3.1

Course Number: CE 365K
Course Title: Hydraulic Engineering Design
Course Instructor: R.J. Charbeneau

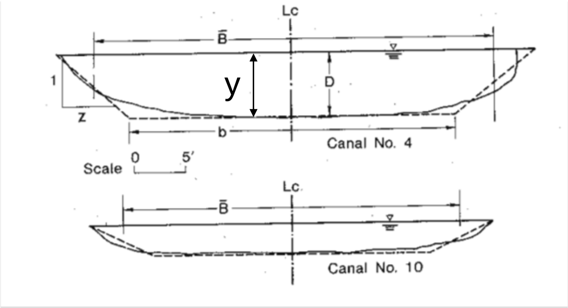
- Subject: ***Open Channel Hydraulics***
- Topics Covered:
  8. Open Channel Flow and Manning Equation
  9. Energy, Specific Energy, and Gradually Varied Flow
  10. Momentum (Hydraulic Jump)
  11. Computation: Direct Step Method and Channel Transitions
  12. Application of HEC-RAS
  13. Design of Stable Channels

3.2

Topic 8: Open Channel Flow

Geomorphology of Natural Channels:

Geomorphology of natural channels concerns their shape and structure. Natural channels are of irregular shape, varying from approximately parabolic to approximately trapezoidal (Chow, 1959).



Trapezoidal fit

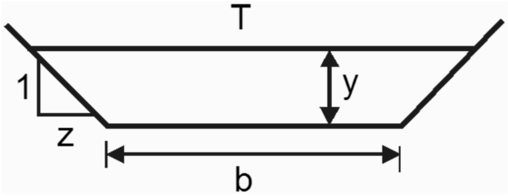
Channel Geometry Characteristics

- Depth, y
- Area, A
- Wetted perimeter, *P*
- Top width, T

Hydraulic Radius, R_h = Area / Wetted perimeter

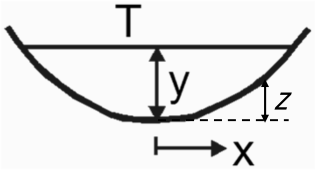
Hydraulic Depth, D_h = Area / Top width

Trapezoidal Channel



$$A = (b + z y) y \quad ; \quad P = b + 2y\sqrt{1+z^2} \quad ; \quad T = b + 2 z y$$

Parabolic Channel



$$z = a\,x^2 \quad ; \quad T = 2\sqrt{\frac{y}{a}} \quad ; \quad A = \frac{4}{3}\sqrt{\frac{1}{a}}\,y^{3/2} = \frac{2}{3}T\,y$$

If $0 < (4\ a\ y)^{1/2} < 1$ Then

$$P = T + \frac{8}{3}\frac{y^2}{T} \quad \rightarrow \quad R_h = \frac{\frac{2}{3}y}{1+\frac{8}{3}\left(\frac{y}{T}\right)^2} \approx \frac{2}{3}y$$

Example #12: Parabolic Channel

A grassy swale with parabolic cross-section shape has top width T = 6 m when depth y = 0.6 m while carrying stormwater runoff. What is the hydraulic radius?

$$a = \frac{4\,y}{T^2} = \frac{4 \times 0.6\,m}{(6m)^2} = 0.067\,m^{-1} \quad \rightarrow$$

$$R_h = \frac{\frac{2}{3}y}{1+\frac{8}{3}\left(\frac{y}{T}\right)^2} = 0.390\,m \cong \frac{2}{3}y$$

Flow in Open Channels: Manning Equation

Manning’s equation is used to relate the average channel (conduit) velocity to energy loss, $S_f = h_f/L$.

Manning equation (<u>metric units</u>: m, s)

$$V = \frac{1}{n} R_h^{\ 2/3} S_f^{\ 1/2} \quad \leftrightarrow \quad Q = \frac{1}{n} A R_h^{\ 2/3} S_f^{\ 1/2}$$

UNITS ?!!? Does “n” have units? Tabulated values?

Manning Equation (Cont.)

$$\frac{V}{R_h^{\ 2/3}} = \frac{Q}{AR_h^{\ 2/3}} = \frac{S_f^{\ 1/2}}{n} \quad \left[\frac{m^{1/3}}{s}\right]$$

To change to US Customary units multiply by

$$\phi = \left(L_R\right)^{1/3} = \left(3.28\,\tfrac{ft}{m}\right)^{1/3} = 1.486$$

General case

$$Q = \frac{\phi}{n} A R_h^{\ 2/3} S_f^{\ 1/2}$$

ϕ = 1 (metric) or 1.486 (English)

3.9

Channel Conveyance, K

For Manning's equation, K combines roughness and geometric characteristics of the channel

$$K = (\phi/n)\ A\ R_h^{2/3}$$

Manning's equation: $Q = K\ S_f^{1/2}$

3.10

Roughness and Manning's n

Equivalence between roughness size (k) and Manning's n:

$$\boxed{n = 0.034\ k^{1/6} \quad \text{(k in ft)}}$$

Strickler (1923)

Examples	n	k (cm)
Concrete (finished)	0.012	0.06
Asphalt	0.016	0.3
Earth channel (gravel)	0.025	5
Natural channel (clean)	0.030	15
Floodplain (light brush)	0.050	300

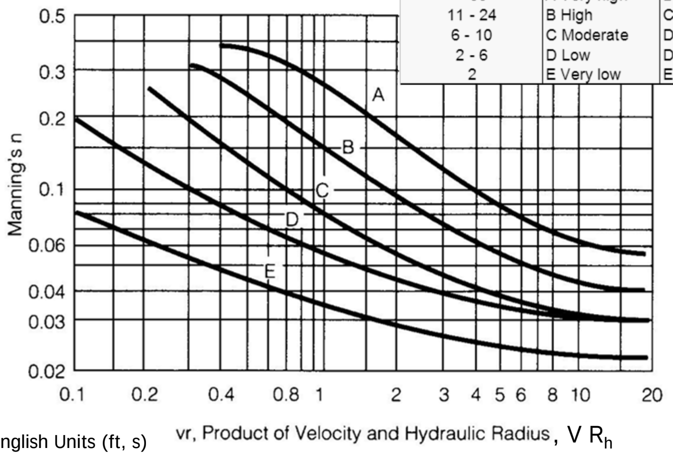
* Compare with Manning's n for sheet flow

Grassed Channels

Vegetal Retardance Class (Table 8.4)

Average Length of Grass (inches)	Grass Density	
	Good	Fair
> 30	A Very high	B High
11 - 24	B High	C Moderate
6 - 10	C Moderate	D Low
2 - 6	D Low	D Low
2	E Very low	E Very low

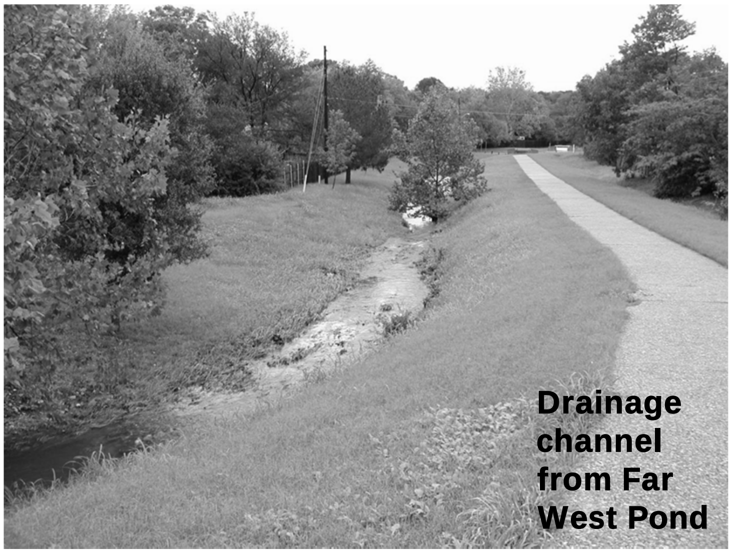
SCS (after Chow, 1959)



English Units (ft, s) , V R_h

Figure 8.11, page 311

Example #13: Estimate the channel discharge capacity for S_o = 0.008



Drainage channel from Far West Pond

3.13

Example #13: Q_{max} (cfs) =

1. For grass channels, use Slide 3.11. Guess an initial value n = 0.05

2. Geometry:

Bottom width, b = 5 ft (~)

Side slope (z:1), z = 2

Maximum depth, y = 4 ft

$\rightarrow$ Q = 240 ft^3/s

Hydraulic radius, R_h = 2.3 ft

Velocity, V = 4.6 ft/s

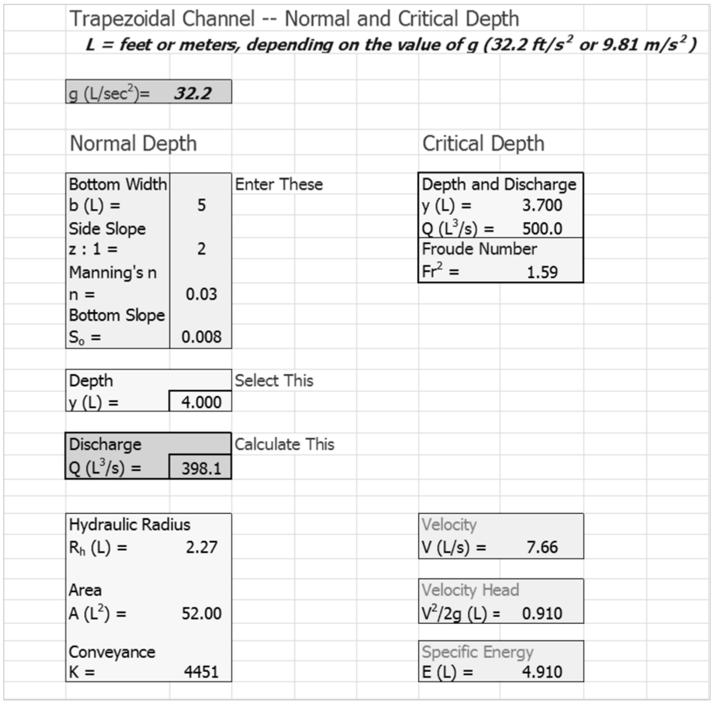
$\rightarrow$ V R_h = 10.6 ft^2/s

3.14

Example #13 (Cont.)

