CE 311K - McKinney

HW-9 Nonlinear Equations

Problem 1. Use fixed-point iteration to locate the root of:

$$f(x) = e^{-x} - x$$

Use an initial guess of x_0=1.0 and iterate until the approximate error is less than 0.01%.

i	xi	xi+1	error
0	1.00000	0.36788	1.7E+00
1	0.36788	0.69220	4.7E-01
2	0.69220	0.50047	3.8E-01
3	0.50047	0.60624	1.7E-01
4	0.60624	0.54540	1.1E-01
5	0.54540	0.57961	5.9E-02
6	0.57961	0.56012	3.5E-02
7	0.56012	0.57114	1.9E-02
8	0.57114	0.56488	1.1E-02
9	0.56488	0.56843	6.2E-03
10	0.56843	0.56641	3.6E-03
11	0.56641	0.56756	2.0E-03
12	0.56756	0.56691	1.1E-03
13	0.56691	0.56728	6.5E-04
14	0.56728	0.56707	3.7E-04
15	0.56707	0.56719	2.1E-04
16	0.56719	0.56712	1.2E-04
17	0.56712	0.56716	6.7E-05

Problem 2. A mass balance for a pollutant in a well-mixed lake can be written as:

$$V\frac{dc}{dt} = W - Qc - kV\sqrt{c}$$

where c is the concentration of the pollutant in g/m^3 the parameter values are V = 1x10^6 m^3, Q = 1x10^5 m^3/yr, W=1x10^6 g/yr, and k = 0.2 m$^{0.5}$/g$^{0.5}$/yr. The steady-state concentration can be found from the equation:

$$W - Qc - kV\sqrt{c} = 0$$

The root can be found using Fixed-Point Iteration using

Method 1: $c_{i+1} = \left(\frac{W - Qc_i}{kV}\right)^2$ Method 2: $c_{i+1} = \frac{W - kV\sqrt{c_i}}{Q}$

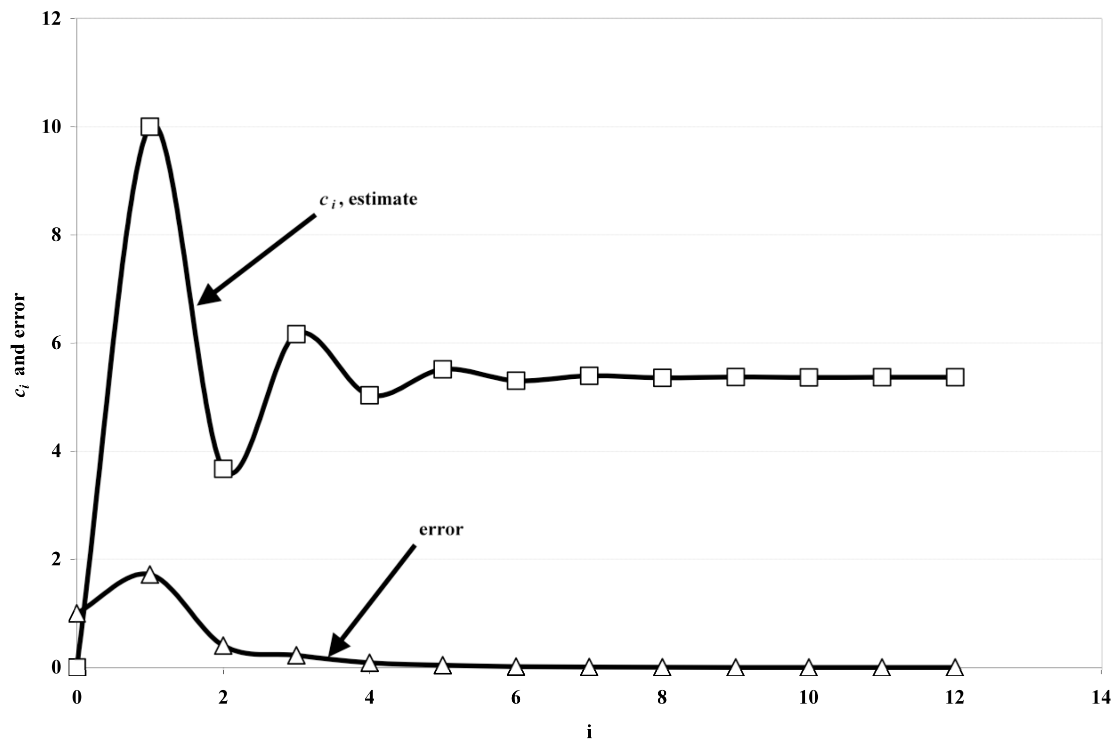
Only one of these equations will work all the time for initial guesses for $c > 1$.

