GIS in Water Resources Exercise #3 Solution

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Question 1: *Hand calculations of slope at point A using each of the three methods and comments on the differences.*

Part 1. Slope Calculations

1.1 Hand Calculation (Point A only):

(i) ESRI Slope

25.4	26.1	27	28.6	27.7
25	26	26.4 A	27.9	27.4
25.1	25.8	26.8 B	28.6	27.6
27.5	28	27.7	30.6	28.3

Cell Referencing

a	b	c
d	e	f
g	h	i

Equations:

c_size = 10m

dz/dx = ((a+2d+g)-(c+2f+i))/8*c_size

= ((26.1+2*26.0+25.8)-(28.6+2*27.9+28.6))/(8*10) = -0.11375

dz/dy = ((g+2h+i)-(a+2b+c))/8*c_size

= ((25.8+2*26.8+28.6)-(26.1+2*27+28.6))/(8*10) = -0.00875

These represent the x and y components of the slope vector shortened as follows

Δx = dz/dx = -0.11375

Δy = dz/dy = 0.00875

slope (rise/run) = sqrt(Δx^2+Δy^2) = sqrt((-0.11375)^2+(0.00875)^2) = 0.114

slope (angle) = atan(slope (rise/run)) = atan(0.114) = 0.1136 rads = 6.508 degrees

Note: degrees = rads * 180/π. Calculators can be set to return rads or degrees. Excel and computer programs usually return rads.

aspect = atan(Δx/Δy) = atan(-0.11375/-0.00875) = 1.494 rads = 85.6 degrees

This is an angle in the SW quadrant since both x and y components are negative. Add 180 degrees to get the angle clockwise from north

aspect = 180+(85.6) = 265.6

The following Excel Object includes the formulae. You can double click on this to open this object in Excel.

(i) ESRI Standard Slope Function								
Grid size	10	m						
25.4	26.1	27	28.6	27.7		dz/dx=	-0.11375	
25	26	**26.4**	27.9	27.4		dz/dy=	-0.0088	
25.1	25.8	26.8	28.6	27.6				
27.5	28	27.7	30.6	28.3		rise/run=	0.114086	
						Slope=	0.113595	radians
							6.508509	**degree**
						Aspect	-1.64757	radians
							-94.3987	degree
			Result as angle clockwise from North				**265.6013**	degree
(This is an Excel Object so you can click on it to see the formulas)								

(ii) The 8 direction pour point model D8

Slope is calculated separately to each adjacent grid cell using the formula
Slope = (Center elevation - Side elevation)/Distance
Distance to diagonal side cells is the diagonal distance $\sqrt{2}$ * cell size

The following Excel object includes these calculations.

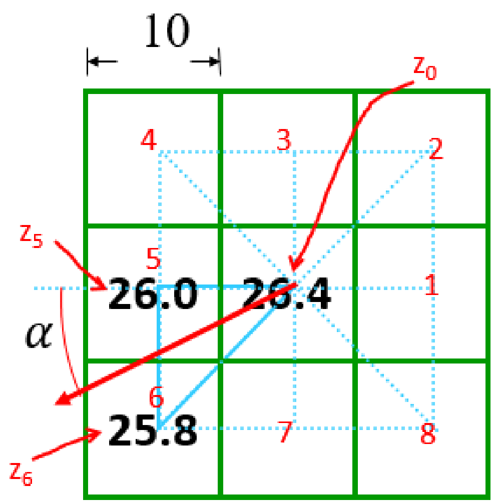
ii) D8	Center cell	26.4						
Distances	Side	10	Diagonal	14.14214				
Direction	Value	Distance	Slope		Direction Encoding			
1	27.9	10	-0.150		32	64	128	
2	28.6	14.142	-0.156		16	←	1	
4	26.8	10	-0.040		8	4	2	
8	25.8	14.142	**0.042**	Maximum (positive down) slope to cell in direction 8				
16	26	10	0.040					
32	26.1	14.142	0.021					
64	27	10	-0.060					
128	28.6	14.142	-0.156					
(This is an Excel Object so you can click on it to see the formulas)								

Note that the steepest 8 direction pour point model slope in direction 16 is:

$$\frac{center\ cell - sice\ cell\ 16}{cell\ size} = \frac{26.4 - 26}{10} = 0.04$$

D8 Slope: 0.042
D8 Direction: 8

(iii) By Dinfinity



$$\alpha = \text{atan}\left(\frac{z_5 - z_6}{z_0 - z_5}\right) = \text{atan}\left(\frac{26 - 25.8}{26.4 - 26}\right)$$