V R_h > 10 ft2/s → n = 0.03 (slide 3.11)

Q_{max} = 400 cfs



Trapezoidal Channel -- Normal and Critical Depth

L = feet or meters, depending on the value of g (32.2 ft/s^2 or 9.81 m/s^2)

g (L/sec^2)= ***32.2***

Normal Depth

Bottom Width
b (L) = 5
Side Slope
z : 1 = 2
Manning's n
n = 0.03
Bottom Slope
S_o = 0.008

Enter These

Depth
y (L) = 4.000

Select This

Discharge
Q (L^3/s) = 398.1

Calculate This

Hydraulic Radius
R_h (L) = 2.27

Area
A (L^2) = 52.00

Conveyance
K = 4451

Critical Depth

Depth and Discharge
y (L) = 3.700
Q (L^3/s) = 500.0
Froude Number
Fr^2 = 1.59

Velocity
V (L/s) = 7.66

Velocity Head
$V^2/2g$ (L) = 0.910

Specific Energy
E (L) = 4.910

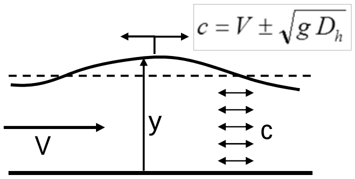
Normal Depth

Normal depth is the depth of ***uniform flow*** in an prismatic open channel. Since the flow is uniform, the depth and discharge are related through Manning's equation with $S_f = S_o$.

$$Q = \frac{\phi}{n} A R_h^{2/3} S_o^{1/2} \quad \rightarrow \quad y_n$$

Given Q, n, $A(y)$, $R_h(y)$ and S_o: solve for y_n

Waves (Small Disturbances) in a Moving Stream



$$c = V \pm \sqrt{g D_h}$$

Wave (disturbance) can move upstream if

$$\boxed{V < \sqrt{g D_h} \quad \left(Fr = \frac{V}{\sqrt{g D_h}} < 1 \right)}$$

Froude Number

Critical Depth – Froude number

Critical flow occurs when the velocity of water is the same as the speed at which disturbances of the free surface will move through shallow water. The speed or *celerity* of disturbances in shallow water is given by c = $(g\ D_h)^{1/2}$, where D_h is the hydraulic depth. Critical flow occurs when v = c, or more generally

$$\boxed{Fr \equiv \frac{V}{\sqrt{g\,D_h}} \quad \rightarrow \quad Fr_c^{\,2} = \frac{(Q/A)^2}{g\left(A/T\right)} = \frac{Q^2\,T}{g\,A^3} = 1 \quad \rightarrow \quad y_c}$$

Importantly, critical depth is independent of the channel slope.

Topic 9: Energy, Specific Energy, and Gradually Varied Flow

1D Energy Equation:

Closed Conduit:

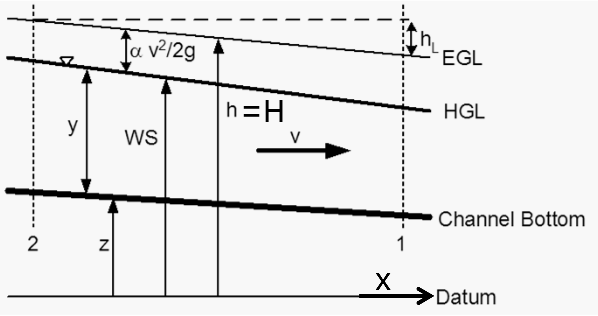
$$\left(\frac{V^2}{2g}+\frac{p}{\gamma}+z\right)_1 = \left(\frac{V^2}{2g}+\frac{p}{\gamma}+z\right)_2 + h_L$$

Open Channel Flow:

$$\left(\frac{V^2}{2g}+y+z_B\right)_1 = \left(\frac{V^2}{2g}+y+z_B\right)_2 + h_L$$

$z_B \rightarrow z$ hereafter

Energy in Open Channels



Friction Slope:

→

S_f = - dH/dx

Channel Slope:

→

S_o = - dz/dx

$$Head\ H = \alpha \frac{V^2}{2g} + y + z = \alpha \frac{Q^2}{2gA^2} + y + z = E + z$$

Kinetic
Energy
Correction
Factor

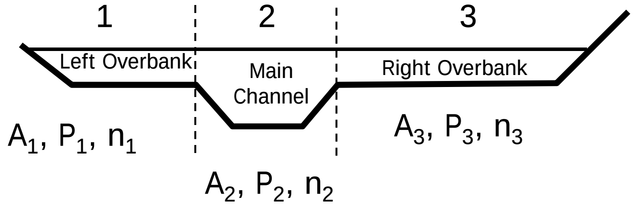
$$\alpha = \frac{1}{A} \iint \left(\frac{V}{V_{ave}} \right)^3 dA$$

Specific
Energy

What do we do with α?

- For simple channels assume α = 1
- For complex channels (main channel plus left and right-bank floodplains), velocity variation at a given station can be significant, and α should be calculated and used in a 1D energy equation (HEC-RAS does this automatically!)

Kinetic Energy Coefficient, α

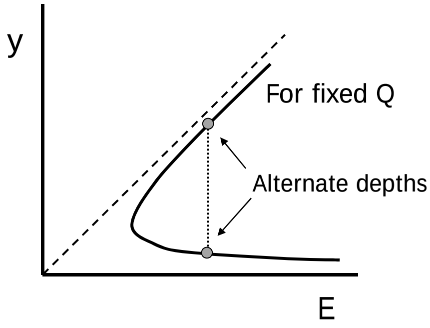


$$\alpha \frac{\overline{V}^2}{2g} = \frac{\sum_i Q_i \left(\alpha_i V_i^2 / 2g\right)}{\sum_i Q_i} \longrightarrow \boxed{\alpha = A^2 \frac{\sum_i \left(\alpha_i K_i^3 / A_i^2\right)}{\left(\sum_i K_i\right)^3}}$$

Specific Energy, E

Hydraulic energy head measured with respect to the local channel bottom, as a function of depth y

$$\boxed{E = \frac{V^2}{2g} + y = \frac{Q^2}{2gA^2} + y}$$



Specific Energy at Critical Flow

Rectangular channel: $D_h = y$

$$E = V^2/2g + y = [(1/2)\ (V^2/gy) + 1]\ y$$

$$= (1/2 + 1)\ y$$

Froude Number

For critical flow (in a rectangular channel):

$y = (2/3)\ E$

$V^2/2g = (1/3)\ E$

Energy Equation

$$\boxed{E_2 + z_2 = E_1 + z_1 + h_{L(2 \rightarrow 1)}}$$

E = Specific Energy = $y + V^2/2g$

Head Loss:

- Major Losses – friction losses along channel
- Minor Losses – channel expansion and contraction

Friction Losses in Open Channel Flow:

Slope of the EGL: $S_f = h_f / L$

Manning's equation: $Q = K S_f^{1/2}$

Bed-friction head loss: $h_f = (Q/K)^2 L$

Minor (Expansion and Contraction) Losses

Energy losses at channel expansions and contractions

$$h_m = C\left|\frac{V_2^{\,2} - V_1^{\,2}}{2g}\right| = \frac{CQ^2}{2g}\left|\frac{1}{A_2^{\,2}} - \frac{1}{A_1^{\,2}}\right|$$

Default values:

Channel Contraction - C = 0.1
Channel Expansion - C = 0.3

Abrupt Expansion: (C = 1)

$$h_m = \frac{V^2}{2g}$$

Gradually Varied Flow Profiles

Physical laws governing the head variation in open channel flow

$$H = \frac{V^2}{2g} + y + z = E + z$$

$$\left(-\frac{d}{dx}\right)H = \left(-\frac{d}{dx}\right)(E+z) \rightarrow S_f = -\frac{dE}{dx} + S_o \rightarrow \boxed{S_o = S_f + \frac{dE}{dx}}$$

1) Gravity (S_o) is the driving force for flow
2) If $S_o = S_f$ then $dE/dx = 0$ and flow is <u>uniform</u> (*normal depth*)
3) Gravity (S_o) is balanced by friction resistance (S_f) and longitudinal adjustment in specific energy (dE/dx)
4) Adjustments in specific energy are constrained through specific energy diagram

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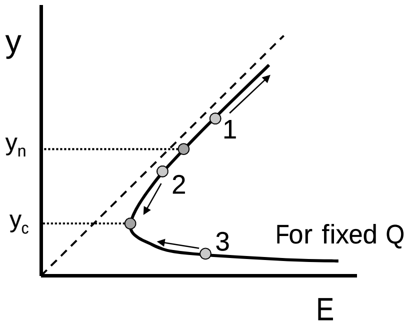
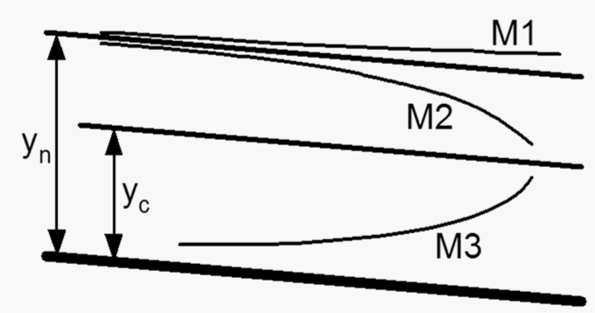
Gradually Varied Flow: Mild Slope ($y_n > y_c$)

$$S_o = S_f + \frac{dE}{dx}$$

1. Point 1 (M1 Curve): $y > y_n$ → $S_f < S_o$ → $dE/dx > 0$ → depth increases downstream
2. Point 2 (M2 Curve): $y < y_n$ → $S_f > S_o$ → $dE/dx < 0$ → depth <u>decreases</u> downstream
3. Point 3 (M3 Curve): $y < y_c < y_n$ → $S_f > S_o$ → $dE/dx < 0$ → depth <u>increases</u> downstream



Mild Slope

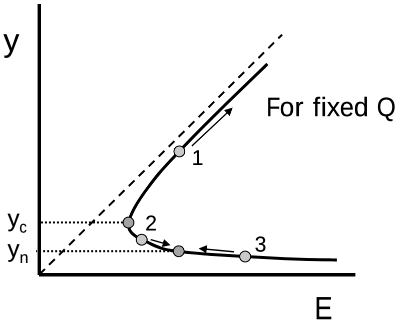


Note that for the M1 and M2 curves, the depth approaches normal depth in the direction of flow computation for subcritical flow.

