## GIS in Water Resources <br> Fall 2018 <br> Homework \#1 Solution

## 1. Coordinate System Parameters

The map below shows the continental United States in geographic coordinates overlaid by a grid which has $5^{\circ} \times 5^{\circ}$ cells. The parameters of the USA Contiguous Albers Equal Area Conic USGS version coordinate system as displayed in ArcGIS are given below. Draw on the map above the Central Meridian, Reference Latitude, and Standard Parallels used in this coordinate system.

## Solution:

Reading from the table on p .2 , the projection parameters are:
Central Meridian: $\mathbf{9 6}^{\circ} \mathbf{W}$
Reference Latitude (Latitude of Origin) $\mathbf{2 3}^{\circ} \mathbf{N}$
Standard Parallel 1: $\mathbf{2 9 . 5}{ }^{\circ} \mathbf{N}$
Standard Parallel 2: $\mathbf{4 5 . 5}{ }^{\circ} \mathbf{N}$
These lines are drawn on the map below.
a) Put a large dot at the intersection of the Central Meridian and Reference Latitude on the map and label this with the $(\mathrm{X}, \mathrm{Y})$ coordinates that this location has in the given coordinate system.
Solution: The False Easting and False Northing are both 0 in this projection, so the (X,Y) coordinates at the origin of this projection are (0,0).


| Coordinate System Details |  |
| :--- | :--- |
| Projected Coordinate System | USA Contiguous Albers Equal Area Conic USGS version |
| Projection | Albers |
| WKID | 102039 |
| Authority | Esri |
| Linear Unit | Meters (1.0) |
| False Easting | 0.0 |
| False Northing | 0.0 |
| Central Meridian | -96.0 |
| Standard Parallel 1 | 29.5 |
| Standard Parallel 2 | 45.5 |
| Latitude Of Origin | 23.0 |
|  |  |
| Geographic coordinate system | GCS North American 1983 |
| WKID | 4269 |
| Authority | EPSG |
| Angular Unit | Degree (0.0174532925199433) |
| Prime Meridian | Greenwich (0.0) |
| Datum | D North American 1983 |
| Spheroid | GRS 1980 |
| Semimajor Axis | 6378137.0 |
| Semiminor Axis | 6356752.314140356 |
| Inverse Flattening | 298.257222101 |

b) What earth surface property does the Albers projection preserve regardless of the projection parameters?

## Solution: Area

c) What earth datum is used with this coordinate system?

## Solution: North America Geographic Coordinate System of 1983

d) The geographic coordinates of Salt Lake City are: $40^{\circ} 45^{\prime} 39^{\prime \prime} \mathrm{N}$ and $111^{\circ} 53^{\prime} 28^{\prime \prime} \mathrm{W}$ The coordinates of New York City are: $40^{\circ} 42^{\prime} 51^{\prime \prime} \mathrm{N}$ and $74^{\circ} 0^{\prime} 22^{\prime \prime} \mathrm{W}$. Calculate the coordinates for each of these in decimal degrees. Express your answers using 5 digits following the decimal point.

Solution: The LatLong spreadsheet used for Exercise 2, is adapted for this problem:

| SitelD | SiteName | Latitude | Longitude | LatDeg | LatMin | LatSec | LongDeg |  |  | LatDD | LongDD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Salt Lake City | $40^{\circ} 45^{\prime} 39^{\prime \prime}$ | $111^{\circ} 53^{\prime \prime} 28^{\prime \prime}$ | 40 | 45 | 39 | 111 | 53 | 28 | 40.76083 | -111.89111 |
| 2 | New York City | $40^{\circ} 42^{\prime} 51^{\prime \prime}$ | $74^{\circ} 0^{\prime} 22^{\prime \prime}$ | 40 | 42 | 51 | 74 | 0 | 22 | 40.71417 | -74.00611 |

For example, for Latitude of Salt Lake City: $\phi=40+45 / 60+39 / 3600)=-40.76083$ and for Longitude of Salt Lake City: $\lambda=-(111+53 / 60+28 / 3600)=-111.89111$. The results for both cities are:

Salt Lake City: $(\phi, \lambda)=(\mathbf{4 0 . 7 6 0 8 3}, \mathbf{- 1 1 1 . 8 9 1 1 1})$
New York City: $(\phi, \lambda)=(40.71417,-74.00611)$
e) Note that the latitude of Salt Lake City and New York City is almost the same. Assuming a spherical earth with radius 6371.0 km , calculate the distance in km that New York is East of Salt Lake City along a latitude parallel.