$\alpha = 0.463$ rad = 26.56 deg

Direction counter clockwise from E is
180+ 26.56 deg = 206.56 deg

$$S = \sqrt{\left(\frac{z_5 - z_6}{\Delta}\right)^2 + \left(\frac{z_o - z_5}{\Delta}\right)^2}$$

$$S = \sqrt{\left(\frac{26 - 25.8}{10}\right)^2 + \left(\frac{26.4 - 26}{10}\right)^2}$$

$$S = 0.04472$$

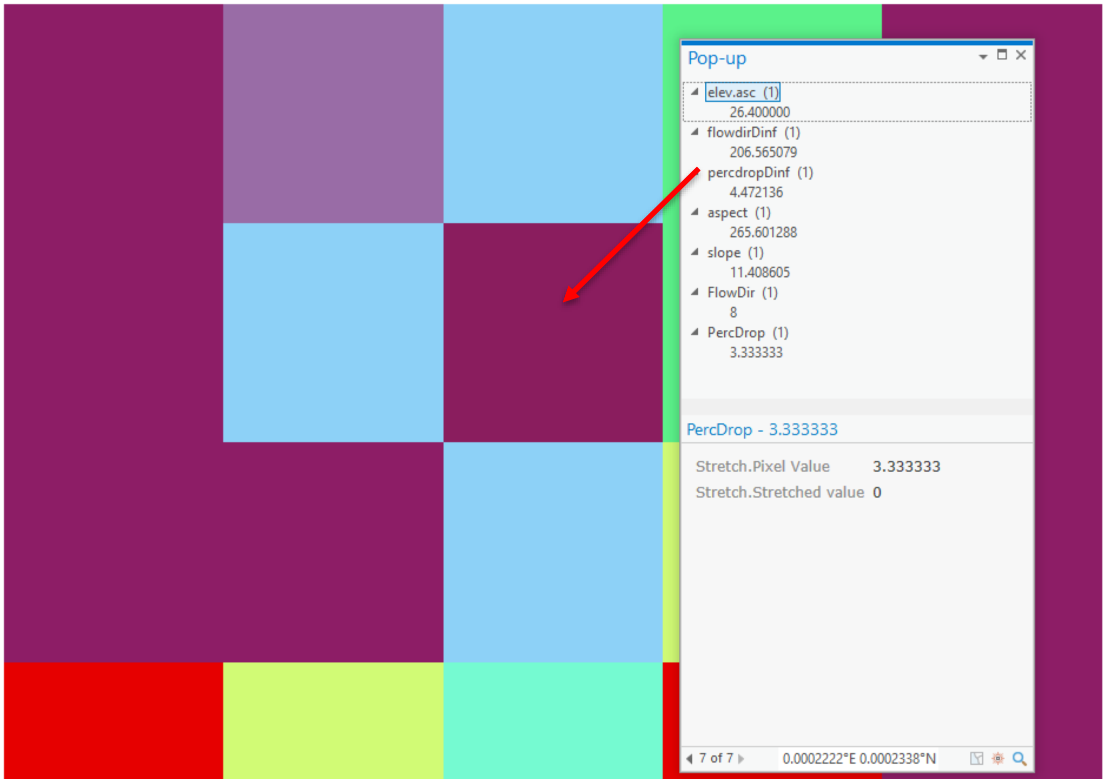
<u>Differences</u>
Represented as an aspect the D8 direction would be 235 degrees, but simply stating the direction as 8 or to the SW is sufficient.
The main difference is that the ESRI slope considers all 8 surrounding grid cell values, and curiously, not the actual grid cell value. It represents the slope of a polynomial surface fit to all these grid cells. The D8 method only considers adjacent elevations lower than the center cell which is consistent with the assumption of where water would flow not being influenced by adjacent neighbors that are higher.

The ESRI slope is probably most appropriate for computation of quantities such as illumination due to sunlight in energy balance calculations where the slope of the surface fit based on all surrounding values seems best, but for the flow of water, the D8 or Dinf method is better.

Question 2: *Table giving slope, aspect, hydrologic slope and flow direction at grid cells A and B. For hydrologic slope report results from both D8 and DINF methods. Include in your solution diagrams or sketches that define or indicate what each of these numbers means for the specific values obtained for cells A and B.*

1.2 <u>Verifying calculations using ArcGIS</u>
The values at cell A from ArcGIS are Slope = 11.4%, Aspect = 265.6 deg, PercDrop = 3.33% and FlowDir=8, Dinfflowdir=206.56 deg and DInfPercDrop= 4.47 %. PercDrop differs from the hand calculations, but other values correspond. I believe that the PercDrop difference is a bug.