Gradually Varied Flow: Steep Slope ($y_n < y_c$)

$$S_o = S_f + \frac{dE}{dx}$$

1. Point 1 (S1 Curve): $y > y_c > y_n$ → $S_f < S_o$ → $dE/dx > 0$ → depth increases downstream
2. Point 2 (S2 Curve): $y > y_n$ → $S_f < S_o$ → $dE/dx > 0$ → depth <u>decreases</u> downstream
3. Point 3 (S3 Curve): $y < y_n$ → $S_f > S_o$ → $dE/dx < 0$ → depth <u>increases</u> downstream



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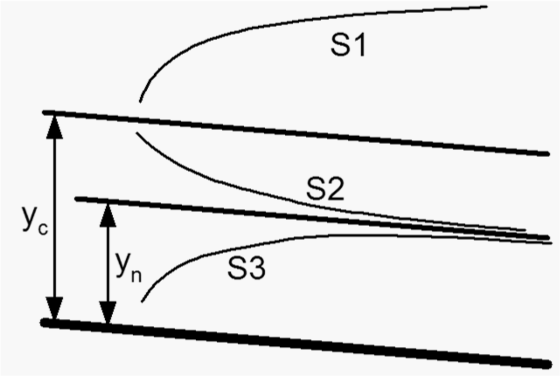
Steep Slope



Note that for curves S2 and S3 the depth approaches normal depth in the direction for flow computation for supercritical flow

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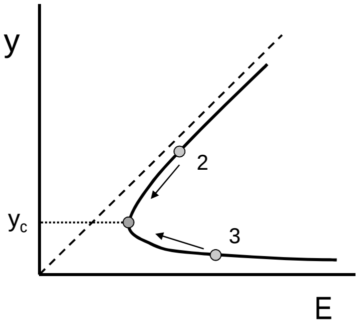
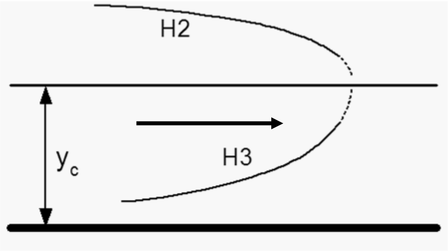
3.32

Horizontal Slope (S_o = 0)

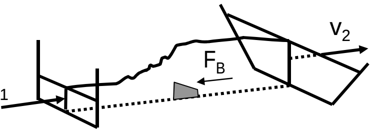
$$S_o = S_f + \frac{dE}{dx} \quad \rightarrow \quad S_f = -\frac{dE}{dy}\frac{dy}{dx} > 0$$





Topic 10: Momentum and Hydraulic Jump

y_p = depth to pressure center
(X-section centroid)



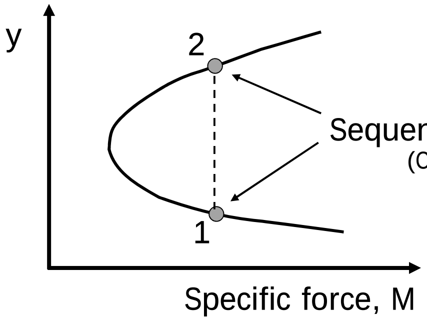
Momentum Equation: $\Sigma F = \rho\, Q\, (V_2 - V_1)$

$$\gamma\, y_{p1}\, A_1 - \gamma\, y_{p2}\, A_2 - F_B = \rho\, Q^2\, (1/A_2 - 1/A_1)$$

Write as: $\boxed{M_1 = M_2 + F_B/\gamma}$

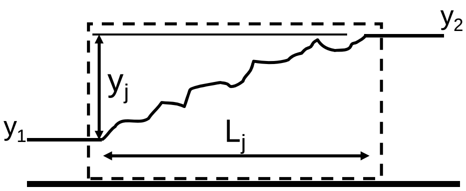
$\boxed{\text{Specific Force: } M = y_p\, A + Q^2/(g\, A)}$

Hydraulic Jump

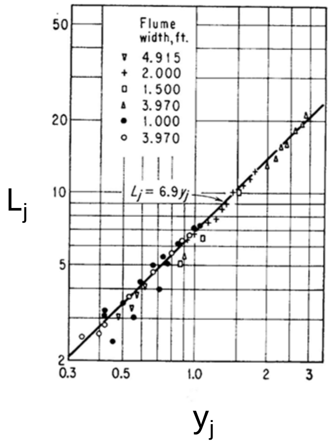


Sequent depths
(Conjugate)

Energy Loss and Length



Energy Loss: $\boxed{\Delta E = \frac{y_j^3}{4\,y_1\,y_2}}$ Rectangular Channel



Exercise: convince yourself that this is the same as $\Delta E = E_1 - E_2$.

Jump Length:

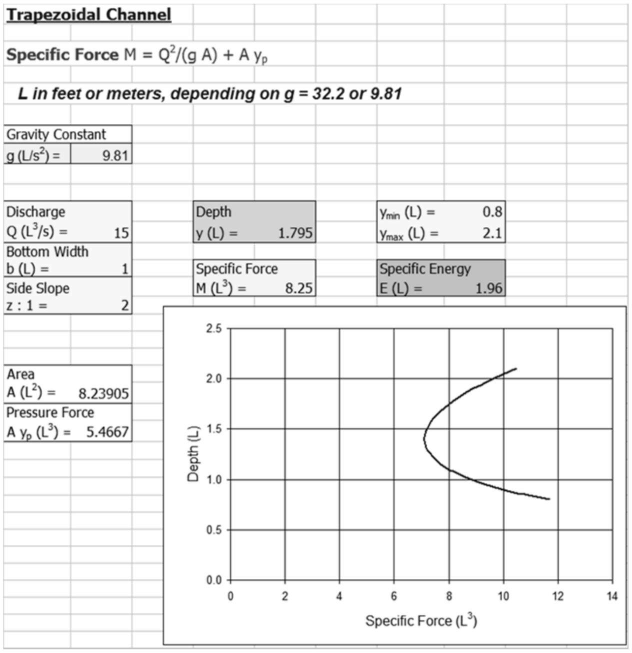
$\boxed{L_j = 6.9\ y_j = 6.9\ (y_2 - y_1)}$

Example #14: Normal depth downstream in a trapezoidal channel is 1.795 m when the discharge is 15 m^3/s. What is the upstream sequent depth?

Answer:
1.065 m

How do you find the energy loss?

$\Delta E = 0.14$ m
Power = 20.6 kW



Trapezoidal Channel

Specific Force $M = Q^2/(g\ A) + A\ y_p$

L in feet or meters, depending on g = 32.2 or 9.81

Gravity Constant	
g (L/s^2) =	9.81

Discharge		Depth		y_{min} (L) =	0.8
Q (L^3/s) =	15	y (L) =	1.795	y_{max} (L) =	2.1
Bottom Width					
b (L) =	1	Specific Force		Specific Energy	
Side Slope		M (L^3) =	8.25	E (L) =	1.96
z : 1 =	2				

Area	
A (L^2) =	8.23905
Pressure Force	
A y_p (L^3) =	5.4667

Topic 11: Direct Step Method and Channel Transitions

Returning to the Energy Equation

$$E_2 + z_2 = E_1 + z_1 + h_L(2 \rightarrow 1)$$

Step calculation

$$z_2 - z_1 = S_o \ \Delta x$$

$$h_L(2 \rightarrow 1) = \overline{S}_f \ \Delta x + C \left| \Delta \frac{Q^2}{2gA^2} \right|$$

Solve for Δx:

$$\Delta x = \frac{E_1 + C \left| \Delta \frac{Q^2}{2gA^2} \right| - E_2}{S_o - \overline{S}_f}$$

Direct Step Method: Approach

$$\Delta x = \frac{E_1 + C \left| \Delta \frac{Q^2}{2gA^2} \right| - E_2}{S_o - \overline{S}_f}$$

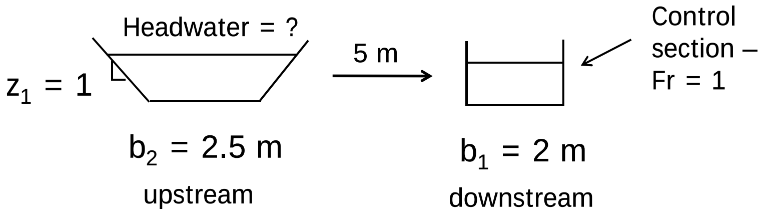
In this case you select a sequence of depths and solve for the distance between them: y_1, y_2, y_3, ...

- With known y: E_1, E_2, $Q^2/2gA^2$, etc. are calculated
- The average friction slope is calculated from

$$\overline{S}_f = \left(\frac{Q_2 + Q_1}{K_2 + K_1} \right)^2$$

Example #15: Channel Transition

What is the headwater upstream of a control section in a downstream box culvert: Q = 10 m^3/s; channel n = 0.025



Headwater = ?

z_1 = 1

5 m

Control section – Fr = 1

b_2 = 2.5 m

upstream

b_1 = 2 m

downstream

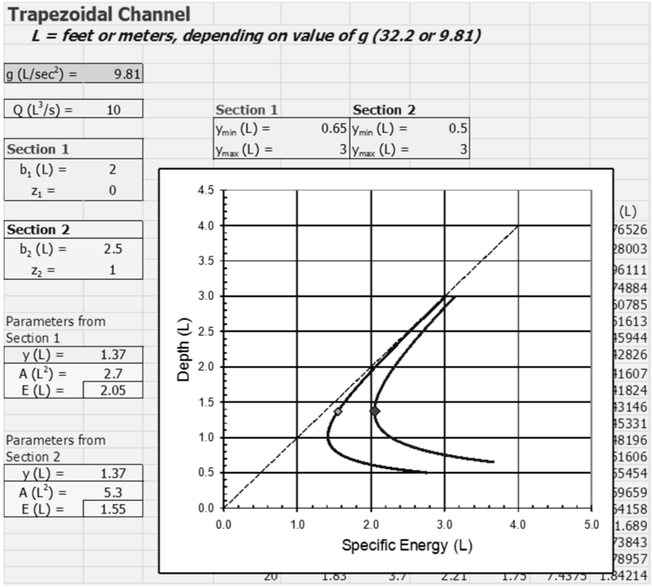
Control Section:

$$V = (g\ D_h)^{0.5} = (g\ y)^{0.5} \rightarrow Q = (g\ y)^{0.5}\ (b\ y)$$

$$y = [Q / b\ g^{0.5}]^{2/3} = 1.37\ \text{m}$$

Example #15 (Cont.): Specific Energy

A 1.37 m depth in the upstream channel (Section 1, blue) corresponds to specific energy E = 1.55 m, compared with E = 2.05 m in the control section. The upstream E_2 must exceed 2.05 m, which requires an increase in depth.



Trapezoidal Channel

L = feet or meters, depending on value of g (32.2 or 9.81)

g (L/sec^2) = 9.81

Q (L^3/s) = 10

Section 1

b_1 (L) = 2

z_1 = 0

Section 2

b_2 (L) = 2.5

z_2 = 1

Section 1

y_{min} (L) = 0.65

y_{max} (L) = 3

Section 2

y_{min} (L) = 0.5

y_{max} (L) = 3

Parameters from Section 1

y (L) = 1.37

A (L^2) = 2.7

E (L) = 2.05

Parameters from Section 2

y (L) = 1.37

A (L^2) = 5.3

E (L) = 1.55

3.41

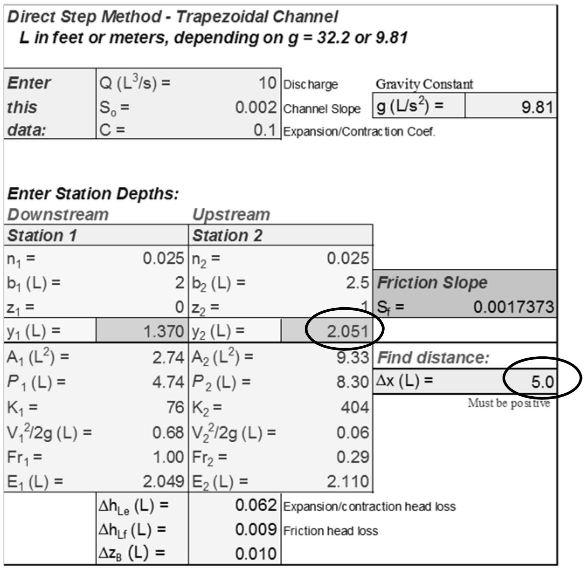
Example #15 (Cont.): Direct Step Method

Assume a 5-m long transition section. Adjust depth until ∆x = 5 m → y_2 = 2.051 m

Resulting upstream specific energy is E_2 = 2.110 m.

