1	Shared Autonomous Vehicle Modeling Considering System Optimization and Simulation
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22	Abstract
23	This paper optimizes the assignment of shared automated vehicles under users' uncertain departure
24	times. Automated vehicles can drive themselves so no staff are needed to relocate vehicles in the
25	one-way carsharing system. To formulate the one-way vehicle sharing system with departure time
26	uncertainties, a two-phase solution method is established. Phase 1 decides the strategic planning
27	of vehicles distributed at stations by using a system optimization approach, followed by Phase 2,
28	which tracks vehicle movements via an agent-based simulation model. The optimization solutions
29	in Phase I serve as inputs for Phase 2. A case study in the six-county Austin, Texas area is
30	conducted to verify the proposed model and corresponding solution approach. Under the base case
31 22	setting, a freet of 8,564 automated venicles is deployed in the study region. Optimization results
32 22	suggest that system profits are optimized when vehicle remains priced at \$1.28 per km (\$0.8 per mile). The number of properties which releasting drops by 0.80% if the releastion exercises exercises are a standard to the releasting exercises
23 24	increases from \$0.006 per km (\$0.06 per mile) to \$0.32 per km (\$0.2 per mile). The prefit of
34 35	serving each trip is \$10.20 when using high-cost vehicles and \$11.60 if using low-cost vehicles
36	Three-hour simulation results show an average person-trip length of 25 km (15.6 miles) with 29.6-
37	minutes of average driving time. If a 24-hour day is simulated the vehicle-occupied time and
38	vehicle-distance traveled are 4 hours and 200 km (125 miles) per vehicle respectively. The low
39	coefficient of variation of satisfied demand across 30 demand scenarios demonstrates the
40	robustness of the proposed two-phase solution method.
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Keywords: Shared autonomous vehicles, Vehicle assignment, Optimization, Simulation

45 **1 Introduction**

Vehicle sharing is becoming popular, allowing users to reserve unoccupied vehicles from 46 traditional vehicle rental services by the minute or hour, instead of by the day. Vehicle users may 47 have more flexibility when choosing a destination to return their vehicle with reduced cost, and 48 49 fleet owners may profit due to higher utilization of the vehicles in their fleet. An existing study shows that vehicle serving time will rise by about 30% (Huang et al., 2020a). It indicates that 50 vehicles need to run half of the time during daily operation, i.e., 6-hour driving time during a 12-51 hour vehicle sharing system service time. Owing to these benefits, many traditional vehicle rental 52 53 companies are beginning to provide hourly vehicle sharing services to expand their market (Hertz 54 24/7, 2021). Popular vehicle sharing operators include companies like Car2Go (SHARE NEW, 55 2021) and Shanghai's EVCARD (EVCARD, 2021).

56 There are two main types of vehicle sharing services, depending on the vehicle return restrictions. Round-trip vehicle sharing requires users to return vehicles to the original departure 57 location. Such a service is less popular among the public, especially for commuting trips. Therefore, 58 one-way vehicle sharing has been popular in recent years, and is now becoming a major vehicle 59 sharing service. Users can pick up and drop off rental vehicles at any allowed parking space (Lu 60 et al., 2018). One-way vehicle sharing can be called station-based vehicle sharing if available 61 62 parking stations are predetermined; otherwise, it is often called free-float vehicle sharing, and any prepaid or free parking spaces can be used (Balac et al., 2019). 63

Existing studies mainly focus on two kinds of problems when designing a vehicle sharing 64 system in a new vehicle rental market: strategic planning and operational decisions (Huang et al., 65 2018). Strategic planning makes long-term decisions that include fleet size, vehicle assignment, 66 67 and station location and capacity. Operational decisions focus on real-time vehicle movements, relocations, and pricing. Strategic planning often serves as the foundation of operational decisions, 68 while the network performance can be leveraged to adjust planning strategies through feedback. 69 Therefore, in a bi-level optimization model that considers both strategic planning and operational 70 71 decisions, strategic planning is often formulated as an upper-level model while the operation 72 decisions are the lower-level.

73 One challenge of designing the vehicle sharing system is the vehicle assignment (or "vehicle initialization") at the predetermined stations at the beginning of daily operations (Li et al., 74 75 2020; Li and Liu; 2021). The vehicle assignment procedure determines the fleet size and the number of vehicles allocated to each parking station. An excessive number of vehicles deployed 76 77 (or initialized) in traffic zones can lead to an underutilized vehicle fleet. However, if inadequate 78 vehicles are deployed across the network, vehicle rental demand cannot be satisfied, leading to a 79 loss of profits. Regarding this, an optimized vehicle assignment at the beginning of day can 80 properly balance the vehicle use and avoid the two kinds of profit loss mentioned above. Existing studies often build mathematical optimization models that take the whole day's demand into 81 account (Huang et al., 2020a; Xu et al., 2018; Xu and Meng, 2019). However, it is difficult to track 82 83 and capture detailed vehicle movements on road with such a model. Simulation models can fill 84 this gap by recording detailed vehicle route choices and decisions (Illgen and Höck, 2020). Even

so, the time-varying travel demand cannot be well reflected in simulation models.