(a) Use Method 2 and demonstrate that it will work for an initial guess of c_0=2.0 g/m^3.
(b) Demonstrate what happens if you try to use Method 1.

$$c_{i+1} = \left(\frac{W - Qc_i}{kV}\right)^2 \quad \text{or} \quad c_{i+1} = \frac{W - kV\sqrt{c_i}}{Q}$$

	Method 1	**Method 2**
i	**c_i**	**c_i**
0	2	2
1	16	7.17E+00
2	9	4.64E+00
3	0.25	5.69E+00
4	23.76563	5.23E+00
5	47.37311	5.43E+00
6	349.1873	5.34E+00
7	28762.01	5.38E+00
8	2.07E+08	5.36E+00
9	1.07E+16	5.37E+00
10	2.85E+31	5.37E+00

Clearly, method 1 does not work, but method 2 works well.



Problem 3. Determine the smallest real root of

$$f(x) = -11 - 22x + 17x^2 - 2.5x^3$$

graphically and (b) using the bisection method using a stopping criterion of 0.05%. (Root is between –1.0 and 0.0).

Iteration #	x_l	x_u	x_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	ε_a	$\varepsilon_a<\varepsilon_s$
1	-1	0	-0.5	30.5	-11	4.5625		
2	-0.5	0	-0.25	4.5625	-11	-4.39844	1	F
3	-0.5	-0.25	-0.375	4.5625	-4.39844	-0.22754	0.333333	F
4	-0.5	-0.375	-0.4375	4.5625	-0.22754	2.088257	0.142857	F
5	-0.4375	-0.375	-0.40625	2.088257	-0.22754	0.910782	0.076923	F
6	-0.40625	-0.375	-0.39063	0.910782	-0.22754	0.336756	0.04	F
7	-0.39063	-0.375	-0.38281	0.336756	-0.22754	0.053396	0.020408	F
8	-0.38281	-0.375	-0.37891	0.053396	-0.22754	-0.08737	0.010309	F
9	-0.38281	-0.37891	-0.38086	0.053396	-0.08737	-0.01707	0.005128	F
10	-0.38281	-0.38086	-0.38184	0.053396	-0.01707	0.018146	0.002558	F
11	-0.38184	-0.38086	-0.38135	0.018146	-0.01707	0.000536	0.00128	F
12	-0.38135	-0.38086	-0.3811	0.000536	-0.01707	-0.00827	0.000641	F
13	-0.38135	-0.3811	-0.38123	0.000536	-0.00827	-0.00387	0.00032	T-STOP

Problem 4. Starting from an initial guess $x_0 = 2.5$, take 2 iterations of Newton's method to find the **maximum value** of the function:

$$f(x) = 2\sin x - \frac{x^2}{8}$$

To find min/max, take the derivative and set it equal to zero:

$$g(x) = f'(x) = 2\cos x - \frac{x}{4} = 0 \qquad \text{and} \qquad g'(x) = -2\sin x - \frac{1}{4}$$

Now apply Newton's method to find the root of this function

$$x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} = x_i - \frac{2\cos x_i - \frac{x_i}{4}}{-2\sin x_i - \frac{1}{4}}$$

where (as Dustin Wiggins pointed out it is a "Maximum" not a "Minimum"!)

Microsoft Excel - F02EX2

x(i)	x(i+1)	g(x)	g'(x)
2.5	0.960696	-2.22729	-1.44694
0.960696	1.440124	0.905726	-1.88918
1.440124	1.395596	-0.09943	-2.23295
1.395596	1.395466	-0.00029	-2.21938
1.395466	1.395466	-2.9E-09	-2.21934
1.395466	1.395466	0	-2.21934