Contraction losses at entrance ~ 6 cm

Friction losses in transition section ~ 1 cm



Direct Step Method - Trapezoidal Channel
L in feet or meters, depending on g = 32.2 or 9.81

Enter	Q (L^3/s) =	10	Discharge
this	S_o =	0.002	Channel Slope
data:	C =	0.1	Expansion/Contraction Coef.

Gravity Constant
g (L/s^2) = 9.81

Enter Station Depths:

Downstream		***Upstream***	
Station 1		***Station 2***	
n_1 =	0.025	n_2 =	0.025
b_1 (L) =	2	b_2 (L) =	2.5
z_1 =	0	z_2 =	1
y_1 (L) =	1.370	y_2 (L) =	2.051
A_1 (L^2) =	2.74	A_2 (L^2) =	9.33
P_1 (L) =	4.74	P_2 (L) =	8.30
K_1 =	76	K_2 =	404
$V_1^2/2g$ (L) =	0.68	$V_2^2/2g$ (L) =	0.06
Fr_1 =	1.00	Fr_2 =	0.29
E_1 (L) =	2.049	E_2 (L) =	2.110

Friction Slope
S_f = 0.0017373

Find distance:
∆x (L) = 5.0
Must be positive

Δh_{Le} (L) =	0.062	Expansion/contraction head loss
Δh_{Lf} (L) =	0.009	Friction head loss
Δz_B (L) =	0.010	

3.42

Example #16: Specific Energy and Channel Transitions

- Trapezoidal channel with b = 8 ft, z = 2, n = 0.030. Normal depth occurs upstream and downstream.
- Rectangular culvert (b = 5 ft, n = 0.012) added with concrete apron extending 10 feet downstream from culvert outlet.
- Develop flow profile, especially downstream of the culvert, for Q = 250 cfs.

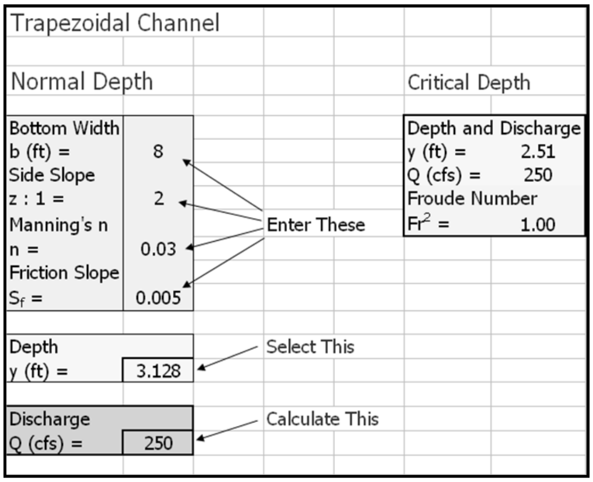
Ex. #16 (Cont.): Normal and Critical Depth in Main Channel

Upstream and Downstream channel have

y_n = 3.13 ft

y_c = 2.51 ft

$y_n > y_c$ → Mild Slope → Downstream Control



Trapezoidal Channel

Normal Depth

Bottom Width
b (ft) = 8
Side Slope
z : 1 = 2
Manning's n
n = 0.03
Friction Slope
S_f = 0.005

Enter These

Depth
y (ft) = 3.128

Select This

Discharge
Q (cfs) = 250

Calculate This

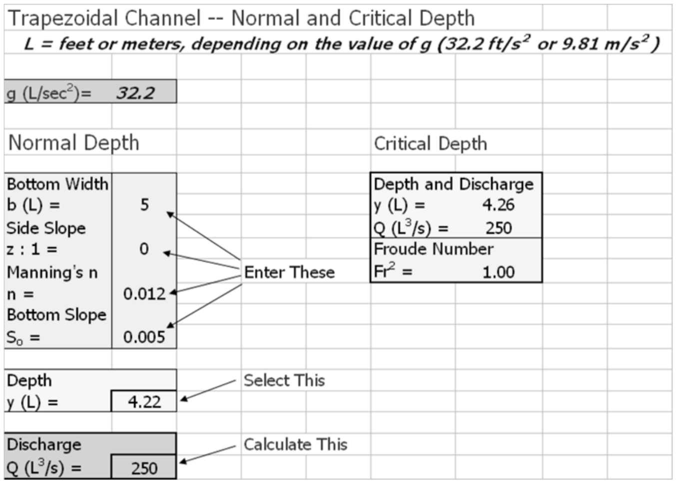
Critical Depth

Depth and Discharge
y (ft) = 2.51
Q (cfs) = 250
Froude Number
Fr^2 = 1.00

Ex. # 16 (Cont.): Normal and Critical Depth in Culvert

y_n = 4.22 ft

y_c = 4.26 ft



Trapezoidal Channel -- Normal and Critical Depth

L = feet or meters, depending on the value of g (32.2 ft/s^2 or 9.81 m/s^2)

g (L/sec^2)= 32.2

Normal Depth

Bottom Width
b (L) = 5
Side Slope
z : 1 = 0
Manning's n
n = 0.012
Bottom Slope
S_o = 0.005

Enter These

Depth
y (L) = 4.22

Select This

Discharge
Q (L^3/s) = 250

Calculate This

Critical Depth

Depth and Discharge
y (L) = 4.26
Q (L^3/s) = 250
Froude Number
Fr^2 = 1.00

$y_n < y_c$ → Steep Slope → Upstream control

3.45

Ex. #16 (Cont.): Specific Energy Diagram

Normal depth:

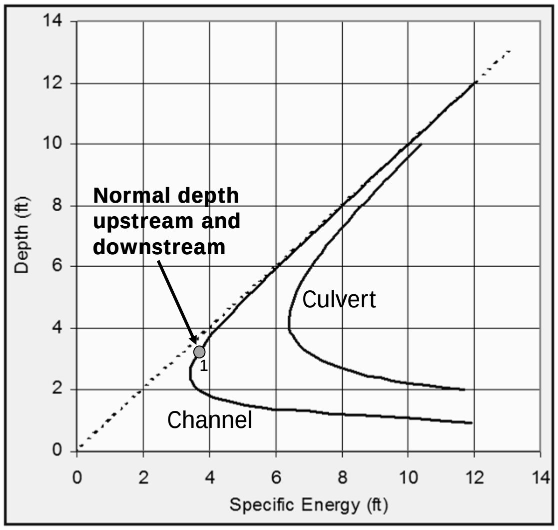
E = 3.13 ft

Minimum specific energy in culvert:

E = 6.40 ft

To enter the culvert from upstream, specific energy increased through M1 curve

(Culvert as "choke")

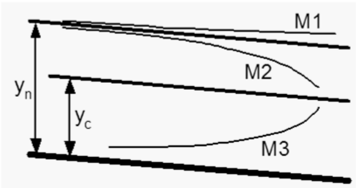


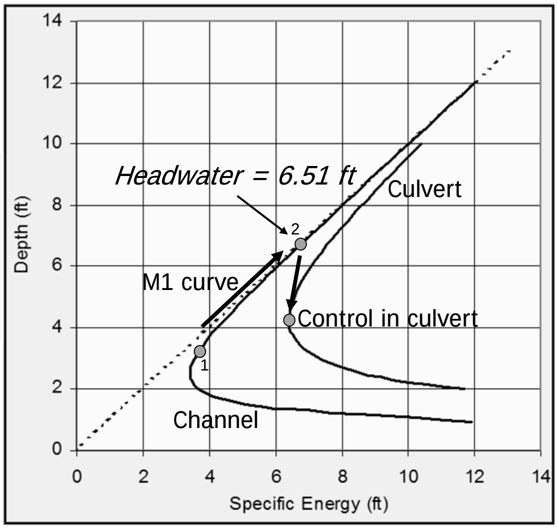
3.46

Ex. #16 (Cont.) Upstream Transition and Entry to Culvert

Trapezoidal Channel

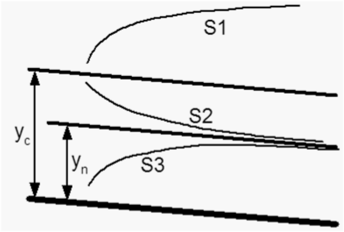
→ M1 Curve



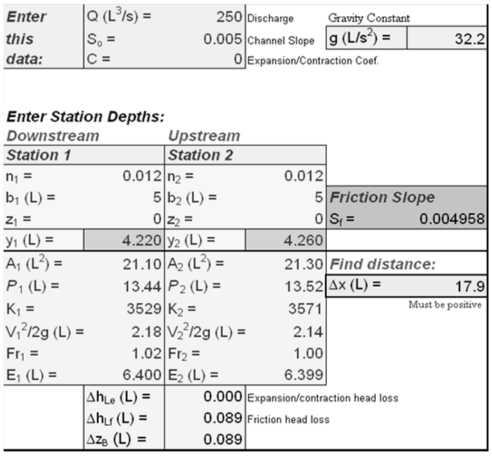


Ex. #16 (Cont.): Control in Culvert

$y_n < y_c$ → Steep Slope → Control at Culvert Entrance



S2 curve in culvert; normal depth reached before culvert end



Enter this data:
Q (L^3/s) = 250 Discharge
S_o = 0.005 Channel Slope
C = 0 Expansion/Contraction Coef.
Gravity Constant
g (L/s^2) = 32.2

Enter Station Depths:

Downstream		*Upstream*	
Station 1		**Station 2**	
n_1 =	0.012	n_2 =	0.012
b_1 (L) =	5	b_2 (L) =	5
z_1 =	0	z_2 =	0
y_1 (L) =	4.220	y_2 (L) =	4.260
A_1 (L^2) =	21.10	A_2 (L^2) =	21.30
P_1 (L) =	13.44	P_2 (L) =	13.52
K_1 =	3529	K_2 =	3571
$V_1^2/2g$ (L) =	2.18	$V_2^2/2g$ (L) =	2.14
Fr_1 =	1.02	Fr_2 =	1.00
E_1 (L) =	6.400	E_2 (L) =	6.399

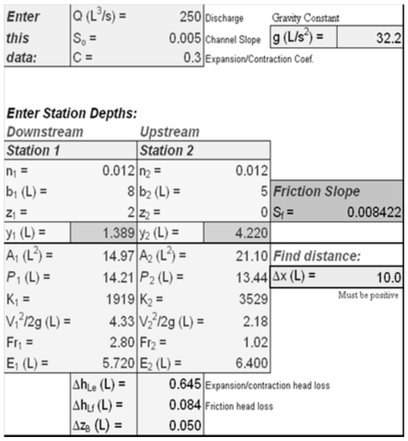
Friction Slope
S_f = 0.004958

Find distance:
Δx (L) = 17.9
Must be positive

Δh_{Le} (L) = 0.000 Expansion/contraction head loss
Δh_{Lf} (L) = 0.089 Friction head loss
Δz_B (L) = 0.089

Ex. #16 (Cont.): Possible Depths at Concrete Apron

Set $y_2 = y_n$ in culvert and adjust $y_1 = y_a$ until Δx = 10 ft for depth at end of concrete apron



Enter this data:
Q (L^3/s) = 250 Discharge
S_o = 0.005 Channel Slope
C = 0.3 Expansion/Contraction Coef.
Gravity Constant
g (L/s^2) = 32.2