The case of different departure times across the day brings challenges to vehicle assignment, and has not been well explored. Most existing studies assume travel demand and departure time to be fixed based on historical or predicted data (Ranjbari et al., 2020; Jorge et al., 2015). In terms of the uncertainty in travel demand, only a few papers explored the case when departure time of vehicles leaving one traffic zone follows a distribution function (Lu et al., 2018; He et al., 2017).

Furthermore, using autonomous vehicles is seen as future development direction of the carsharing system. It can largely reduce the personnel cost because imbalance problem between travel demand and vehicle supply is a key problem. The current method to address such a problem is to employ drivers to relocate vehicles or provide payment discount. When using autonomous vehicles (AVs), the operation cost can be reduced (Chen and Liu, 2022). Also, for the carsharing operator, the complex route planning of personnel movements is removed as AVs can drive to working place to pick up waiting clients easily.

The main contributions of the paper are as follows. (I) We build a combined method of system optimization and simulation to handle the carsharing system planning and operation problem. It can exactly decide the vehicle allocation in the upper level based on mathematical optimization model, and track the vehicle movements in the lower level based on agent-based simulation. (II) The departure time uncertainty is considered in this paper. The subsequent challenge is to establish two-phase solution method with random departure time. (III) AVs are used to provide carsharing service which can avoid the personnel movement management problem.

This paper aims to address the research gaps in dealing with the gasoline vehicle 105 assignment problem under departure time uncertainties. Shared autonomous vehicles (SAVs) are 106 107 used such that no staff are hired to drive and relocate vehicles. A Bi-level Mixed-integer Linear Programming (MILP) model is proposed, with an objective of maximum profit from the vehicle 108 operator's perspective. Decision variables include vehicle assignment patterns (i.e., number of 109 vehicles initialized at each station) across traffic zones in the strategic planning level, and vehicle 110 111 movements in the operational decision level with uncertain departure times. A two-phase solution method is established to solve this problem. Phase 1 is a mathematical optimization model that 112 solves for the vehicle assignment pattern that maximizes profit, based on the base case setting. 113 Leveraging the assignment pattern in Phase 1, the simulation model in Phase 2 is used to track 114 115 exact vehicle movements. The scenarios with uncertain departure time are tested one-by-one. To demonstrate the proposed research methodology, this paper applies a case study of the 6-county 116 Austin area with 2,210 traffic analysis zones (TAZs) and 3-hour morning peak travel demand. 117

The remainder of this paper is organized as follows. In Section 2, the related literature in system optimization and simulation studies are reviewed. The problem setting with a general model is proposed in Section 3. The two-level method, mathematical optimization and simulation, are presented in Section 4. Section 5 details a case study in Austin's 6-county area. Conclusions and future research directions are given in the last section.

124 **2 Literature Review**

The literature of studies on fleet size optimization for shared vehicles is summarized in Table 1. This table differentiates studies using gasoline vehicles (GVs) and electric vehicles (EVs), as well as two solution methods, namely mathematical optimization, and system simulation. Moreover, related studies on demand distribution are reviewed in Table 1. The population' total travel between OD pairs follows a distribution model. When deciding fleet size in an urban area, operators should consider the day-to-day demand fluctuation.

131

132 Most papers adopt mathematical optimization models to plan vehicle assignment. With given travel demand, Kek et al. (2009), Fan (2014), Nourinejad and Roorda (2014), Repoux et al. 133 (2015), Weikl and Bogenberger (2015), Vasconcelos et al. (2017), Lu et al. (2018), Xu et al. (2018), 134 Li et al. (2019), Xu and Meng (2019) and Lu et al. (2020) optimized vehicle distribution among 135 stations during daily operations. The vehicle purchase costs are often set in the objective function, 136 137 and the known station capacity is set as a constraint. Fleet size, station location and station capacity are joined together in some other studies, such as Nair and Miller-Hooks (2011), Correia and 138 Antunes (2012), Jorge et al. (2012), Jorge et al. (2014), Boyaci et al. (2015), Jorge et al. (2015), 139 Deng and Cardin (2018), Huang et al. (2018) and Huang et al. (2020a). Joint optimization can 140 141 avoid parking space waste and maximize the operator's profit. However, the mathematical optimization assumes the travel demand occurs at the beginning of one time period, such as 10 or 142 30 minutes. The vehicle movements cannot be exactly tracked during the optimization process. 143