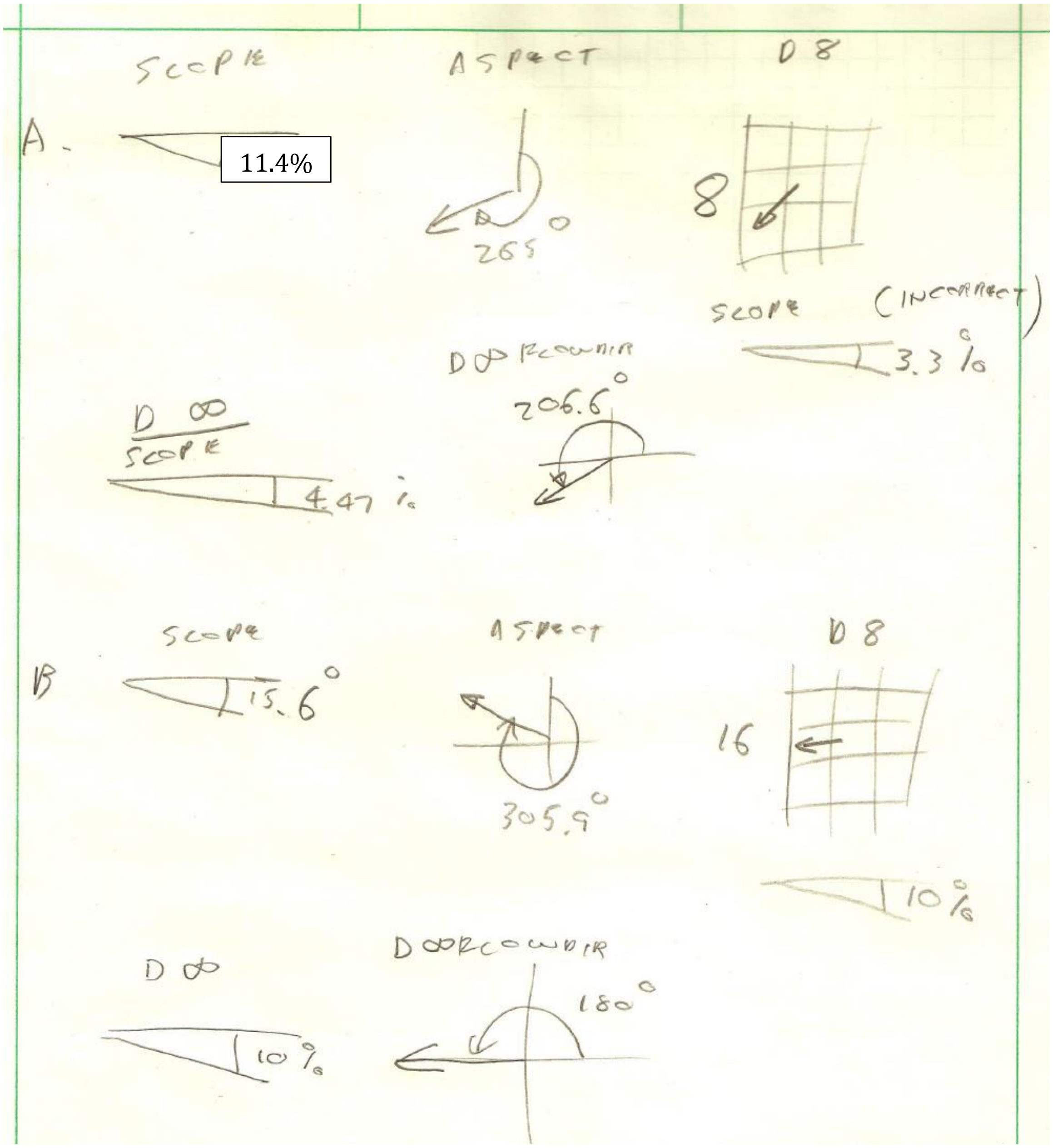
At cell A



<u>ArcGIS Calculated:</u>

Pt A	Pt B
Pop-up elev.asc (1) 26.400000 slope (1) 11.408605 aspect (1) 265.601288 FlowDir (1) 8 PercDrop (1) 3.333333 FlowDirDinf (1) 206.565079 PercDropDinf (1) 4.472136	Pop-up elev.asc (1) 26.799999 slope (1) 15.577432 aspect (1) 305.858368 FlowDir (1) 16 PercDrop (1) 10.000000 FlowDirDinf (1) 180.000000 PercDropDinf (1) 10.000000