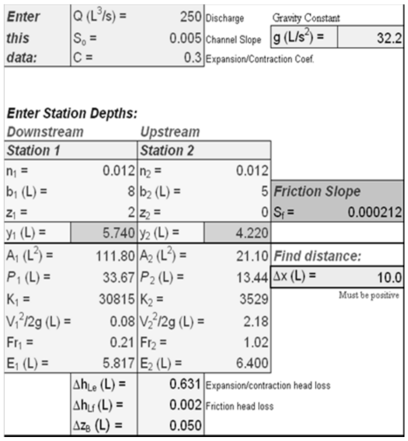
Enter Station Depths:

Downstream		*Upstream*	
Station 1		**Station 2**	
n_1 =	0.012	n_2 =	0.012
b_1 (L) =	8	b_2 (L) =	5
z_1 =	2	z_2 =	0
y_1 (L) =	1.389	y_2 (L) =	4.220
A_1 (L^2) =	14.97	A_2 (L^2) =	21.10
P_1 (L) =	14.21	P_2 (L) =	13.44
K_1 =	1919	K_2 =	3529
$V_1^2/2g$ (L) =	4.33	$V_2^2/2g$ (L) =	2.18
Fr_1 =	2.80	Fr_2 =	1.02
E_1 (L) =	5.720	E_2 (L) =	6.400

Friction Slope
S_f = 0.008422

Find distance:
Δx (L) = 10.0
Must be positive

Δh_{Le} (L) = 0.645 Expansion/contraction head loss
Δh_{Lf} (L) = 0.084 Friction head loss
Δz_B (L) = 0.050



Enter this data:
Q (L^3/s) = 250 Discharge
S_o = 0.005 Channel Slope
C = 0.3 Expansion/Contraction Coef.
Gravity Constant
g (L/s^2) = 32.2

Enter Station Depths:

Downstream		*Upstream*	
Station 1		**Station 2**	
n_1 =	0.012	n_2 =	0.012
b_1 (L) =	8	b_2 (L) =	5
z_1 =	2	z_2 =	0
y_1 (L) =	5.740	y_2 (L) =	4.220
A_1 (L^2) =	111.80	A_2 (L^2) =	21.10
P_1 (L) =	33.67	P_2 (L) =	13.44
K_1 =	30815	K_2 =	3529
$V_1^2/2g$ (L) =	0.08	$V_2^2/2g$ (L) =	2.18
Fr_1 =	0.21	Fr_2 =	1.02
E_1 (L) =	5.817	E_2 (L) =	6.400

Friction Slope
S_f = 0.000212

Find distance:
Δx (L) = 10.0
Must be positive

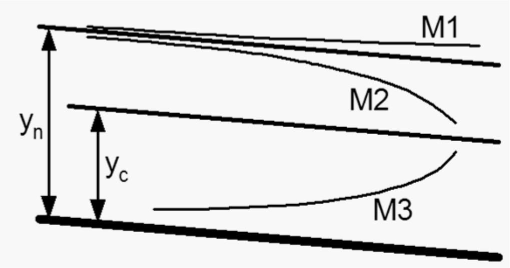
Δh_{Le} (L) = 0.631 Expansion/contraction head loss
Δh_{Lf} (L) = 0.002 Friction head loss
Δz_B (L) = 0.050

Possible depths for y_a = 1.389 ft or 5.74 ft

Ex. #16 (Cont.): Which Depth for y_a?

How do you determine which depth at the end of the culvert apron is correct?

The flow profile downstream of the apron must follow an M-curve (since $y_c < y_n$ in the downstream channel) and the depth must end in normal depth.



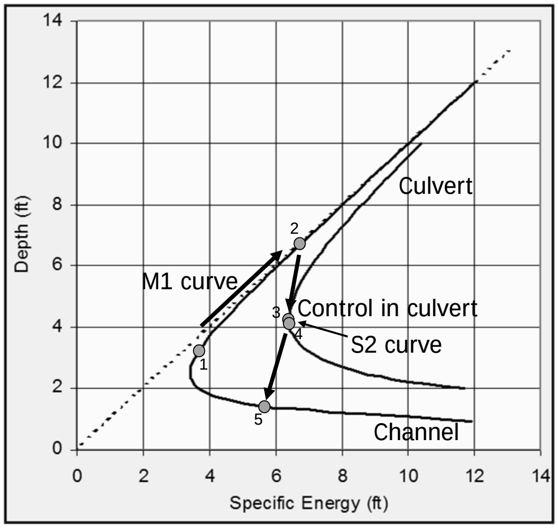
Normal depth is $y_n = 3.13$ ft while $y_a = 1.39$ ft or 5.74 ft.

Cannot reach normal depth from y = 5.74 ft following an M1 curve.

Correct depth is $y_a = 1.39$ ft, which will follow an M3 curve leading to a hydraulic jump, which must end at normal depth (no M-curves approach normal depth in the downstream direction).

Ex. #16 (Cont.): Transition to Apron (Pt. 5)

End of culvert apron at Pt. 5. M3 Curve leading to Hydraulic Jump, which in turn exits at normal depth for the downstream channel



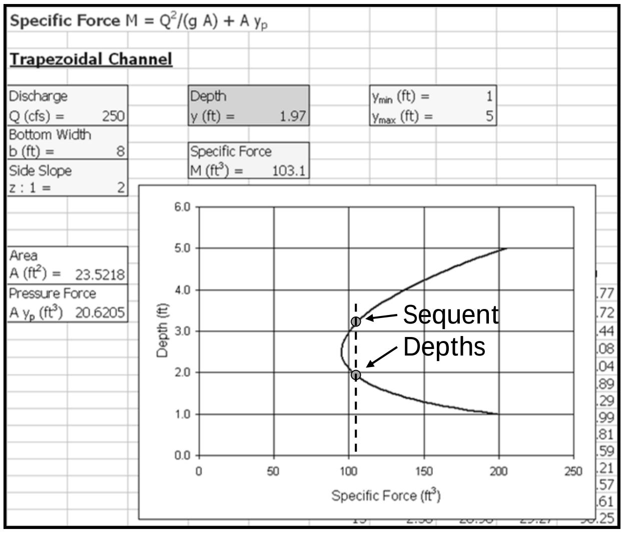
Ex. #16 (Cont.): M3 Profile to Hydraulic Jump

- How far does the flow profile follow the M3 curve downstream of the concrete apron?
- From this supercritical flow profile, the flow must reach subcritical flow at normal depth through a hydraulic jump.
- The flow profile must follow the M3 curve until the depth is appropriate for the jump to reach y_n (this is the sequent depth to y_n).

Ex. #16 (Cont.): Estimation of Sequent Depth

Plot y versus specific force values. For y = 3.13 ft, the specific force (M) value is 103.1 ft^3. Look for the sequent depth on the supercritical limb of the specific-force curve, to find y_s = 1.97 feet.

Conclusion: the M3 curve is followed from a depth of y_a = 1.39 ft to a depth of y_s = 1.97 ft, which marks the entry to a hydraulic jump.



Specific Force $M = Q^2/(g\ A) + A\ y_p$

Trapezoidal Channel

Discharge
Q (cfs) = 250

Bottom Width
b (ft) = 8

Side Slope
z : 1 = 2

Depth
y (ft) = 1.97

Specific Force
M (ft^3) = 103.1

y_{min} (ft) = 1
y_{max} (ft) = 5

Area
A (ft^2) = 23.5218

Pressure Force
A y_p (ft^3) 20.6205

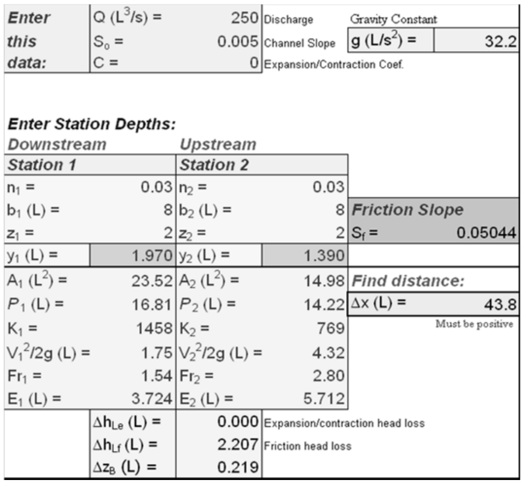
Ex. #16 (Cont.): Distance along M3 Curve and Jump Length

The Direct Step method may be used to estimate the distance along the M3-curve from the culvert apron to the entrance of the hydraulic jump.

Taking a single step, this distance is estimated to be 44 feet.

The jump length is calculated as

L_j = 6.9 (3.13 – 1.97) = 8 ft



Enter	Q (L^3/s) =	250	Discharge	Gravity Constant	
this	S_o =	0.005	Channel Slope	g (L/s^2) =	32.2
data:	C =	0	Expansion/Contraction Coef.		

Enter Station Depths:

Downstream		*Upstream*			
Station 1		***Station 2***			
n_1 =	0.03	n_2 =	0.03		
b_1 (L) =	8	b_2 (L) =	8	***Friction Slope***	
z_1 =	2	z_2 =	2	S_f =	0.05044
y_1 (L) =	1.970	y_2 (L) =	1.390		
A_1 (L^2) =	23.52	A_2 (L^2) =	14.98	***Find distance:***	
P_1 (L) =	16.81	P_2 (L) =	14.22	Δx (L) =	43.8
K_1 =	1458	K_2 =	769		Must be positive
$V_1^2/2g$ (L) =	1.75	$V_2^2/2g$ (L) =	4.32		
Fr_1 =	1.54	Fr_2 =	2.80		
E_1 (L) =	3.724	E_2 (L) =	5.712		

Δh_{Le} (L) =	0.000	Expansion/contraction head loss
Δh_{Lf} (L) =	2.207	Friction head loss
Δz_B (L) =	0.219	

Ex. #16 (Cont.): Transition Example

1 → Upstream normal

2 → Upstream culvert

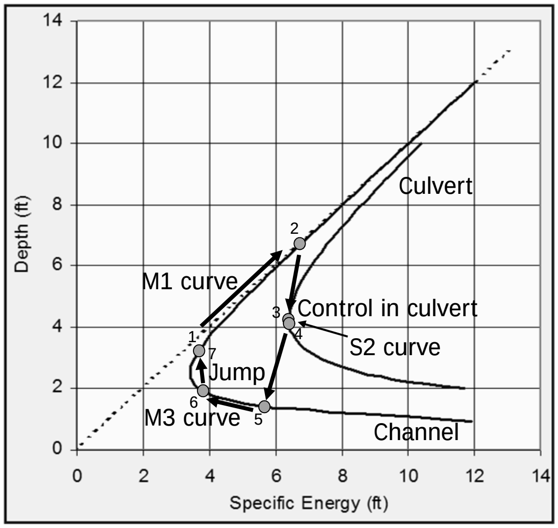
3 → Culvert entrance (control)