The simulation model can track vehicle behavior and location, but only a few studies exist 144 (Huang et al., 2022a; Huang et al., 2022b; Yan et al., 2020). When deciding strategic planning, 145 146 those existing simulation models use the given fleet size and vehicle assignment without considering the operations. When conducting the simulation, it needs to give the number of 147 allocated vehicles by using heuristics algorithms. Combining optimization and simulation can 148 avoid the limitation. For example, Monteiro et al. (2021) built an agent-based simulation model to 149 150 obtain the system performance of vehicle sharing, while the fleet size is simply optimized by a mathematical model. Furthermore, the simulation model can track the battery capacity of EVs 151 (Huang et al., 2020a; Xu et al., 2018; Xu and Meng, 2019; Zhao et al., 2018). Limited battery 152 capacity and long charging times result in the solution difficulties. They will bring many challenges 153 154 for vehicle sharing organization. In many studies, the charging station location problem is proposed when deciding electric vehicle fleet size in an urban area. 155

When taking the uncertain departure times into account, vehicle sharing demand is usually 156 assumed to follow a distribution, such as a Poisson or normal distribution (Lu et al., 2018; Li et 157 158 al., 2019). Most existing studies focus on the uncertain demand but not departure time. For the uncertain demand, the total demand is assumed to follow a distribution model during the operation 159 time. However, few studies consider the choice of departure time. Yang and Tang (2018) and Tang 160 et al. (2020) explored the departure time choice behavior. A bi-level model is established when 161 considering the mass transit railway system optimization. Few studies focus on shared vehicles 162 163 with departure time uncertainty, and there is no mature methodology in this field.

165 **3 Problem Setting**

This paper formulates a SAV system optimization problem under departure time uncertainty, in 166 which the upper level is to decide the vehicle assignment and the lower level is to track the vehicle 167 168 movements. The objective is to maximize the total profits of the vehicle sharing operator. The selected study region includes |I| traffic zones with a certain zone denoted as $i \in I$. The daily 169 operation time periods are equally divided into |T| time steps (or intervals) and one time period is 170 defined as $t \in T$. In the strategic level, number of vehicles $\overline{V_i}$ in each traffic zone *i* are the decision 171 variables. The optimal solution of vehicle assignments is used as an input to decide the vehicle 172 sharing services Q_{iit} and relocations N_{iit} at the operational level. 173

174 Travelers' departure time is uncertain and may change from day to day. Let Ξ be the set of 175 daily departure time scenarios (can be seen as demand scenarios) and the probability of scenario 176 $\omega \in \Xi$ is $\mathbf{E}_{\omega} > 0$ with $\sum_{\omega \in \Xi} \mathbf{E}_{\omega} = 1$. The daily operational decisions are only decided based on the 177 scenario $\omega \in \Xi$.

- 178
- 179 3.1 Notation

180 This paper uses *I* as the set of zones, with a certain zone donated as $\{i\}, \{j\} \in I$. *T* presents 181 the set of time steps (or intervals) and one time period is defined as $t \in T$. $\Xi: \{\omega\}$ is the set of 182 demand scenarios with varying departure time from day to day.

The parament references in this paper are shown below. c_f define the fixed costs per vehicle per day including depreciation, maintenance and insurance costs, while c_e define the costs of power consumption of a vehicle running for one mile (0.625 km) and c_r define the costs of relocating a vehicle per mile (0.625 km). p_{pu} is the pick-up payment per trip. Travel distance from traffic zone $i \in I$ to traffic zone $j \in I$ is denoted by g_{ij} . We use q_{ijt} to denote travel demand upper bound from traffic zone $i \in I$ to traffic zone $j \in I$ where $i \neq j$ at time step $t \in T . s$ is average speed of shared vehicles. \mathbf{E}_{ω} is probability of demand scenario $\omega \in \Xi$.

The decision variables references in this paper are shown below. We use *P* to denote the shared automated vehicle payment per mile (0.625 km). \overline{Q}_{ijt} is number of served travel requests from traffic zone $i \in I$ to traffic zone $j \in I$ where $i \neq j$ at the beginning of time step $t \in T$, and $Q_{ijt}(\omega)$ is \overline{Q}_{ijt} in scenario $\omega \in \Xi$. In a similar way, \overline{N}_{ijt} is number of proactive vehicle relocations from traffic zone $i \in I$ to traffic zone $j \in I_i$ where $i \neq j$ at the beginning of time step $t \in T$, and $N_{ijt}(\omega)$ is \overline{N}_{ijt} in scenario $\omega \in \Xi$. $\overline{V_i}$ is number of vehicles in traffic zone $i \in I$ at the beginning of daily operation, \overline{V}_{it} is \overline{V}_i at the beginning of time step $t \in T$, and $V_{it}(\omega)$ is \overline{V}_{it} in scenario $\omega \in \Xi$.

197 The auxiliary variables references in this paper are shown below. \overline{W}_{it} is number of vehicles 198 idling in traffic zone *i* at time instant *t*, and $W_{it}(\omega)$ is \overline{W}_{it} in scenario $\omega \in \Xi . \overline{U}_{ijt}$ is number of 199 vehicles leaving from traffic zone *i* to traffic zone *j* at time instant *t*, and $U_{iit}(\omega)$ is \overline{U}_{ijt} in

200 scenario $\omega \in \Xi$.