## Solution:

If A is Salt Lake City and B is New York City, then the distance along a latitude parallel $\phi$, is given by $\mathbf{A B}=\operatorname{Re}^{*} \operatorname{Cos} \phi * \Delta \lambda$. $\mathrm{Re}=6371.0 \mathrm{~km}$. Let's take the average of the two latitudes, $\phi=(40.76083+40.71417) / 2=40.73750$ in decimal degrees $=40.73750 *(\pi / 180)$ radians $=$ 0.71100 radians. Similarly, $\Delta \lambda=-74.00611-(-111.89111)=37.88500^{\circ}=37.88500^{*}(\pi / 180)$ radians $=\mathbf{0 . 6 6 1 2 2}$ radians. $\mathbf{A B}=6371.0 * \operatorname{Cos}(0.71100) *(0.66122)=3191.9 \mathbf{k m}$.
f) Determine the distance that Salt Lake City is north of the Latitude of Origin $\left(23^{\circ} \mathrm{N}\right)$ in km .

## Solution:

If the Origin is O and Salt Lake City is A, then the distance North on OA is OA $=\boldsymbol{\operatorname { R e }} * \Delta \phi$ where $\Delta \phi=40.76083-23=17.76083^{\circ}=17.76083^{*} \mathrm{PI}() / 180$ radians $=0.30999$ radians. Hence distance North on $\mathrm{OA}=6371.0 * 0.30999=\mathbf{1 9 7 4 . 9} \mathbf{~ k m}$.
g) Calculate the great circle distance between New York and Salt Lake City in km assuming a spherical earth with radius 6371.0 km .

## Solution:

The formula for great circle distance is shown below, as is a solution worked out in Excel, as explained in class.

$$
\text { Dist }=R_{e} \operatorname{Arc} \operatorname{Cos}\left[\operatorname{Sin} \phi_{A} \operatorname{Sin} \phi_{B}+\operatorname{Cos} \phi_{A} \operatorname{Cos} \phi_{B} \operatorname{Cos}\left(\lambda_{A}-\lambda_{B}\right)\right]
$$

| Location | Point | Phi | SinPhi | CosPhi | Lamda | LA - LB | Cos(LA-LB) |
| :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Salt Lake City | A | 0.7114 | 0.6529 | 0.7574 | -1.9529 | -0.6612 | 0.7892 |
| New York | B | 0.7106 | 0.6523 | 0.7580 | -1.2917 |  |  |
|  |  |  |  |  |  |  |  |
| Parenthesis | 0.879001 |  |  |  |  |  |  |
| Distance $(\mathrm{km})$ | 3166.601 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

## Great Circle Distance $=\mathbf{3 1 6 6 . 0} \mathbf{~ k m}$

h) Comment on the difference between the answers to (e) and (g) and the potential for fuel saving by airlines flying on great circle routes.

In this case, the Great Circle (g) lies almost along the parallel line used in part (e), so the difference in the distance between them $=3191.9-3166.0=25.9 \mathrm{~km}=0.8 \%$ difference is quite small.
i) Use ArcGIS Pro to determine the precise coordinates of Salt Lake City and New York City in the USA Contiguous Albers Equal Area Conic USGS version coordinate system.

This can be done as follows
Use Excel to create a table with decimal degree Latitude and Longitude (This should be your answer to (d) above.

| A | B | C |  |
| :---: | :--- | :---: | :---: |
| 1 | Name | Latitude | Longitude |
| 2 | Salt Lake City | 40. | -111. |
| 3 | New York City | 40. | -74. |

Open ArcGIS Pro and create a new project with a Map. Use Add Data to add your Excel Sheet. Right click on the Sheet Table and Display XY Data


In the XY Table to Point Geoprocessing tool that opens set the Spatial Reference to NAD 1983 in the North America, USA and territories group. Here we are assuming the latitude and longitude information provided are with respect to the NAD 1983 datum.