4 → Culvert exit

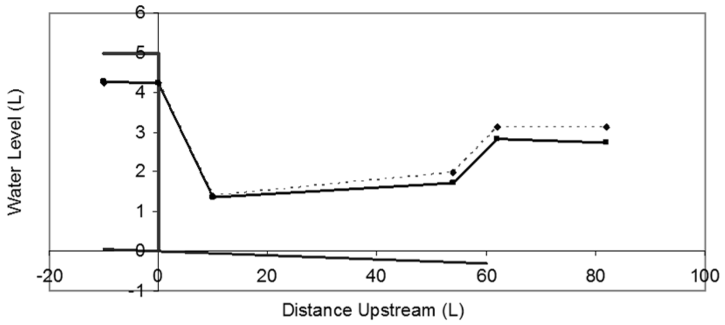
5 → Culvert apron

6 → Jump upstream

7 → Jump downstream (normal depth)



Ex. #16 (Cont.): Hydraulic Profile of Channel Transition



Ex. #16 (Cont.): Energy Loss through Jump

$$\boxed{E_2 + z_2 = E_1 + z_1 + h_{L(2 \rightarrow 1)}}$$

$$h_L = E_u - E_d + z_u - z_d = E_u - E_d + S_o \Delta x$$

$$h_L = 3.724 - 3.62 + 0.005 \times 8 = 0.144 \text{ ft}$$

Very weak hydraulic jump

3.57

Ex. #16 (Cont.): Discussion

- Tired yet??
- Advantages of hand/spreadsheet calculation include control of each step in calculations
- Disadvantages include 1) tedious, and 2) requires some level of expert knowledge
- Alternatives? Computer application using HEC-RAS (River Analysis System)

3.58

Topic 12: Solve Energy (and Momentum) Equations Using HEC-RAS

$$\boxed{E_2 + z_2 = E_1 + z_1 + h_{L(2 \rightarrow 1)}}$$

E = Specific Energy = $y + v^2/2g$

Head Loss:

- Major Losses – friction losses along channel
- Minor Losses – channel expansion and contraction

Computation Problem

For subcritical flow, compute from downstream to upstream.
Discharge may vary from station to station, but are assumed known.
The depth, area, etc. at station 1 (downstream) are known.

Energy Equation:

$\bar{S}_f$

$$\left(\alpha\frac{Q^2}{2gA^2}+WS\right)_2=\left(\alpha\frac{Q^2}{2gA^2}+WS\right)_1+\left(\frac{Q_2+Q_1}{K_2+K_1}\right)^2 L$$

WS = Water Surface = y + z

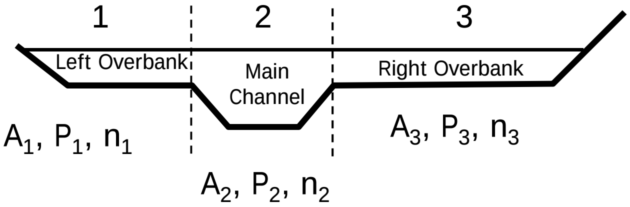
$$+\,C\left|\left(\frac{\alpha Q^2}{2gA^2}\right)_2-\left(\frac{\alpha Q^2}{2gA^2}\right)_1\right|$$

This is what HEC-RAS does.

Unknowns at Station 2: *WS, A, K, (R_h)*

Compound Channel Section

For one-dimensional flow modeling, the slope of the EGL is uniform across the channel section



$$Q=\frac{\phi}{n}A\,R_h^{\,2/3}\,S^{1/2}=\sum_i\frac{\phi}{n_i}A_i\,R_{hi}^{\,2/3}\,S^{1/2}$$

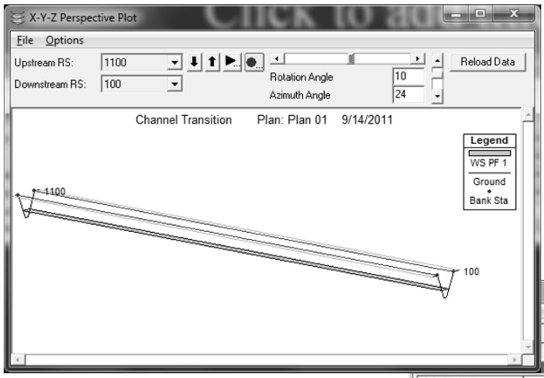
Thus

$$\boxed{Q=\left(\sum_i K_i\right)S^{1/2}}\longleftrightarrow\boxed{\bar{v}=\frac{Q}{A}=\frac{\left(\sum_i K_i\right)S^{1/2}}{A}}$$

Example # 17: Same problem (Ex. #16) using HEC-RAS

- Create a Project
- Enter “Reach” on Geometric Data
- Enter X-Section information for stations
- Enter Discharge and Boundary Conditions
- Compute

Ex. #17 (Cont.): Simple Run on Straight Channel



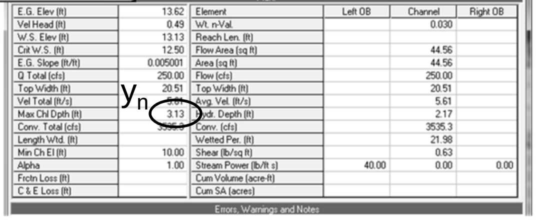
Upstream RS: 1100

Downstream RS: 100

Rotation Angle 10

Azimuth Angle 24

Entry Data:
Q = 250 cfs
BC: Normal depth downstream with S_o = 0.005



Profile: PF 1

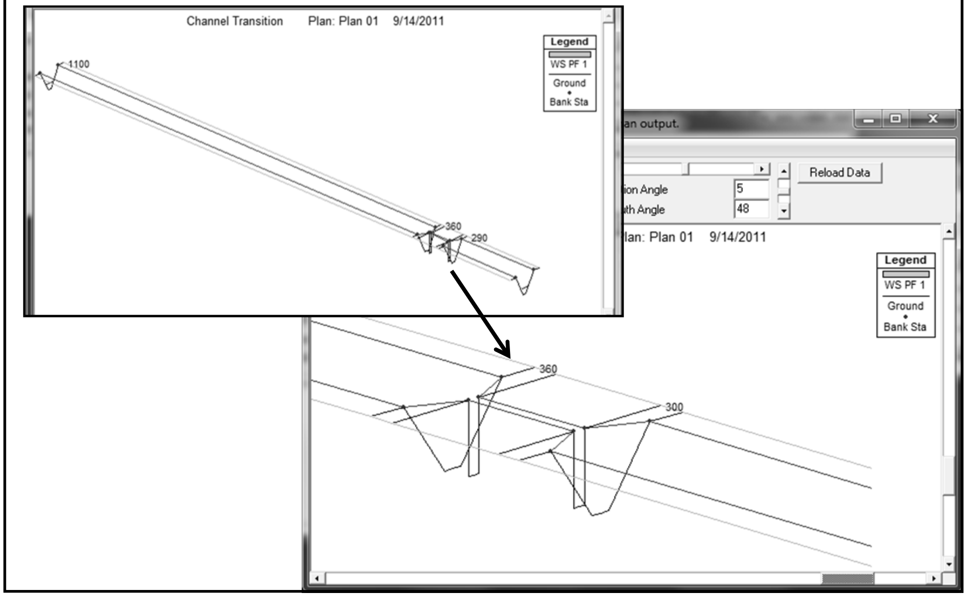
RS: 100

Plan:

		Element	Left OB	Channel	Right OB
E.G. Elev (ft)	13.62	Wt. n-Val.		0.030	
Vel Head (ft)	0.49	Reach Len. (ft)			
W.S. Elev (ft)	13.13	Flow Area (sq ft)		44.56	
Crit W.S. (ft)	12.50	Area (sq ft)		44.56	
E.G. Slope (ft/ft)	0.005001	Flow (cfs)		250.00	
Q Total (cfs)	250.00	Top Width (ft)		20.51	
Top Width (ft)	20.51	Avg. Vel. (ft/s)		5.61	
Vel Total (ft/s)	5.61	Hydr. Depth (ft)		2.17	
Max Chl Dpth (ft)	3.13	Conv. (cfs)		3535.3	
Conv. Total (cfs)	3535.3	Wetted Per. (ft)		21.98	
Length Wtd. (ft)		Shear (lb/sq ft)		0.63	
Min Ch El (ft)	10.00	Stream Power (lb/ft s)	40.00	0.00	0.00
Alpha	1.00	Cum Volume (acre-ft)			
Frctn Loss (ft)		Cum SA (acres)			
C & E Loss (ft)					

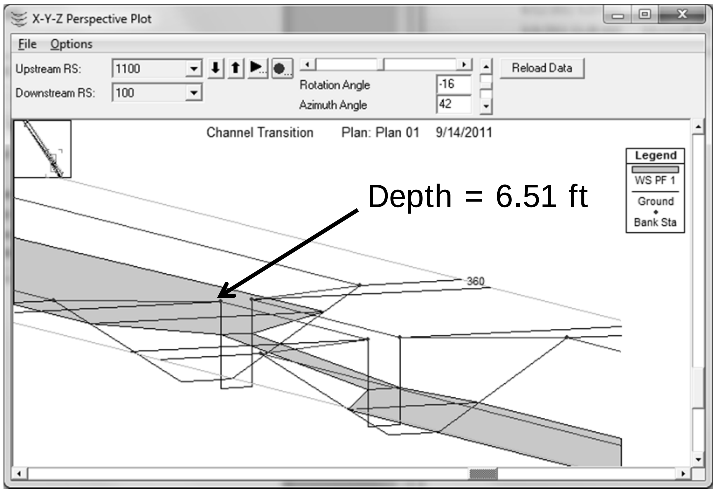
Conclusion: HEC-RAS correctly computes normal depth

Ex. #17 (Cont.): Add X-Section Data for Culvert



ion Angle 5
th Angle 48

Ex. #17 (Cont.): Compute for channel with culvert section



Upstream RS: 1100
Downstream RS: 100
Rotation Angle -16
Azimuth Angle 42

Depth = 6.51 ft

Headwater = 6.51 ft

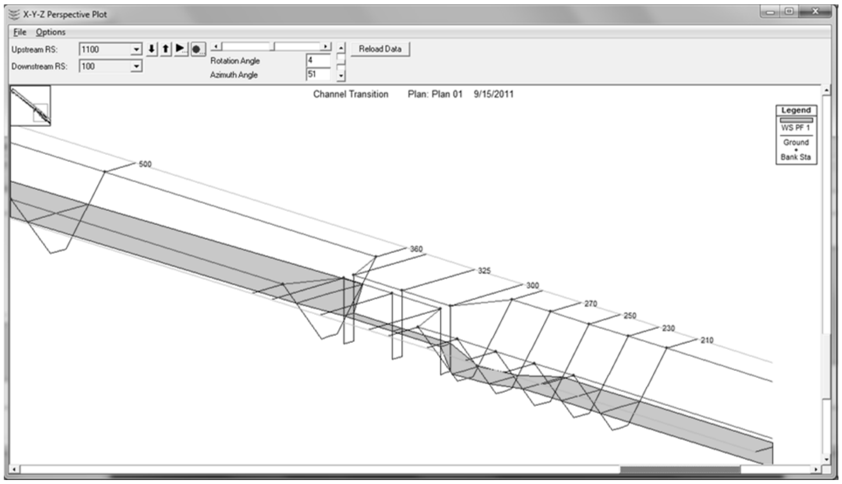
Tailwater = 3.13 ft

Conclusion: Headwater correctly calculated but tailwater uniform at normal depth → need more X-sections