201 3.2 Price based elastic demand function

202 The elastic demand of vehicle sharing is affected by the payment of users. Hence, elastic demand

- function Equation (1) is introduced. The potential demand will be lower when increasing the payment because users will be less inclined to use shared vehicles. On the contrary, a lower price
- 205 will encourage more users to choose vehicle sharing.

$$\sum_{t \in T} \sum_{j \in J} \sum_{i \in J} Q_{ijt}(\omega) \le \sum_{t \in T} \sum_{j \in J} \sum_{i \in J} q_{ijt}(\omega) (1 + \alpha P) \qquad \forall \omega \in \Xi$$
(1)

For example, we can set α as -0.2. When the price rises, the demand will drop. Figure 1 shows the elastic demand with varying price. A linear relationship is assumed and no users are expected at a very high cost of \$8 per km (\$5 per mile). Such an elastic demand function can be extended to other forms, like exponential or log-linear functions.

212 Figure 1 The elastic demand with varying price

213

211

206

214 3.3 Integrated model

215
$$\mathbf{P0} \max_{U,V,N,Q} \phi = -\sum_{i \in I} c_f \overline{V_i} + \mathbf{E}_{\omega \in \Xi} \left[\sum_{t \in T} \sum_{i \in I} \sum_{j \in I} \left[\left(P - c_e \right) g_{ij} + p_{pu} \right] Q_{ijt}(\omega) - \sum_{t \in T} \sum_{i \in I} \sum_{j \in I} c_r g_{ij} N_{ijt}(\omega) \right]$$
(2)

216 Constraints (1), plus:

217
$$V_{i1}(\omega) = \overline{V_i}$$
 $\forall \omega \in \Xi$ (3)

218
$$Q_{ijt}(\omega) \le q_{ijt}(\omega) \qquad \forall i \in I, j \in I, t \in T, \omega \in \Xi$$
(4)

219
$$\sum_{j \in I} Q_{ijt}(\omega) + \sum_{j \in I} N_{ijt}(\omega) \le V_{it}(\omega) \qquad \forall i \in I, t \in T, \omega \in \Xi$$
(5)

220
$$U_{ijt}(\omega) = Q_{ijt}(\omega) + N_{ijt}(\omega) \qquad \forall i \in I, j \in I, t \in T, \omega \in \Xi$$
(6)

221
$$W_{it}(\omega) = V_{it}(\omega) - \sum_{j \in I} U_{ijt}(\omega) \qquad \forall i \in I, t \in T, \omega \in \Xi$$
(7)

222
$$V_{it+1}(\omega) = W_{it}(\omega) + \sum_{j \in I} U_{jim}(\omega) \qquad \forall i \in I, t \in |T| - 1, m = max \{0, t+1 - \lceil g_{ji} / s \rceil\}, \omega \in \Xi$$
(8)

223
$$N_{ijt}(\omega), Q_{ijt}(\omega), \overline{V_i}, V_{it}(\omega), U_{ijt}(\omega), W_{ijt}(\omega) \in \mathbb{Z}^0 \qquad \forall i \in I, j \in I, t \in T, \omega \in \Xi$$
(9)

The objective function (2) is to maximize profit for the vehicle sharing operator. It is equal to the revenue minus the vehicle fixed cost, power consumption cost caused by the vehicle sharing service and proactive relocation. The revenue and cost of vehicle movements are obtained in random demand scenarios, so that the probability $\mathbf{E}_{\omega \in \Xi}$ [] is used. In the following explanation, we omit (ω) in all the decision variables for notation simplicity.

Constraints (3) require the number of vehicles at the beginning of daily operation to follow a static vehicle assignment planning. In Constraints (4), the number of served vehicle sharing requests Q_{ijt} cannot be larger than total demand q_{ijt} . Constraints (5) require that the number of served requests cannot exceed the number of available vehicles (either at the station or relocated

- from other zones) in traffic zone *i* at the beginning of time step *t*. Constraints (6) calculate U_{iit} ,
- the total number of vehicles moved from traffic zone i to traffic zone j at time instant t.
- 235 Constraints (7) calculate the total number of vehicles W_{it} idling in traffic zone *i* at time step *t*.
- 236 Constraints (8) calculate the total number of available vehicles in traffic zone *i* at next time instant
- 237 t+1. W_{it} indicates the number of idling vehicles. $\sum_{j \in I} U_{jim}$ indicates the number of vehicles
- arriving *i* between time step *t* and t+1. Constraints (9) specify the domain of the decision variables.
- 240
- 241 3.4 Non-linear challenge
- 242 The established model **P0** is not positive definite (PSD), because the term $\left[\left(P-c_{e}\right)g_{ii}+p_{pu}\right]Q_{iit}(\omega)$
- in the objective function is a square equation. Hence, model **P0** cannot be solved by using a commercial solver. One of the solution methods is to use a dynamic iteration that sets a fixed value of price and then solve the optimization model.
- 246