Point	**Slope (%)**	**Aspect (deg)**	**D8 Slope (%)**	**Flow Dir (D8)**	**DinfPercDrop (%)**	**DinfFlowDir (deg)**
A	11.4	265.6	3.33	8	4.47	206.6
B	15.6	305.9	10	16	10	180



Note that if you look at the data underlying D8 slope at A you have

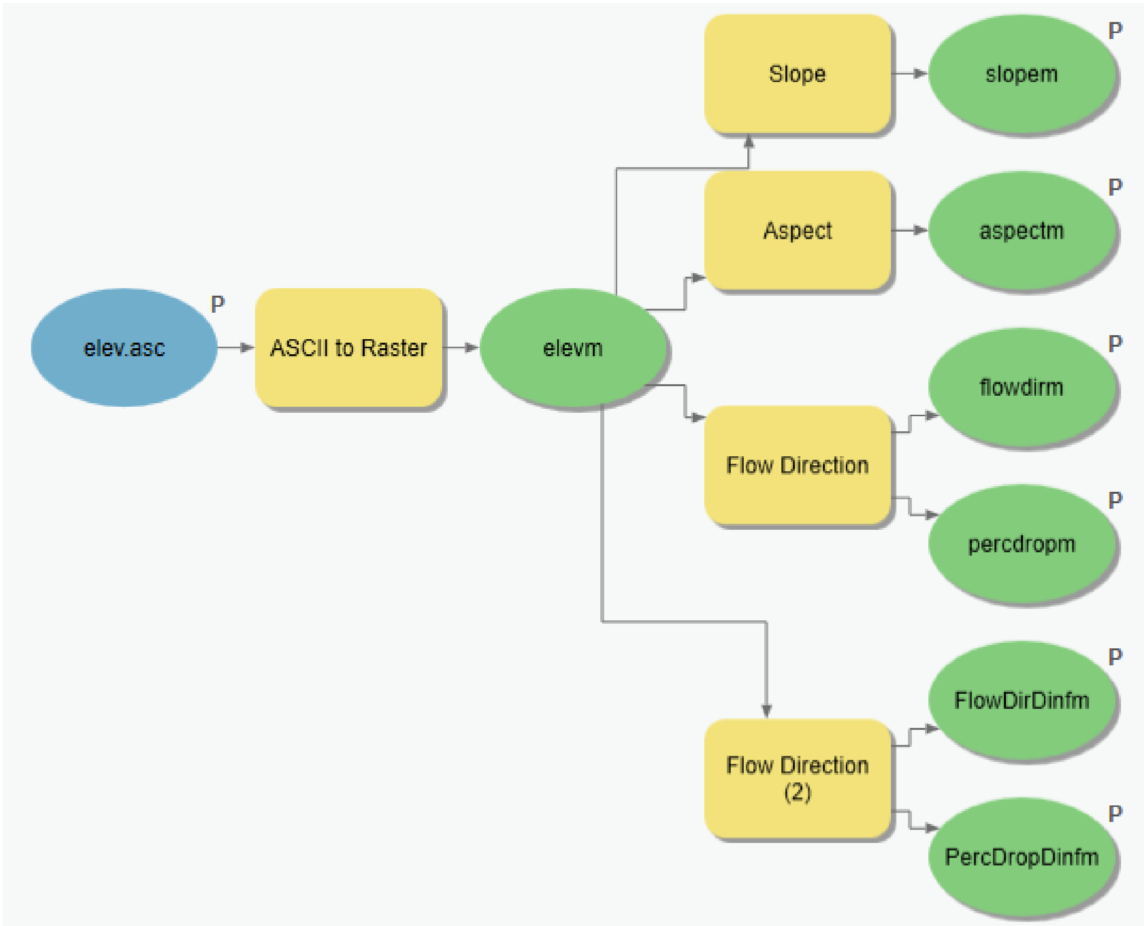
26.1	27	28.6
26.0	26.4	27.9
25.8	26.8	28.6

The percentage drop in direction 8 (indicated with arrow) should thus be (26.4-25.8)/(SQRT(2)*10)=0.0424 = 4.24%

The fact that the ArcGIS function is reporting 3.33% is, I believe, a bug. Buyer beware!