3.65

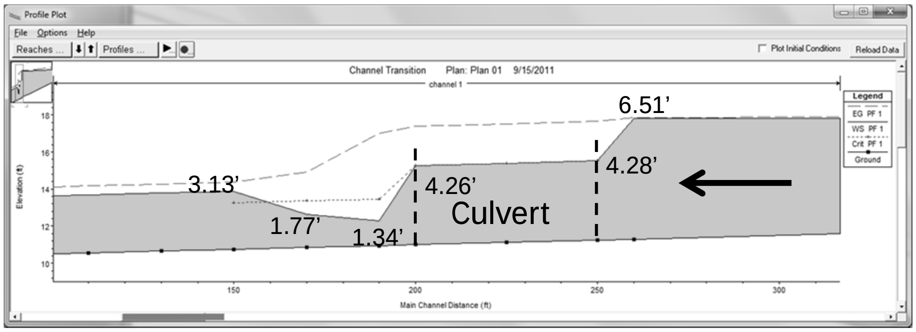
Ex. #17 (Cont.): Add X-sections and Compute

Need to change computation method to “mixed flow”



3.66

Ex. #17 (Cont.): Results - 1



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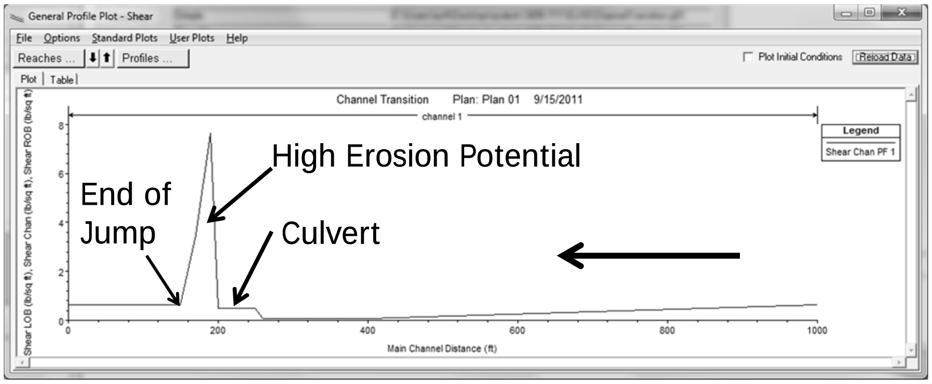
Ex. #17 (Cont.): Results - 2

Bed Shear Stress is Important in assessing channel stability to erosion



High Erosion Potential

End of Jump

Culvert

3.68

Ex. #17 (Cont.): Discussion

- Hand/Spreadsheet calculation and HEC-RAS calculation give equivalent results
- Both are a “pain” (1st for computation and 2nd for set-up)
- Life as a Hydraulic Engineer is a “pain”
- HEC-RAS CAN SIMULATE FLOWS IN COMPLEX CHANNELS THAT CONNOT BE ADDRESSED THROUGH SIMPLE ALTERNATIVES

3.69

Topic 13: Design of Stable Channels

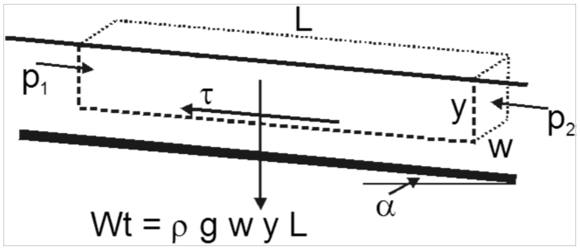
- A stable channel is an unlined channel that will carry water with banks and bed that are not scoured objectionably, and within which objectionable deposition of sediment will not occur (Lane, 1955).
- Objective: Design stable channels with earth, grass and riprap channel lining under design flow conditions

3.70

(Two) Methods of Approach

- ***Maximum Permissible Velocity –*** maximum mean velocity of a channel that will not cause erosion of the channel boundary.
- ***Critical Shear Stress*** – critical value of the bed and side channel shear stress at which sediment will initiate movement. This is the condition of incipient motion. Following the work of Lane (1955), this latter method is recommended for design of erodible channels, though both methods are still used.

Bed Shear Stress

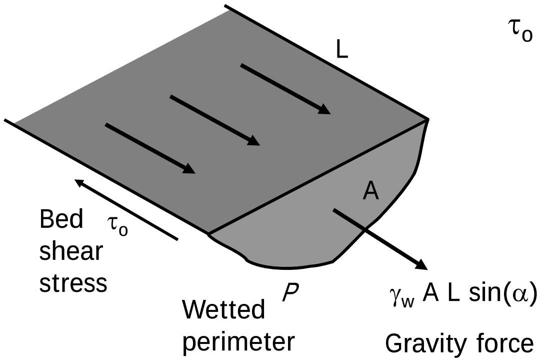


Force balance in downstream direction:

$$p_1\ w\ y - p_2\ w\ y + \rho_w\ g\ w\ y\ L\ \sin(\alpha) - \tau\ w\ L = 0$$

$$\boxed{\tau = \rho_w\ g\ y\ \sin(\alpha) = \gamma_w\ y\ S_o}$$

Bed Shear Stress – Balance Between Gravity and Bed Shear Forces



$$\tau_o\ P\ L = \gamma_w\ A\ L\ \sin(\alpha)$$

$$\boxed{\tau_o = \gamma_w\ R_h\ S}$$

3.73

Calculation of Local Bed Shear Stress

Uniform Flow: $\tau_o = \gamma R_h S$

Local Bed Shear Stress: $\gamma y S_f$

Manning Equation: $S_f = V^2 n^2/(\phi^2 y^{4/3})$

Result: $\tau_o = \gamma V^2 n^2/(\phi^2 y^{1/3})$

3.74

Distribution of Tractive Force

- Varies with side slope, channel width, and location around the channel perimeter
- Following criteria for trapezoidal channels (Lane, 1955)

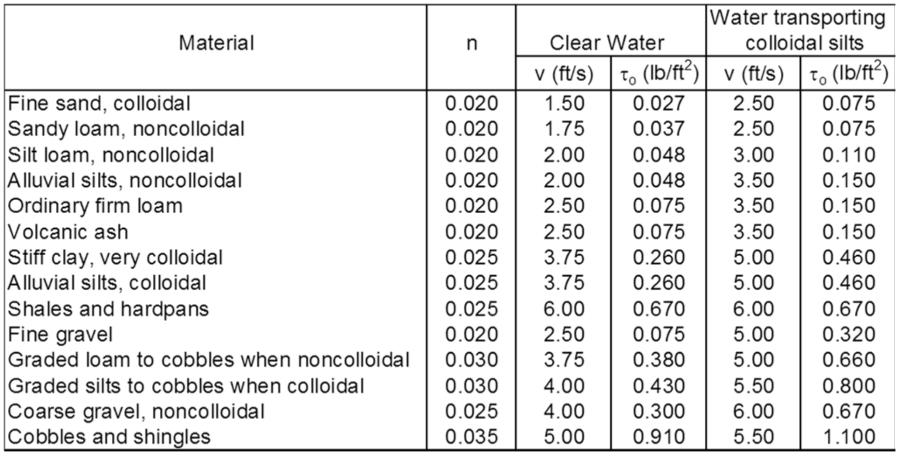
Channel bottom - $(\tau_o)_{max} = \gamma_w y S_o$

Channel sides - $(\tau_o)_{max} = 0.75 \gamma_w y S_o$

(See Figure 8.6, page 303)

Maximum Permissible Velocity – Unlined (Earth-lined) Channel

WB Table 9.1 (Fortier and Scobey, 1926)



Material	n	Clear Water		Water transporting colloidal silts	
		v (ft/s)	τ_o (lb/ft^2)	v (ft/s)	τ_o (lb/ft^2)
Fine sand, colloidal	0.020	1.50	0.027	2.50	0.075
Sandy loam, noncolloidal	0.020	1.75	0.037	2.50	0.075
Silt loam, noncolloidal	0.020	2.00	0.048	3.00	0.110
Alluvial silts, noncolloidal	0.020	2.00	0.048	3.50	0.150
Ordinary firm loam	0.020	2.50	0.075	3.50	0.150
Volcanic ash	0.020	2.50	0.075	3.50	0.150
Stiff clay, very colloidal	0.025	3.75	0.260	5.00	0.460
Alluvial silts, colloidal	0.025	3.75	0.260	5.00	0.460
Shales and hardpans	0.025	6.00	0.670	6.00	0.670
Fine gravel	0.020	2.50	0.075	5.00	0.320
Graded loam to cobbles when noncolloidal	0.030	3.75	0.380	5.00	0.660
Graded silts to cobbles when colloidal	0.030	4.00	0.430	5.50	0.800
Coarse gravel, noncolloidal	0.025	4.00	0.300	6.00	0.670
Cobbles and shingles	0.035	5.00	0.910	5.50	1.100

For well-seasoned channels of small slopes and for depths of flow less than 3 ft. Tractive force calculated from $\tau_o = 30\ n^2\ V^2$.

Example #18: Unlined Channel

An earthen channel is to be excavated in a soil that consists of colloidal graded silts to cobbles. The channel is trapezoidal with side slope 2:1 and bottom slope 0.0016. If the design discharge is 400 cfs, determine the size for an unlined channel using the maximum permissible velocity.

From the table we have V = 4 ft/s and n = 0.030.

Required channel cross-section area: A = Q/V = 100 ft^2

Manning equation to find hydraulic radius:

$$R_h = \left(\frac{V\,n}{\phi\, S_o^{1/2}} \right)^{3/2} = \left(\frac{4 \times 0.030}{1.486\,\sqrt{0.0016}} \right)^{3/2} = 2.87\ ft$$

Wetted perimeter: P = A/R$_h$ = 100/2.87 = 34.8 ft

Example #18 (Cont.)

Trapezoidal Channel:

$$A = (b + z\,y)\,y \qquad P = b + 2\,y\sqrt{1+z^2}$$

Combine into the quadratic equation (eliminating b)

$$\left(2\sqrt{1+z^2} - z\right)y^2 - P\,y + A = 0$$

Solution:

$$y = \frac{P \pm \sqrt{P^2 - 4\,A\left(2\sqrt{1+z^2} - z\right)}}{2\left(2\sqrt{1+z^2} - z\right)}$$

Example #18 (Cont.)