247 **4 Solution Method**

- 248 4.1 Optimization-based method
- In the optimization-based method, the demand in the base case setting is used to optimize the longterm vehicle assignment. The vehicle sharing operator should consider the average demand, but not a special day. Hence, all randomness $\omega \in \Xi$ is removed. New variables of \overline{Q}_{ijt} , \overline{N}_{ijt} , \overline{V}_{it} and
- 252 \overline{Q}_{it} are introduced in this model. The following shows the mathematical model.
- 253 254

$$\mathbf{P1}\max_{U,V,N,Q}\phi = -\sum_{i\in I}c_{f}\overline{V_{i}} + \sum_{t\in T}\sum_{i\in I}\sum_{j\in I}\left[\left(p-c_{e}\right)g_{ijt} + p_{pu}\right]\overline{Q}_{ijt} - \sum_{t\in T}\sum_{i\in I}\sum_{j\in I}c_{r}g_{ijt}\overline{N}_{ijt}$$
(10)

- 255 Subject to:
- 256 Constraints (3)-(8), plus:
- 257 $\overline{N}_{ijt}, \overline{Q}_{ijt}, \overline{V}_i, \overline{V}_{it}, \overline{W}_{ijt} \in \mathbb{Z}^0$ $\forall i \in I, j \in I, t \in T$ (11)

The objective function (10) maximizes profit for the vehicle sharing operator with the expected demand. Constraints (11) specify the domain of the decision variables.

- 260
- 261 4.2 Simulation-based method
- 262 *SUMO simulation setup*

Based on the values of \overline{V}_{i1} from the optimization model, the simulation model further offers a realistic representation of traffic flow. The simulation exercise is conducted using simulation of urban mObility (SUMO), which is an agent-based simulation tool that enables detailed real-time vehicle and passenger tracking. The mesoscopic version of SUMO is used, to offer faster computation speeds for the vehicle-sharing demand, while still capturing the key system performance. The link transmission model is leveraged instead of the car-following model and lane-changing model in the mesoscopic version for the traffic flow model.

- The vehicles are initialized based on the pattern suggested in the optimization model. A person is assumed to make a request at the parking lot, so the vehicle can pull out immediately. The vehicle then heads towards the destination parking lot using the shortest travel time path. After the vehicle arrives at the destination, the person parks and leaves the vehicle. The vehicle is then available for others to use. When there is no vehicle available in the parking lot, the request is considered a failed match, and the revenue is lost.
- The parking lots are initialized at an existing edge of the responding TAZ, and this edge is considered as the only destination of all the trips entering/leaving this TAZ. The parking lots are assumed to have unlimited capacity but when all the available vehicles have left a certain parking lot, this parking lot cannot offer vehicles to serve more people. The vehicle sharing clients in one TAZ are free to use the available vehicles at the edge of the current TAZ.
- 281
- 282 Simulation framework

283 The simulation starts with the optimization model to obtain the profit-maximizing vehicle assignment pattern, which is fed into the simulation model. These vehicles will be initialized at the 284 parking lots and serve riders for the morning peak period. At each timestep, the desired departures 285 and arrivals are examined. For the desired departures, the list of the people who would like to use 286 287 the vehicles at this timestep are checked. If there are available vehicles at the parking lot, a random vehicle is assigned to serve this person. In terms of arrivals, the list of all vehicle arrivals at each 288 time step is examined so that destination parking lots are identified, and the parking lot information 289 is updated accordingly. Finally, the vehicle and system performance are collected after the 290 simulation ends. 291

292

293 **5 Case Study**

294 5.1 Austin traffic network

The proposed methodology is adopted in the traffic network of Austin, Texas. All 6 counties of the Austin area are involved, including 2,210 traffic analysis zones (TAZs) and 23,576 links. Figure 2 illustrates the traffic network. As the purpose of this study is to explore the vehicle sharing system optimization, the shortest routes among TAZs are selected.

299

300 Figure 2. SUMO simulation traffic network in Austin, Texas

- 301
- 302 *K-means clustering*
- 303 The optimization-based method faces a challenge due to the large-scale network and discrete 304 operation time. For the decision variables of satisfied demand \overline{Q}_{iit} or relocation operations \overline{N}_{ijt} ,
- the optimization-based method will have 4,884,100 (2,210*2,210) variables for just one time step.
- 306 To reduce the computation burden, TAZ clustering is desired. In this paper, a k-means clustering
- 307 method is proposed such that a total of 2,210 TAZs are aggregated into 100 traffic zones (shown
- 308 in Figure 3). K-means is a classic method in data clustering analysis. The local optimal group size
- 309 is based on the Euclidian distance between the cluster center and each TAZ center. In the Austin

- 310 network, after 52 iterations, the optimal clusters are obtained.
- 311
- 312 Figure 3. K-mean clustering for the 6-county Austin network
- 313
- 314 Demand generation with dynamic travel time

The travel demand from 7 am to 10 am is used in this case study. The initial optimization time step

size tested 1 second as a unit, which led to a long computation time. Therefore, we divide the 3-

hour operation period into 36 time steps, with 5 minutes per time step. Vehicle movements are

- allowed at the beginning of each time period.
- 319
- 320 5.2 Optimization results
- 321 Base case

In this paper, the parameter settings were adopted from existing studies (Huang et al., 2018 & 2020a & 2020b; Loeb and Kockelman, 2019; Segal and Kockelman, 2016; Gurumurthy and Kockelman, 2020). The optimization model is solved on a laptop with Gurobi 9.1.1 solver on an i7 processor @3.60GHz, 32GB RAM in a Windows 10 64 bit operating system. Since the non-linear constraints have been relaxed by optimization and simulation solution method, Gurobi is a preferred choice of solver.