Question 3: *A screen capture of your final model builder model.*

1.3 Model Builder Output



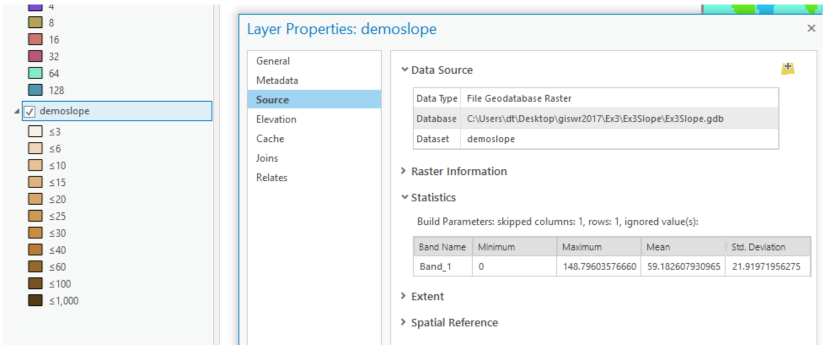
Question 4. *A table giving the minimum and maximum values of each of the six outputs Slope, Aspect, Flow Direction and Hydrologic Slope (Percentage drop) by D8 and Flow Direction and Hydrologic Slope (Percentage drop) by DINF for the digital elevation model in demo.asc. Also turn in a screen shot of the FlowDirDinf raster calculated using this model*

Table: Summary of Demo.asc Ouputs

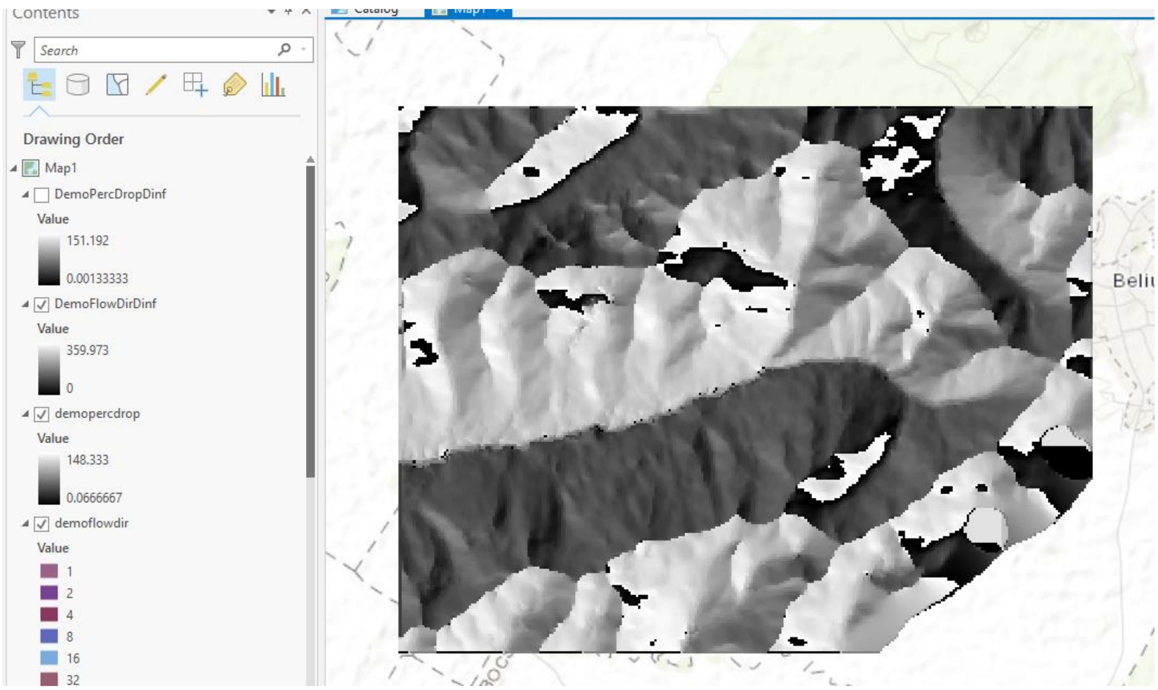
Layer	Min	Max
Slope	0	149
Aspect	-1	360
Flow Dir	1	128
PercDrop	0.066	148.3
DinfFlowDir (deg)	0	360
Dinf Perc Drop	0.001	151.2

-1 for aspect is used to represent flat grid cells

Note that using the scale that ArcGIS automatically uses to symbolize layers that suggests and upper bound of 1000 for slope is insufficiently precise which is why it is better to get this information from Layer Properties > Source > Statistics



Screen shot of DemoFlowDirDinf. Note that this appears rather similar to a hillshade due to the nature of Dinf flow directions being angles from 0 to 360 degrees.



