CE 319 F
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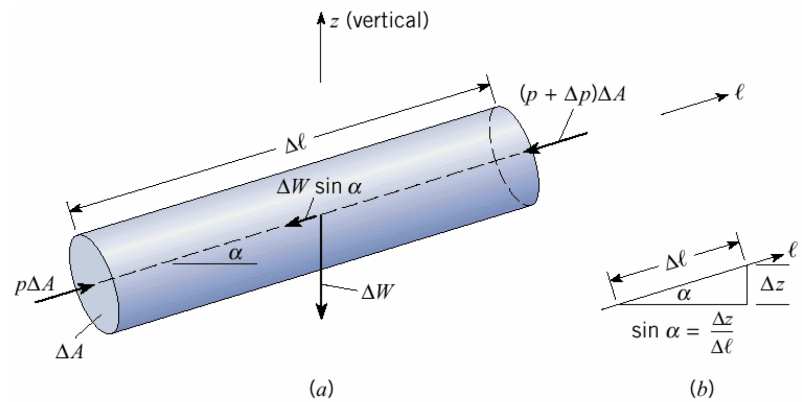
Elementary Mechanics of Fluids

Bernoulli Equation

Euler Equation

- Fluid element accelerating in l direction & acted on by pressure and weight forces only (no friction)
- Newton's 2nd Law



$$\sum F_l = Ma_l$$

$$p\Delta A - (p + \Delta p)\Delta A - \Delta W \sin\alpha = \rho \Delta l \Delta A a_l$$

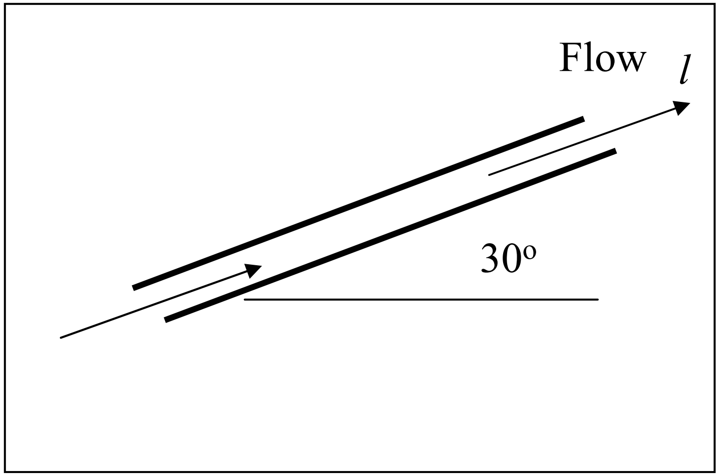
$$p - (p + \Delta p) - \gamma \Delta l \sin\alpha = \rho \Delta l a_l$$

$$-\frac{dp}{dl} - \rho g \frac{dz}{dl} = \rho a_l$$

$$-\frac{d}{dl}\left(\frac{p}{\gamma} + z\right) = \frac{a_l}{g}$$

Ex (5.1)

- **Given:** Steady flow. Liquid is decelerating at a rate of 0.3g.
- **Find:** Pressure gradient in flow direction in terms of specific weight.



$$-\frac{d}{dl}\left(\frac{p}{\gamma}+z\right)=\frac{a_l}{g}$$

$$\frac{dp}{dl}=-\frac{\gamma}{g}a_l-\gamma\frac{dz}{dl}$$

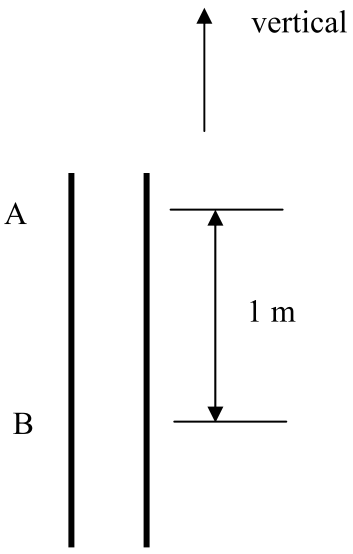
$$=-\frac{\gamma}{g}(-0.3g)-\gamma\sin 30^o$$

$$=\gamma(0.3-0.5)$$

$$\frac{dp}{dl}=-0.2\gamma$$

EX (5.3)

- **Given:** $\gamma = 10\ \text{kN/m}^3$, $p_B - p_A = 12\ \text{kPa}$.
- **Find:** Direction of fluid acceleration.