The basic setting assumes a fare of \$1.6 per (traveler-occupied) km (\$1 per mile), fuel cost of \$0.096 per km (\$0.06 per mile), vehicle relocation cost of \$0.192 per km (\$0.12 per mile), and vehicle depreciation, maintenance & insurance cost of \$0.64 per km (\$0.40 per mile). To reflect the uncertainty and variations (across settings) in the real world, sensitivity analyses are conducted on the cost assumptions, with 3 levels of price, relocation costs, and depreciation, maintenance & insurance costs. The optimization results are shown in Table 2.

334 The base case scenario demonstrates a profit of \$10.86 per vehicle over the 3-hour morning 335 peak period, with a total fleet size of 11,000 vehicles satisfying 67.12% of the vehicle-sharing travel demand. When increasing the vehicle rental price from \$1.28 per km (\$0.80 per mile) to 336 \$1.92 per km (\$1.20 per mile), the average profits per vehicle decrease to \$7.91 during the 3-hour 337 morning peak period. If extending to 24-hour operation, the average profit per vehicle is \$63.30 338 339 and the overall fleet size decreases by 22.0%. The percentage of served/satisfied requests drops 340 significantly with the increase of fare, thanks to the smaller fleet size. Results indicate that arranging 11,963 vehicles with a rental price of \$1.28 per km (\$0.80 per mile) in the Austin area 341 is a stable strategic planning that will not cut off too many vehicle sharing requirements. 342

343 Moreover, we conduct the relocation cost comparative analysis by fixing the price and 344 vehicle depreciation, maintenance & insurance cost to examine how the relocation cost affects the vehicle sharing system performance. With the increase of vehicle relocation cost, the number of 345 relocation operations drops lightly. Another finding is that the vehicle relocation cost does not 346 affect the fleet size decision. Also, the sensitivity of vehicle fixed cost is provided to analyze the 347 profits and fleet size changes. With the increase of vehicle fixed cost, the fleet size drops by about 348 349 26.1%, and number of relocations increases by 20.0%. In this way, the satisfied demand falls from 350 75.1% to 59.0%.

352 *Vehicle allocations*

353 Vehicle allocation results in this section are based on the base case settings of price, fuel cost, 354 vehicle relocation cost and vehicle maintenance cost in the above section. The total profits are 355 \$119,489 by serving 67.12% vehicle sharing demand. A total of 11,000 vehicles are assigned to 356 Austin in the morning peak hours, and 58 proactive vehicle relocations are conducted to move vehicles to high-demand areas. Vehicle in-service rate is 23%, as indicated by 41.52 minutes 357 driving time during the 3-hour morning peak hours. Table 3 shows the vehicle distributions at the 358 359 beginning of the daily operation. The average number of vehicles in traffic zones is 110, with the maximum value being 770, although some zones are not initialized with any vehicles. It shows 360 that traffic zones #26, #37, #57, #80, #83 and #88 are high-demand zones that might be residential 361 362 zones.

363

364 5.3 Simulation results

Based on the output of vehicle arrangement in optimization results, 30 scenarios are tested in this case study. The departure time of trips leaving one station during 3-hour morning peak hours follows a uniform distribution. Table 4 shows the simulation results.

The system performance in terms of service rate is quite robust with mean values varying within 0.10%. The low coefficient of variation (CoV) in service rate, average travel distance and average service time (all less than 1%) also prove that the proposed vehicle sharing operation mechanism is robust across 30 demand scenarios. Such results indicate that the proposed twophase solution method can ensure a steady service rate and handle random departure scenarios via flexible vehicle sharing operations.

Due to the steady performance in average service rate, 30 scenarios are shown to be 374 sufficient to represent the departure time uncertainties in the case study. Going through the 375 376 literature in vehicle sharing studies under uncertainty, we found that most studies also adopted a 377 relatively small number of scenarios due to computation resource constraints. For example, Brandstätter et al. (2017) and Fan (2014) adopted 7 scenarios to simulate the stochastic demand in 378 379 a week. He et al. (2017) chose 30 demand scenarios with the operation data between March and 380 April 2014, and Biondi et al. (2016) used 46 scenarios with the dataset covering the period from 381 May 17 to July 1, 2015. Only Lu et al. (2018) adopted 1000 scenarios to explore the profitability 382 and quality of service in vehicle sharing systems. In this Austin case study, a total of 100 zones 383 and 36 time steps together are considered. A larger number of scenarios will certainly further 384 improve the accuracy in system performance evaluation, yet at the cost of huge computation time.