You should see two dots on your map, one for Salt Lake City and one for New York.

In Geoprocessing locate the Project tool and Project your Point Layer to a new Feature Class with USA Contiguous Albers Equal Area Conic USGS version coordinate system (in the Projected, Continental, North America group)

| Geoprocessing | - $7 \times$ |
| :---: | :---: |
| $\oplus$ Project | 三 |
| Parameters \| Environments (?) |  |
| Input Dataset or Feature Class |  |
| Sheet1_XYTableToPoint | - |
| Output Dataset or Feature Class |  |
| SLCNY |  |
| Output Coordinate System |  |
| USA_Contiguous_Albers_Equal_Area_Conic_USGS_version | - (4) |
| -veograpmic Tiansionmationt |  |

Coordinate System $\times$


XY Coordinate Systems Available Search $0 \quad \ln$


Click on Details for the coordinate system selected to verify that the details of this coordinate system are the same as specified above on page 2 .

Locate the Add Geometry Attributes Tool and Run it for this Projected Feature Class


Open the attribute table of the Feature class and report the values of Point_X and Point_Y fields.

Solution: After following the instructions above, the map on the next page was created from which the coordinates are found from the Attribute table of the NAD 83, USGS Albers Projection as:

| City | X-Coordinate (m) | Y-Coordinate (m) |
| :--- | :--- | :--- |
| Salt Lake City | $\mathbf{- 1 3 2 4 3 0 4 . 0}$ | $\mathbf{2 0 8 3 1 8 9 . 4}$ |
| New York City | $\mathbf{1 8 2 6 2 5 9 . 8}$ | $\mathbf{2 1 7 9 2 5 8 . 9}$ |

Table 1. Coordinates in USGS Albers Projection

j) Reconcile your answer to (f) with results from (i) and comment on any differences.

## Solution:

The answer to question (f) was $\mathbf{1 9 7 4 . 9} \mathbf{~ k m}$, as the distance that Salt Lake City is north of the Origin. The corresponding figure from part (i) is the Point_Y value of Salt Lake City, which is $2083.2 \mathbf{k m}$. The difference between these values is $\mathbf{1 0 8} \mathbf{~ k m}$, which is about a $5 \%$ difference. That is a fairly significant amount. The USGS Albers Equal Area Conic Projection preserves area but it is apparent that this leads to some significant distance distortion, especially over long distances.
k) Use the projected coordinates from (i) to calculate the distance from Salt Lake City to New York.

## Solution:

Using Pythagoras Theorem with the coordinates below, and values to the nearest km

| City | X-Coordinate $(\mathbf{m})$ | Y-Coordinate $(\mathbf{m})$ |
| :--- | :--- | :--- |
| Salt Lake City | $\mathbf{- 1 3 2 4 3 0 4 . 0}(\mathbf{- 1 3 2 4 . 3} \mathbf{~ k m})$ | $\mathbf{2 0 8 3 1 8 9 . 4}(\mathbf{2 0 8 3 . 2} \mathbf{~ k m})$ |
| New York City | $\mathbf{1 8 2 6 2 5 9 . 8}(\mathbf{1 8 2 6 . 3} \mathbf{~ k m})$ | $\mathbf{2 1 7 9 2 5 8 . 9}(\mathbf{2 1 7 9 . 3} \mathbf{~ k m})$ |

Distance $=\sqrt{(1826.3-(-1324.3))^{2}+(2179.3-(2083.2))^{2}}=\mathbf{3 1 5 2 . 1} \mathbf{~ k m}$

1) Explain why the distances from answers (e), (g) and (k) are different and provide a brief interpretation of these differences.