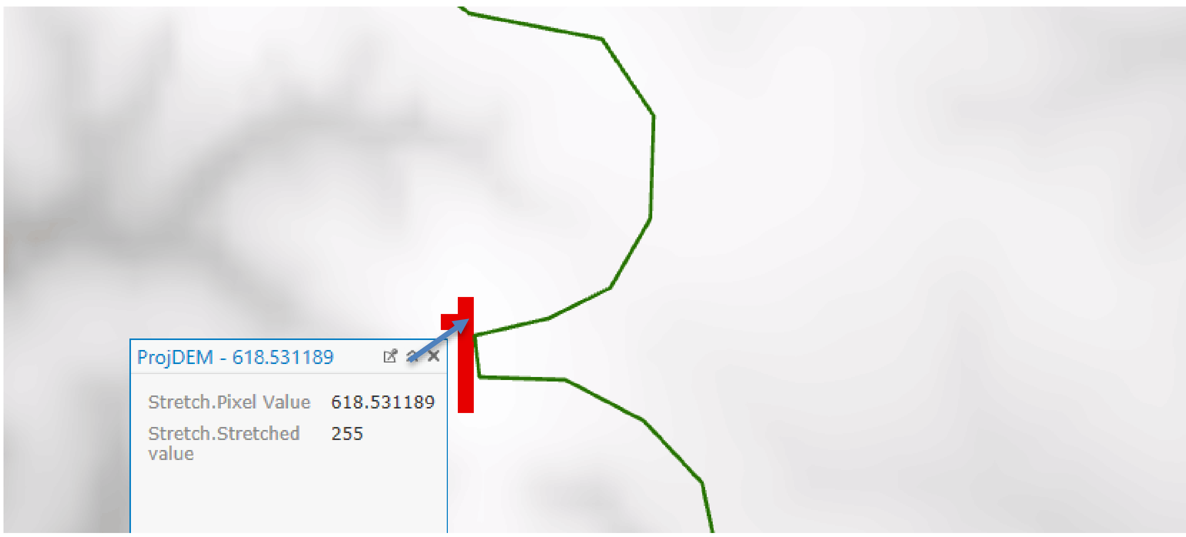
Part 2 San Marcos

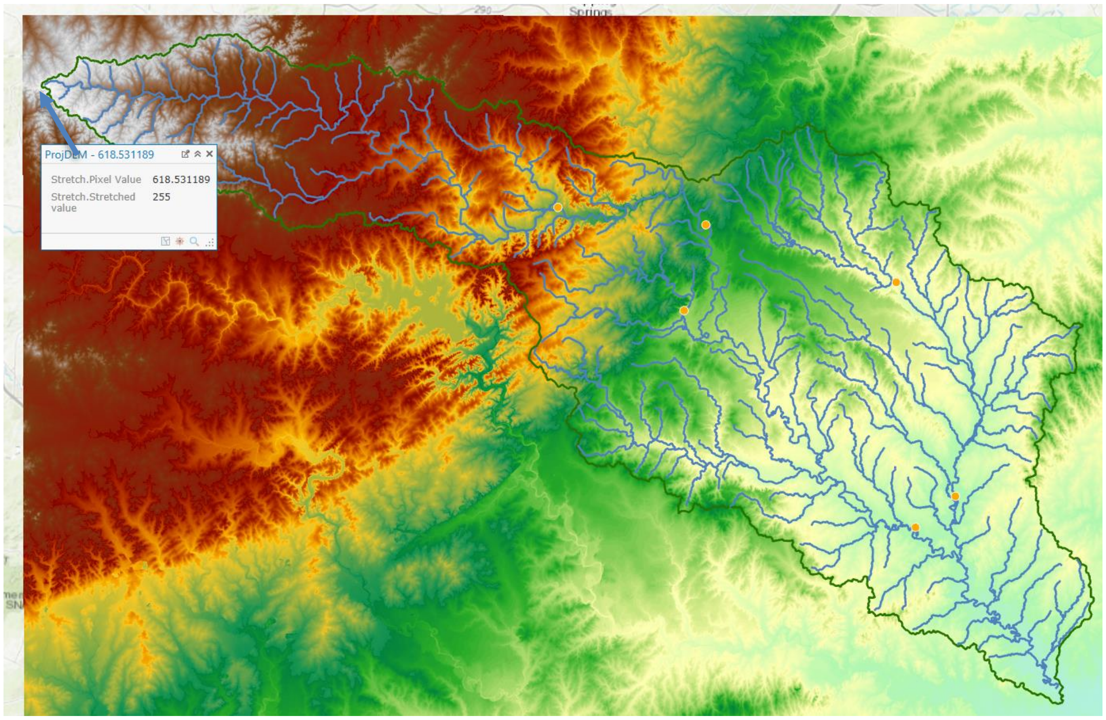
Question 5. *The number of columns and rows in the projected DEM. The cell size of the projected DEM. The minimum and maximum elevations in the projected DEM.*

