
3	0	28	136	53	64	78	209
4	310	29	26	54	25	79	51
5	125	30	52	55	18	80	770
6	28	31	18	56	4	81	12
7	45	32	4	57	681	82	172
8	33	33	255	58	54	83	652
9	90	34	46	59	105	84	217
10	31	35	67	60	13	85	40
11	28	36	39	61	49	86	13
12	363	37	535	62	54	87	102
13	96	38	12	63	28	88	556
14	258	39	242	64	163	89	49
15	47	40	12	65	170	90	19
16	168	41	67	66	77	91	7
17	272	42	29	67	108	92	53
18	45	43	26	68	33	93	73
19	18	44	185	69	6	94	0
20	78	45	28	70	66	95	163
21	253	46	32	71	95	96	0
22	43	47	1	72	221	97	1
23	153	48	61	73	77	98	0
24	169	49	63	74	2	99	89
25	23	50	11	75	97	100	12

Simulation results

Based on the output of vehicle arrangement in optimization results, 30 scenarios are tested in this case study. The departure time of each trip among 30 scenarios follows a uniform distribution. Table 5 shows the simulation results.

The system performance in terms of service rate is quite robust with mean values varying within 0.10%. The low coefficient of variation (CoV) in service rate, average travel distance and average service time (all less than 1%) also prove that the proposed vehicle sharing operation mechanism is robust across 30 demand scenarios. Such results indicate that the proposed two-phase stochastic optimization method can ensure a steady service rate and handle random departure scenarios via flexible vehicle sharing operations.

Table 5 Vehicle distributions in 100 traffic zones

Scenarios	Satisfied demand (%)	Average travel distance (Mile)	Average service time (min)
1	85.99	15.51	29.34
2	86.19	15.58	29.60
3	86.14	15.68	29.73
4	86.22	15.56	29.69
5	86.22	15.69	29.91
6	86.22	15.57	29.62
7	86.13	15.51	29.52
8	86.17	15.52	29.44
9	86.02	15.63	29.59
10	86.22	15.62	29.63
11	86.18	15.58	29.52
12	86.04	15.53	29.30
13	86.02	15.66	29.76
14	86.10	15.56	29.45
15	86.25	15.59	29.72
16	86.03	15.63	29.53
17	86.14	15.62	29.61
18	86.05	15.70	29.94
19	86.01	15.59	29.58
20	86.16	15.56	29.59
21	86.01	15.51	29.33
22	86.09	15.66	29.61
23	86.12	15.64	29.74
24	86.02	15.60	29.61
25	86.15	15.61	29.69
26	86.18	15.51	29.52
27	85.93	15.64	29.43
28	86.06	15.57	29.43
29	85.95	15.65	29.62
30	85.99	15.66	29.53
Minimum value	85.93	15.51	29.30
Mean value	86.10	15.60	29.59
Maximum value	86.25	15.70	29.94
CoV	0.10%	0.37%	0.51%

Due to the steady performance in average service rate, 30 scenarios are shown to be sufficient to represent the departure time uncertainties in the case study. Going through the literature in vehicle sharing studies under uncertainty, we found that most studies also adopted a relatively small number of scenarios due to computation resource constraints. For example, Brandstätter et al. (2017) and Fan (2014) adopted 7 scenarios to simulate the stochastic demand in a week. He et al. (2017) and He et al. (2020) chose 30 demand scenarios with the operation data between March and April 2014, and Biondi et al. (2016) used 46 scenarios with the dataset covering the period from May 17 to July 1, 2015. Only Lu et al. (2018) adopted 1000 scenarios to explore the profitability and quality of service in vehicle sharing systems. In this Austin case study, a total of 100 zones and 36 time steps together are considered. A larger number of scenarios will certainly further improve the accuracy in system performance evaluation, yet at the cost of huge computation time.

Comparative analysis

In this section, comparative analyses are conducted between two solution methods: Scenario 1 in Phase 2 is tested using a mathematical model and SUMO simulation. The vehicle arrangements among 100 traffic zones are given based on the solutions of Phase 1.

Satisfied demand reaches 80.75% in the simulation model, while it reaches 75.7% in the optimization model. The simulation model has better performance in terms of tracking vehicle movements in the operational level. It also demonstrates that the proposed methodology of using an agent-based simulation model in Phase 2 is compatible in the study framework. The average vehicle-traveled miles and service time of vehicles increase by 47.52% and 49.76%, respectively. This may be because serving long-distance easily leads to larger profits.

CONCLUSION

This study explores the vehicle arrangement problem in SAV systems under departure time uncertainty. A two-phase stochastic optimization programming is proposed to address the uncertainty problem. In Phase 1, a mathematical optimization model is established to decide the vehicle arrangement at the beginning of the day, using mean values of vehicle sharing demand. Based on the optimization results, the agent-based simulations under 30 demand scenarios are conducted in Phase 2 to analyze the system performance. The case study is conducted on the 6-county Austin traffic network with uncertain departures at 3-hour morning peak hours.

The optimization results show that maximum profits can be obtained when deploying 11,000 shared vehicles in the Austin area under the base case setting. Higher profits can be obtained when vehicle rental price is set as $0.80 per mile, compared to prices at $1 or $1.20 per mile. When increasing the relocation cost from $0.06 to $0.20 per mile, the number of vehicle relocations decrease from 61 to 55. The profit of serving each trip is $10.20 when using a high-cost vehicle, while using a low-cost vehicle can bring profits of $11.60 for each trip. Furthermore, the SUMO simulation results indicate 86% of requests to be satisfied. With more satisfied demand, the average service time is 30 minutes in the SUMO simulation, which is longer than the average value of 23 minutes of all travelers in the optimization model. The low CoV proves that the proposed stochastic optimization has strong robustness. The proposed optimization-simulation solution performs better than the existing optimization method. Using an agent-based model in Phase 2 can improve the service rate by 6.7%.

The departure time uncertainty problem is proposed and explored in this paper, but the solution to the problem can be improved in future research. One of the limitations lies in the departure time centralization in Phase 1's optimization problem. When calculating the average departure time in 30 demand scenarios, the values are more likely to occur around the middle of the time horizon. This led to a higher peak demand than expected, leading to a larger fleet size. Moreover, the limited computation memory allows the scenario of only 100 traffic zones with 2,210 TAZs, which might cut off some optimal solutions. Solving the large-scale system optimization problem is another possible research direction.

AUTHOR CONTRIBUTION STATEMENT

The authors confirm the contribution to the paper as follows: Kai Huang: Conceptualization, Optimization, Writing. Yantao Huang: Simulation, Writing. Kara Kockelman: Supervision, Reviewing. All authors reviewed the results and approved the final version of the manuscript.

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