385

386 5.4 Comparative analysis

387 To verify the advantage of the proposed simulation-based method, a comparative analysis is

388 provided. The first case is to use SUMO simulation in Phase 2, while the second case is to conduct

389 mathematical optimization in Phase 2. The vehicle arrangements among 100 traffic zones are given

based on the solutions of Phase 1, and travel demand in Scenario 1 is used in Phase 2.

- Satisfied demand reaches 80.75% in the simulation model, while it reaches 75.7% in the optimization model. The simulation model has better performance in terms of tracking vehicle movements in the operational level. It also means that the proposed methodology of using an agentbased simulation model in Phase 2 is compatible in the study framework. The average vehicletraveled distance and service time of vehicles increase by 47.52% and 49.76%, respectively. This may be because serving long-distance easily leads to larger profits.
- 397
- 398 5.5 Practical applications

In practice, the departure time of vehicle sharing clients are not fixed. For example, clients would depart earlier or later than peak hours to avoid traffic congestion. Only a few studies explore the influence of flexible departure time on the SAV system optimization. Hence, to address the research gaps, this paper optimizes the vehicle assignment problem and tracks exact vehicle movements under departure time uncertainties.

It is important for the profits and operation costs of SAV operation. The traditional method is to make strategic planning decisions based on the demand of a special day. Such a method cannot obtain the optimal solution. Decisions of vehicle assignment made for holidays would not be suitable for working days. Hence, this paper proposes an innovative optimization-simulation method to combine the advantages of two aspects: the optimization model can make macroscopic planning decisions, and the simulation model can track microscopic vehicle movements. Such a method can help the SAV operator make more effective decisions.

411

412 Conclusion

This study explores the SAV systems optimization under departure time uncertainty. A two-phase solution method is proposed to address the uncertainty problem. In Phase 1, a mathematical optimization model is established to decide the vehicle arrangement at the beginning of the day, using mean values of vehicle sharing demand. Based on the optimization results, the agent-based simulations under 30 demand scenarios are conducted in Phase 2 to analyze the system performance. The case study is conducted on the 6-county Austin traffic network with uncertain departures at 3-hour morning peak hours.

The optimization results show that maximum profits can be obtained when deploying 420 421 11,000 shared vehicles in the Austin area under the base case setting. Higher profits can be obtained 422 when vehicle rental price is set as \$1.28 per km (\$0.80 per mile), compared to prices at \$1.6 per 423 km (\$1 per mile) or \$1.92 per km (\$1.20 per mile). When increasing the relocation cost from \$0.096 per km (\$0.06 per mile) to \$0.32 per km (\$0.20 per mile), the number of vehicle relocations 424 425 decreases from 61 to 55. The profit of serving each trip is \$10.20 when using a high-cost vehicle, while using a low-cost vehicle can bring profits of \$11.60 for each trip. Furthermore, the SUMO 426 427 simulation results indicate 86% of requests are satisfied. With more satisfied demand, the average service time is 30 minutes in the SUMO simulation, which is longer than the average value of 23 428 429 minutes of all travelers in the optimization model. The low CoV proves that the proposed two-430 phase solution method has strong robustness. The proposed optimization-simulation solution 431 performs better than the existing optimization method. Using an agent-based model in Phase 2 can

432 improve the service rate by 6.7%.

433 The departure time uncertainty problem is proposed and explored in this paper, but the solution to the problem can be improved in future research. One of the limitations lies in the 434 435 departure time centralization in Phase 1's optimization problem. When calculating the average 436 departure time in 30 demand scenarios, the values are more likely to occur around the middle of the time horizon. This led to a higher peak demand than expected, leading to a larger fleet size. 437 Moreover, the limited computation memory allows the scenario of only 100 traffic zones with 438 439 2,210 TAZs, which might cut off some optimal solutions. Solving the large-scale system optimization problem is another possible research direction. Another limitation is that we have not 440 fulfilled the stochastic optimization. A direction for future studies would be to build a loop 441 framework between the two phases. In this way, an optimal solution can be obtained, as the 442 feedback from Phase 2 can affect the optimization in Phase 1. 443

444 Data Availability Statement

445 Some or all data, models, or code that support the findings of this study are available from the 446 corresponding author upon reasonable request.