<u>DEM Summary (projdem.tif)</u>
Rows: 2745
Columns: 4222
Cell Size: 30 x 30 m
Min: 69.7651
Max: 618.532

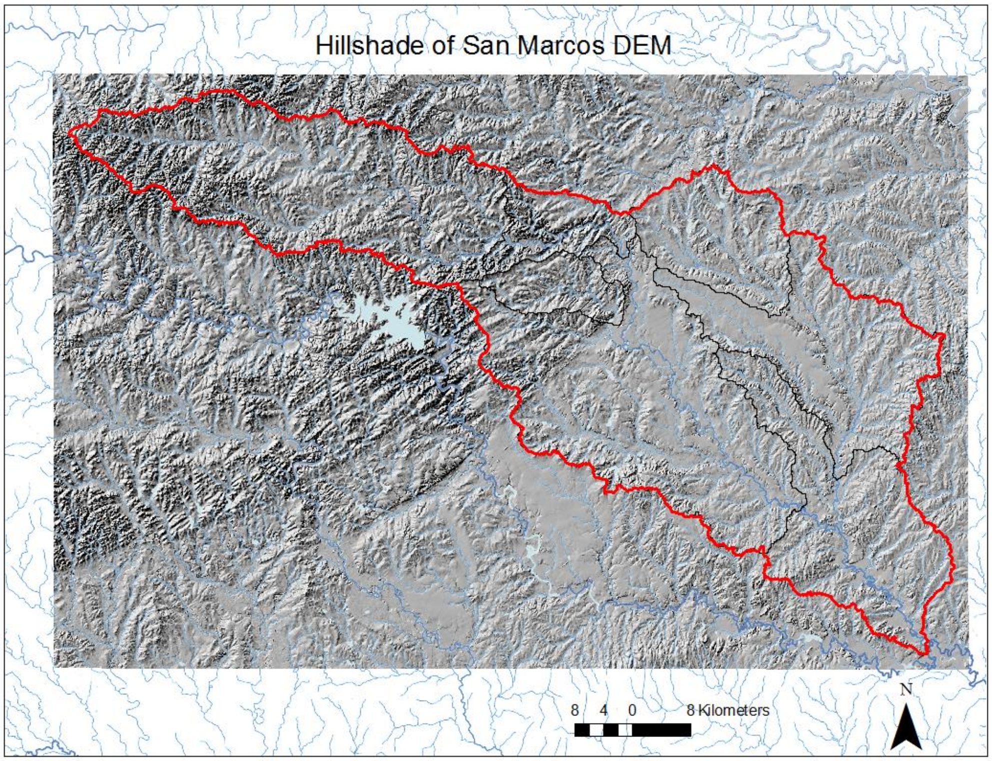
Question 6. *A screen shot showing the location of the highest elevation value in the San Marcos DEM.*

Below is a screen shot showing the map with ProjDEM layer active. The red grid cells are from Raster Calculation ProjDEM > 618. Clicking on cells reveals the cell indicated as the highest with a value of 618.53. The second image below shows where this is overall in the domain





Question 7. *A layout with a depiction of topography either with elevation, contour or hillshade in nice colors. Include the streams from NHDPlus and Basin and sub-watersheds from the SanMarcos.gdb Basemap feature dataset.*



Question 8. *A table giving the HydroID, Name, mean elevation, and elevation range for each subwatershed in the SanMarcos Subwatershed feature class. Which subwatershed has the highest mean elevation? Which subwatershed has the largest elevation range?*

Subwatershed Elevation Summary

HydroID	SiteName	Elev. Range (m)	Elev Mean (m)
330	Plum Ck at Lockhart, Tx	137.72	189.86
331	Blanco Rv at Wimberley, Tx	372.98	418.48
332	Blanco Rv nr Kyle, Tx	216.82	288.57
333	San Marcos Rv at San Marcos, Tx	218.47	266.11
334	Plum Ck nr Luling, Tx	115.88	151.94
335	San Marcos Rv at Luling, Tx	311.96	183.53