$$-\frac{d}{dz}\left(\frac{p}{\gamma}+z\right)=\frac{a_z}{g}$$

$$a_z = -g\left(\frac{1}{\gamma}\frac{dp}{dz}+\frac{dz}{dz}\right)$$

$$a_z = -g\left(\frac{p_A - p_B}{\gamma}+1\right)$$

$$a_z = -g\left(\frac{-12{,}000}{10{,}000}+1\right)$$

$$a_z = g(1.2-1) > 0 \qquad \text{(acceleration is up)}$$

HW (5.7)

- Ex (5.6) What pressure is needed to accelerate water in a horizontal pipe at a rate of 6 m/s^2?

$$-\frac{d}{dl}\left(\frac{p}{\gamma}+z\right)=\frac{a_l}{g}$$

$$-\frac{dp}{dl}-\gamma\frac{dz}{dl}=\rho a_l$$

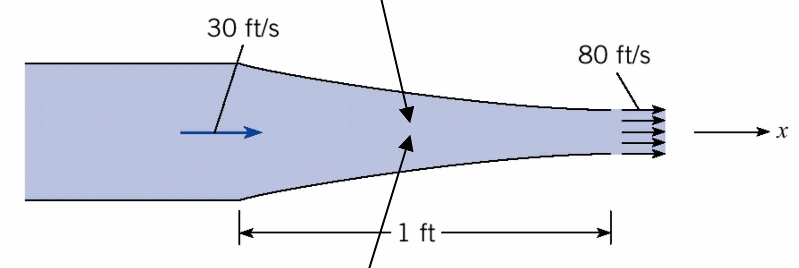
$$-\frac{dp}{dl}=\rho a_l=1000\,kg/m^3*6m/s^2$$

$$\frac{dp}{dl}=-6000\,N/m^3$$

Ex (5.10)

- **Given:** Steady flow. Velocity varies linearly with distance through the nozzle.
- **Find:** Pressure gradient ½-way through the nozzle

$V_{1/2}$=(80+30)/2 ft/s = 55 ft/s



dV/dx = (80-30) ft/s /1 ft = 50 ft/s/ft

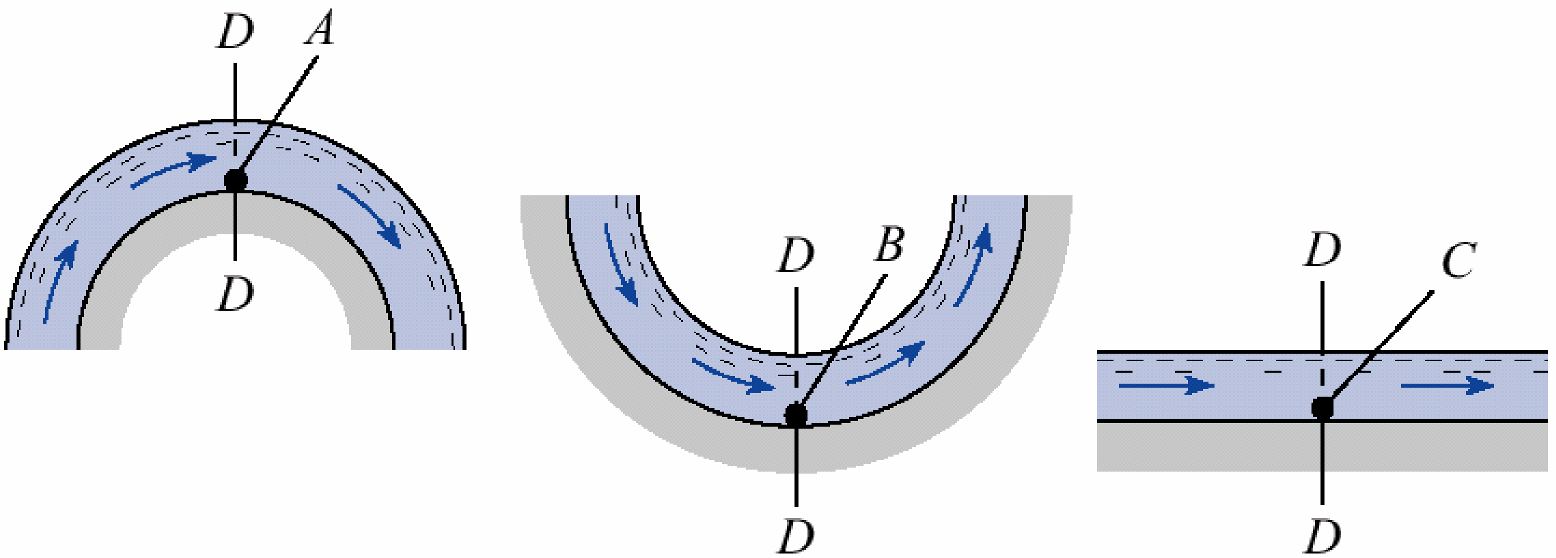
$$-\frac{d}{dx}\left(\frac{p}{\gamma}+z\right)=\frac{a_x}{g}$$

$$\frac{dp}{dx}=-\rho a_x=-\rho\left(V\frac{dV}{dx}\right)$$

$$=(-1.94\, slugs/ft^3)*(55\, ft/s)*(50\, ft/s/ft)$$

$$=-5{,}355\, lbf/ft^2/ft$$

HW (5.11)