For this problem

$$y = \frac{34.8 \pm \sqrt{34.8^2 - 4 \times 100 \times \left(2\sqrt{1+2^2} - 2\right)}}{2\left(2\sqrt{1+2^2} - 2\right)} = 10.0 \; or \; 4.0 \; ft$$

Channel base width (use y = 4 ft since y = 10 ft gives negative b)

$$b = P - 2\,y\sqrt{1+z^2} = 17.0 \; ft$$

Bed shear stress

$$\tau_o = \gamma\, y_{\max}\, S_f = 62.4 \times 4.0 \times 0.0016 = 0.40 \; lb/ft^2$$

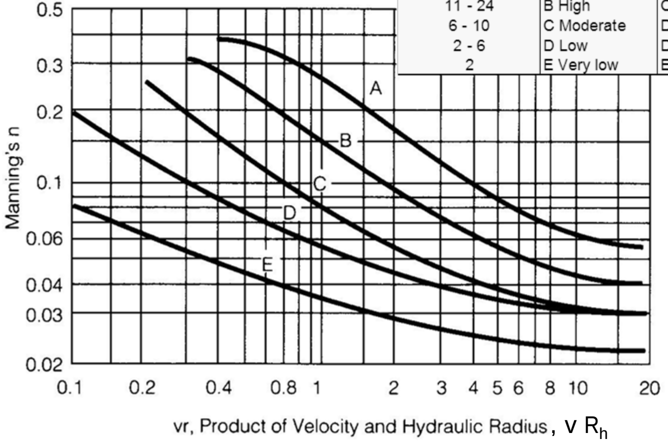
Note: small velocity values are required for stable unlined channels

Grassed Channels

Vegetal Retardance (Table 9.2)

Average Length of Grass (inches)	Grass Density	
	Good	Fair
> 30	A Very high	B High
11 - 24	B High	C Moderate
6 - 10	C Moderate	D Low
2 - 6	D Low	D Low
2	E Very low	E Very low

SCS (after Chow, 1959)



WB, Figure 9.3, page 332

Grassed Channels

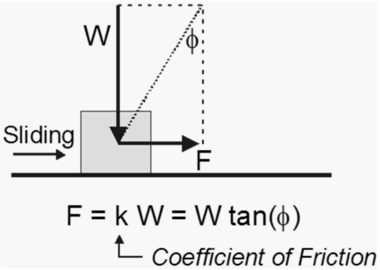
Permissible Velocities for Channels Lined with Grass (SCS, 1941)

Cover	Slope range, %	Permissible velocity, ft/s	
		Erosion-resistant soils	Easily eroded soils
Bermuda grass	0 - 5	8	6
	5 - 10	7	5
	> 10	6	4
Buffalo grass, Kentucky bluegrass, smooth brome, blue grama	0 - 5	7	5
	5 - 10	6	4
	> 10	5	3
Grass mixture	0 - 5	5	4
	5 - 10	4	3
	Do not use on slopes steeper than 10%		
Lespedeza sericea, weeping love grass, ischaemum (yellow bluestem), kudzu, alfalfa, crabgrass	0 - 5	3.5	2.5
	Do not use on slopes steeper than 5%, except for side slopes in a combination channel		
Annuals - used on mild slopes or as tempory protection until permanent covers established, common lespedeza, Sudan grass	0 - 5	3.5	2.5
	Use on slopes steeper than 5% is not recommended		

WB, Table 9.3 (from Chow, 1959)

Movement of Sediment – Incipient Motion

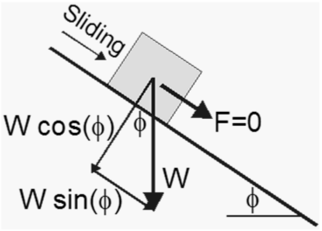
Coefficient of Friction



Force F necessary to move object

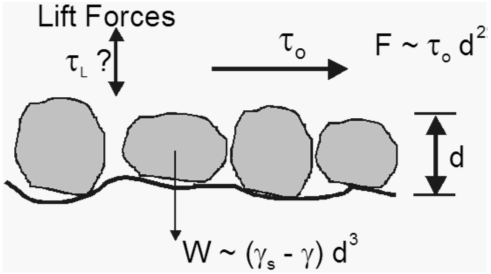
φ = Angle of Repose

Sliding Downhill – F = 0



$$\frac{Tangent}{Normal} = \frac{W\sin(\phi)}{W\cos(\phi)} = \tan(\phi) = k$$

Application to Incipient Motion



Shield's Number, Sh

$$\frac{F_c}{W} \cong \frac{\tau_c\, d^2}{(\gamma_s - \gamma)d^3} = \frac{\tau_c}{(\gamma_s - \gamma)d} \equiv Sh$$

The Shield's number is analogous to the angle of internal friction

Shield’s Curve – Sh(Re*)

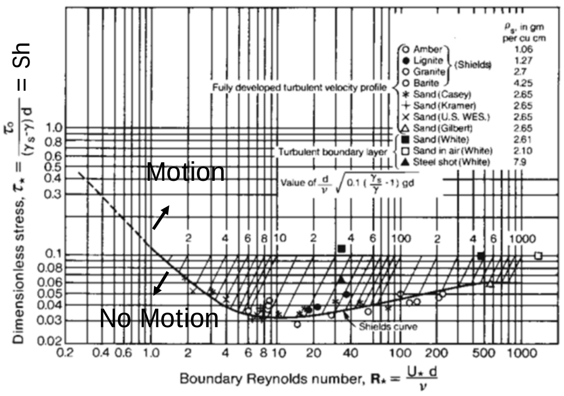
Sh depends on

τ_o, d, ρ, μ

Shear Reynolds number:

$$Re^* = \frac{u^* d}{\nu}$$

$$u^* = \sqrt{\tau_o / \rho}$$



Shield’s Number, Sh

The Shield’s parameter Sh

$$\boxed{Sh = \frac{\tau_o}{(\gamma_s - \gamma)d} = \frac{\gamma R_h S_o}{(\gamma_s - \gamma)d} = \frac{R_h S_o}{(SG - 1)d}}$$

As long as Sh < $Sh_{critical}$ (found from Shield’s Curve), there will be no motion of bed material

Mobility of bed material depends on particle diameter and weight (specific gravity), and on channel flow depth (hydraulic radius) and slope

Design of Riprap Lining

For Re* > 10, the *critical* Shield's number satisfies

$$0.03 \le \frac{\tau_c}{(\gamma_s - \gamma)d} \le 0.06$$

For problems in Stormwater Management, Re* is very large (1000's) and $Sh_{critical}$ = 0.06. For design purposes, a value Sh_c = 0.05 or 0.04 is often selected.

Increased bed material size (riprap) results in decreased Sh (improved bed stability) but increases bed roughness (Manning's n)

Design of Riprap Lining

An approach to design of stable channels using riprap lining:

1. For specified discharge, channel slope and geometry, select a test median particle diameter d_{50} (the designation d_{50} means that 50 percent of the bed material has a smaller diameter)
2. Use Strickler's equation to relate material size to Manning's n: $n = 0.034\ d_{50}^{1/6}$
3. Use Manning's equation to find the depth and R_h
4. Check bed stability using Shield's curve

Example #19: Riprap Lining

What size riprap is required for a channel carrying a discharge Q = 2,500 ft^3/s on a slope S_o = 0.008. The channel has trapezoidal cross section with bottom width b = 25 ft and side slope z:1 = 3:1?

Solution:

1. Select d_{50} = 3 inches (0.25 ft).
2. n = 0.034 $(0.25)^{1/6}$ = 0.0270
3. For the discharge and slope, Manning's equation gives y = 5.24 ft; R_h = 3.67 ft.
4. These values give Sh = 3.67 x 0.008/(1.65 x 0.25) = 0.071; Re* = $(g\ R_h\ S_o)^{1/2}\ d_{50}/\nu$ = $(32.2 \times 3.67 \times 0.008)^{1/2} \times 0.25/10^{-5}$ = 24,000. From the Shield's curve, Sh > Sh_c, and the bed will erode.

Ex. #19 (Cont.): Riprap Lining

2.1 Try d_{50} = 6 inches (0.5 ft)

2.2 n = 0.034 $(0.5)^{1/6}$ = 0.0303

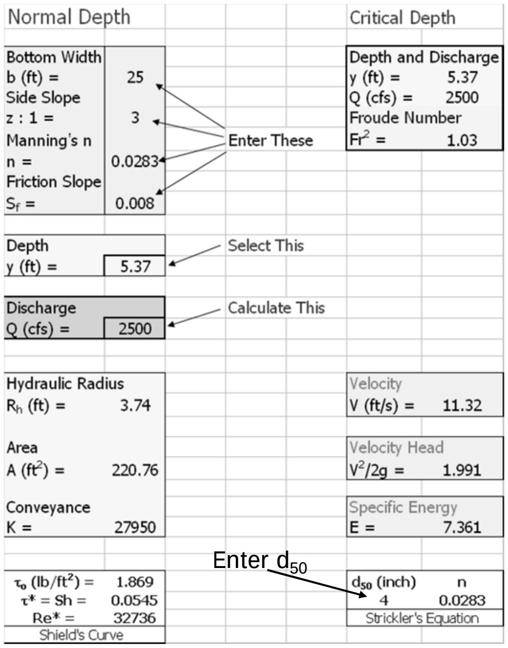
2.3 From Manning's equation, y = 5.56 ft; Rh = 3.85 ft

2.4 Sh = 0.037, Re* = 50,000 → OK

3.1 Try d_{50} = 4 inches

Channel Hydraulics.xls

Check values here



Normal Depth

Bottom Width
b (ft) = 25
Side Slope
z : 1 = 3
Manning's n
n = 0.0283
Friction Slope
S_f = 0.008

Enter These

Depth
y (ft) = 5.37

Select This

Discharge
Q (cfs) = 2500

Calculate This

Hydraulic Radius
R_h (ft) = 3.74

Area
A (ft^2) = 220.76

Conveyance
K = 27950

τ_o (lb/ft^2) = 1.869
τ^* = Sh = 0.0545
Re* = 32736
Shield's Curve

Critical Depth

Depth and Discharge
y (ft) = 5.37
Q (cfs) = 2500
Froude Number
Fr^2 = 1.03

Velocity
V (ft/s) = 11.32

Velocity Head
$V^2/2g$ = 1.991

Specific Energy
E = 7.361

Enter d_{50}

d_{50} (inch) n
4 0.0283
Strickler's Equation

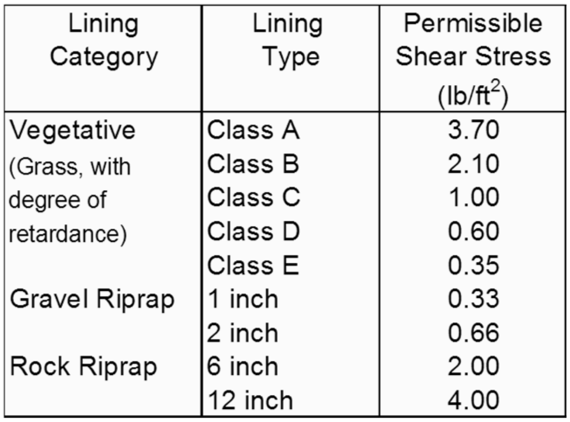
Design of Channel Lining

For Riprap Lining: Recommended Grading – allows voids between larger rocks to be filled with smaller rocks (Simons and Senturk, 1992)

$d_{20} = 0.5\ d_{50}$

$d_{100} = 2\ d_{50}$

Permissible Shear Stress for Lining Materials (US DOT, 1967)



Lining Category	Lining Type	Permissible Shear Stress (lb/ft^2)
Vegetative	Class A	3.70
(Grass, with	Class B	2.10
degree of	Class C	1.00
retardance)	Class D	0.60
	Class E	0.35
Gravel Riprap	1 inch	0.33
	2 inch	0.66
Rock Riprap	6 inch	2.00
	12 inch	4.00