## Solution:

The distances between Salt Lake City and New York City from the three solutions are given in the table below. The Great Circle distance is the value between the other two and because of the way it is calculated, it is probably the most reliable value.

| Question | Method | Distance (Km) |
| :--- | :--- | :--- |
| (e) | Along the parallel | 3191.9 |
| (g) | Great Circle | 3166.0 |
| (h) | Projected to USGS Albers | 3152.1 |

m) Noting that the UTM system has zones $6^{\circ}$ wide and are counted from zone 1 immediately east of the international dateline ( $-180^{\circ}$ to $-174^{\circ}$ ) with central meridian $-177^{\circ}$, determine the UTM zones for each of Salt Lake City and New York City.

## Solution:

Starting at $180^{\circ} \mathrm{W}$ and proceeding eastwards in $6^{\circ}$ decrements, means that $(180-111.9) / 6=$ 11.35. This means we cross 11 complete zones and Salt Lake City lies in UTM Zone 12.

Similarly for New York City, $(180-74.0) / 6=17.6$. This means that New York City is in UTM Zone 18.

## 2. Sizes of DEM Cells

The National Map Elevation Products (3DEP) have data available at 1 arc-sec resolution for most of North America.

The geographic coordinates of UT Austin are: $30^{\circ} 17^{\prime} 10^{\prime \prime} \mathrm{N}$ and $97^{\circ} 44^{\prime} 22^{\prime \prime} \mathrm{W}$. The geographic coordinates of Logan Utah are: $41^{\circ} 44^{\prime} 45^{\prime \prime} \mathrm{N}$ and $111^{\circ} 48^{\prime} 30^{\prime \prime} \mathrm{W} 41$.

Assuming the earth is spherical with a radius of 6371 km , determine the lengths of the lines AB and BC in meters at these locations.


Determine the area of a 1 arc sec grid cell in Logan and in Austin in $\mathrm{m}^{2}$.

## Solution:

Distance $\mathbf{A B}=\mathbf{R e}^{*} \mathbf{C o s} \phi * \Delta \lambda$
Distance $\mathbf{B C}=\operatorname{Re}^{*} \Delta \phi$
In both cases, $\Delta \phi=\Delta \lambda=1 "=(1 / 3600)^{\circ}=(1 / 3600) * \pi / 180=4.848 \times 10^{-6}$
$\mathrm{Re}=6371.0 \mathrm{~km}=6371000 \mathrm{~m}$
The conversion of latitude and longitude to decimal degrees is given below:

| City | LatDeg | LatMin | LatSec | LongDeg | LongMin | LongSec | LatDD | LongDD |  |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| Austin |  | 30 | 17 | 10 | 97 | 44 | 22 | 30.28611 | -97.73944 |
| Logan |  | 41 | 44 | 45 | 111 | 48 | 30 | 41.74583 | -111.80833 |

For Austin, $\phi=30.28611 * \pi / 180=0.5286, \operatorname{Cos}(\phi)=0.8635, \mathbf{A B}=$
$\boldsymbol{\operatorname { R e }} * \boldsymbol{\operatorname { C o s }} \phi * \Delta \lambda=6371000 * 0.8635 * 4.848 \times 10^{-6}=26.67 \mathrm{~m}$ and $\mathbf{B C}=$
$\mathbf{R e}^{*} \Delta \phi=6371000 * 4.848 \times 10^{-6}=30.89 \mathrm{~m}$
The computations are done similarly for Logan, with results given in the table below.

| City | Distance AB (m) | Distance BC (m) | Area = AB*BC (m²) |
| :--- | :--- | :--- | :--- |
| Austin | $\mathbf{2 6 . 6 7}$ | $\mathbf{3 0 . 8 9}$ | $\mathbf{7 1 1 . 8 1}$ |
| Logan | $\mathbf{2 3 . 0 5}$ | $\mathbf{3 0 . 8 9}$ | $\mathbf{8 2 3 . 8 2}$ |

The general conclusion to be drawn from this analysis is that 1 arc-second $=1$ " is approximately 30 m , and that as the latitude moves north, the actual earth area covered by 1" x 1" Digital Elevation Models gets progressively smaller.