447

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- 453 454

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T ?4	Vehic	le type	Solution	method	
Literature	GV	EV	Optimization	Simulation	Uncertain demand
Barrios and Godier (2014)	Х			Х	
Boyaci et al. (2015)	Х		Х		
Correia and Antunes (2012)	Х		Х		
Correia et al. (2014)	Х		Х		
Deng and Cardin (2018)	Х		Х		Х
Fan (2014)	Х		Х		Х
Huang et al. (2018)	Х		Х		
Huang et al. (2020a)		Х	Х		
Jorge et al. (2012)	Х		Х	Х	
Jorge et al. (2014)	Х		Х		
Jorge et al. (2015)	Х		Х		
Kek et al. (2009)	Х		Х		
Lu et al. (2018)	Х		Х		Х
Lu et al. (2020)	Х		Х		
Li et al. (2019)	Х		Х		Х
Monteiro et al. (2021)	Х			Х	
Nair and Miller-Hooks (2011)	Х		Х		
Nourinejad and Roorda (2014)	Х		Х		
Repoux et al. (2015)	Х		Х		
Xu et al. (2018)		Х	Х		
Xu and Meng (2019)		Х	Х		
Vasconcelos et al. (2017)	Х		Х		
Weikl and Bogenberger (2015)	Х		Х		
Zhao et al. (2018)		Х	Х		

Table 1 Literature of vehicle sharing fleet size study

settings)				gs)				
-	Items	Price and cost (\$ per mile, \$ per 0.625km)	\$ Profits per SAV	No. of SAVs	VMT per SAV (mile, 0.625km)	VHT per SAV (min)	Satisfied demand (%)	Proactive relocations
-		\$0.8	\$14.04 /SAV	11,963 SAVs	19.88 mi/SAV	39.17 min/SAV	71.77 (%)	69 SAVs
	Key price	1	10.86	11,000	21.62	41.52	67.12	58
-		1.2	7.91	9,330	23.49	45.10	58.78	53
	Vehicle relocation cost	0.06	10.87	10,999	21.63	41.52	67.11	61
		0.12	10.86	11,000	21.62	41.52	67.12	58
		0.20	10.86	10,997	21.63	41.52	67.10	55
	Vehicle	0.30	11.60	12,664	20.44	39.25	75.12	72
	depreciation, maintenance &	0.40	10.86	11,000	21.62	41.52	67.12	58
	insurance cost	0.50	10.23	9,358	23.46	45.03	58.98	60

Table 2 Shared automated vehicle fleet performance during 3-hour peak hours (with various price and cost settings)

573

574 Note: bolded values are base case settings on these three variables; price is in a unit of \$/mile paid by vehicle users;

575 vehicle relocation cost is in a unit of \$/mile paid by the fleet operator; vehicle fixed cost includes depreciation, 576 maintenance, and insurance cost, in a unit of \$/mile paid by the fleet operator.

Traffic zone	No. of vehicles
#1	46 SAVs
2	41
3	0
4	310
5	125
6	28
7	45
8	33
9	90
10	31
11	28
12	363
13	96
14	258
15	47
16	168
17	272
18	45
10	18
20	78
20	253
21	42
22	45
25	155
24	109
25	23
20	204
27	1
28	130
29	26
30	52
31	18
32	4
33	255
34	46
35	67
36	39
37	535
38	12
39	242
40	12
41	67
42	29
43	26
44	185
45	28
46	32

47	1
48	61
49	63
50	11
51	55
52	50
52	59
53	64
54	25
55	18
56	4
57	681
58	54
59	105
60	13
61	49
62	54
63	28
64	163
65	170
66	77
67	108
68	33
69	6
70	66
70	05
71	95
72	221
75	
74	2
15	97
/6	4/
//	94
78	209
79	51
80	770
81	12
82	172
83	652
84	217
85	40
86	13
87	102
88	556
89	49
90	19
91	7
92	53
93	73
94	0

95	163
96	0
97	1
98	0
99	89
100	12

5	7	Q
2	1)

Table 4 Vehicle distributions in 100 traffic zones

Scenarios	Satisfied demand (%)	Average travel distance (Mile, 0.625km)	Average service time (min)
1	85.99	15.51	29.34
2	86.19	15.58	29.60
3	86.14	15.68	29.73
4	86.22	15.56	29.69
5	86.22	15.69	29.91
6	86.22	15.57	29.62
7	86.13	15.51	29.52
8	86.17	15.52	29.44
9	86.02	15.63	29.59
10	86.22	15.62	29.63
11	86.18	15.58	29.52
12	86.04	15.53	29.30
13	86.02	15.66	29.76
14	86.10	15.56	29.45
15	86.25	15.59	29.72
16	86.03	15.63	29.53
17	86.14	15.62	29.61
18	86.05	15.70	29.94
19	86.01	15.59	29.58
20	86.16	15.56	29.59
21	86.01	15.51	29.33
22	86.09	15.66	29.61
23	86.12	15.64	29.74
24	86.02	15.60	29.61
25	86.15	15.61	29.69
26	86.18	15.51	29.52
27	85.93	15.64	29.43
28	86.06	15.57	29.43
29	85.95	15.65	29.62
30	85.99	15.66	29.53
Minimum value	85.93	15.51	29.30
Mean value	86.10	15.60	29.59
Maximum value	86.25	15.70	29.94
CoV	0.10%	0.37%	0.51%

- 581 Figure 1. The elastic demand with varying price
- 582 Figure 2. SUMO simulation traffic network in Austin, Texas
- 583 Figure 3. K-mean clustering for the 6-county Austin network