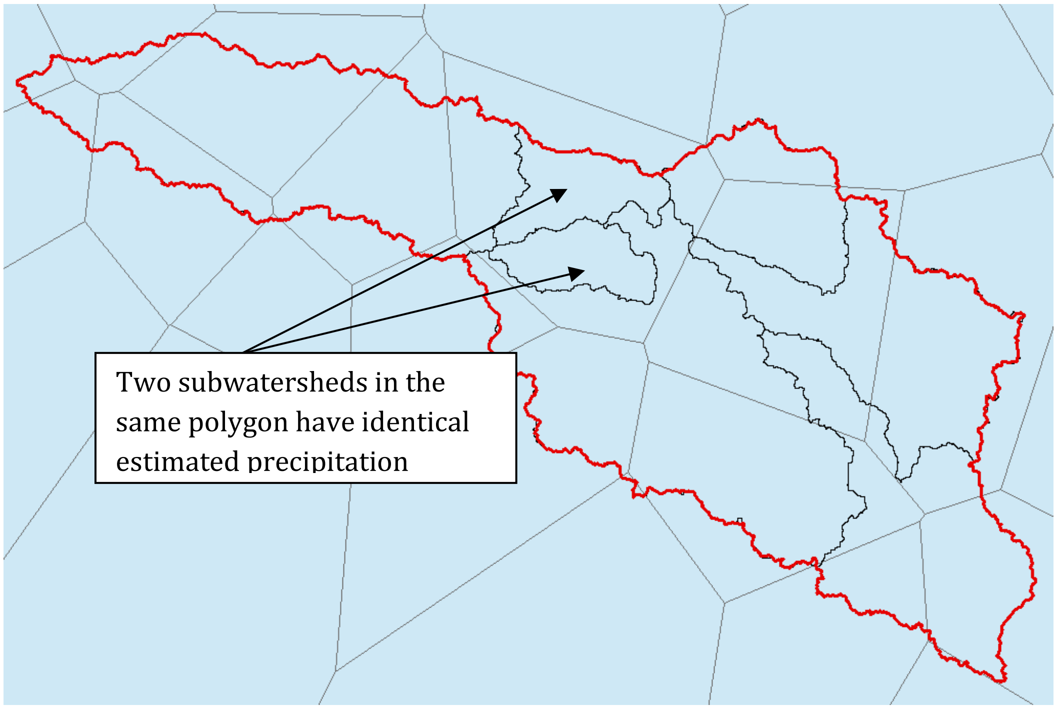
Highest: Blanco Rv at Wimberley, TX
Largest Range: Blanco Rv at Wimberley, TX

Question 9: *A table giving the HydroID, Name, and mean precipitation by the Thiessen method for each subwatershed in the SanMarcos Subwatershed feature class. Which subwatershed has the highest mean precipitation?*

Area Average Precipitation using Thiessen Polygons

HydroID	SiteName	SubW Precip (in)
330	Plum Ck at Lockhart, Tx	36.37
331	Blanco Rv at Wimberley, Tx	37.83
332	Blanco Rv nr Kyle, Tx	40.48
333	San Marcos Rv at San Marcos, Tx	40.48
334	Plum Ck nr Luling, Tx	36.52
335	San Marcos Rv at Luling, Tx	37.59

The highest mean precipitation is found for the San Marcos River at San Marcos and Blanco River near Kyle watersheds. These are identical, because they are both in the same polygon.



Question 10. *A table giving the HydroID, Name, and mean precipitation by the Tension Spline method for each subwatershed in the SanMarcos Subwatershed feature class. Which subwatershed has the highest mean precipitation using a Tension Spline interpolation?*

Area average mean annual precipitation using Spatial Interpolation/Surface fitting (Tension Spline Method)

HYDROID	SiteName	Precip (inches)
330	Plum Ck at Lockhart, Tx	36.22
331	Blanco Rv at Wimberley, Tx	37.89
332	Blanco Rv nr Kyle, Tx	39.79
333	San Marcos Rv at San Marcos, Tx	39.66
334	Plum Ck nr Luling, Tx	36.46
335	San Marcos Rv at Luling, Tx	37.99

Blanco Rv nr Kyle, TX has the highest mean precipitation estimated from Tension Spline Interpolation.