Bernoulli Equation

$$-\frac{d}{ds}\left(\frac{p}{\gamma}+z\right)=\frac{1}{g}a_t$$

$$=\frac{1}{g}V\frac{dV}{ds}$$

$$=\frac{d}{ds}\left(\frac{V^2}{2g}\right)$$

$$\frac{d}{ds}\left(\frac{p}{\gamma}+z+\frac{V^2}{2g}\right)=0$$

$$\frac{p}{\gamma}+z+\frac{V^2}{2g}=\text{Constant}$$

- Consider steady flow along streamline
- *s* is along streamline, and *t* is tangent to streamline

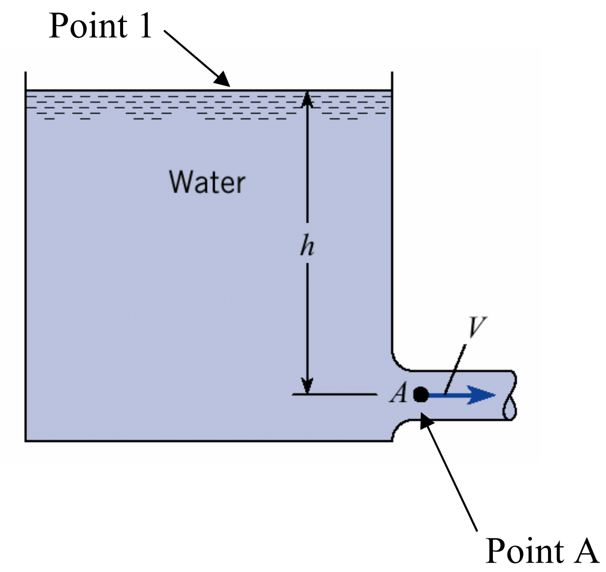
$$\frac{p}{\gamma}+z=Piezometric\ head$$

$$\frac{V^2}{2g}=Velocity\ (dynamic)\ head$$

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

Ex (5.47)

- **Given:** Velocity in outlet pipe from reservoir is 6 m/s and h = 15 m.
- **Find:** Pressure at A.
- **Solution:** Bernoulli equation



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g}$$

$$\frac{0}{\gamma} + h + \frac{0}{2g} = \frac{p_A}{\gamma} + 0 + \frac{V_A^2}{2g}$$

$$p_A = \gamma\left(h - \frac{V_A^2}{2g}\right) = 9810\left(15 - \frac{18}{9.81}\right)$$

$$p_A = 129.2\,kPa$$

Example

- **Given:** D=30 in, d=1 in, h=4 ft
- **Find:** V_A

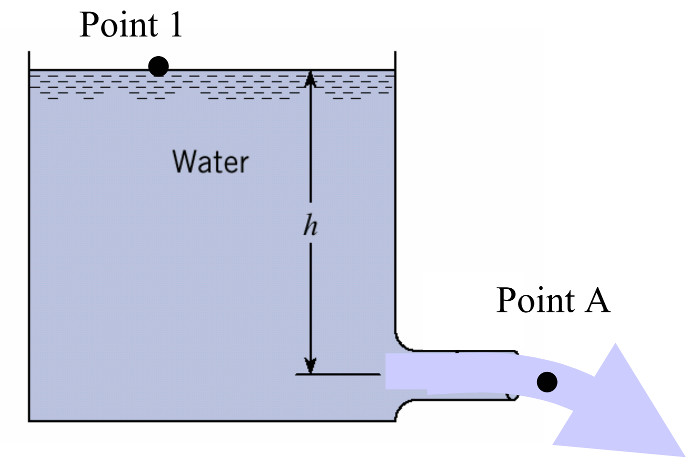
- **Solution:** Bernoulli equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g}$$

$$\frac{0}{\gamma} + h + \frac{0}{2g} = \frac{0}{\gamma} + 0 + \frac{V_A^2}{2g}$$

$$V_A = \sqrt{2gh}$$

$$= 16\ ft/s$$



Example – Venturi Tube

- **Given:** Water 20°C, V_1=2 m/s, p_1=50 kPa, D=6 cm, d=3 cm
- **Find:** p_2 and p_3
- Solution: **Continuity Eq.**

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D}{d}\right)^2$$

- **Bernoulli Eq.**

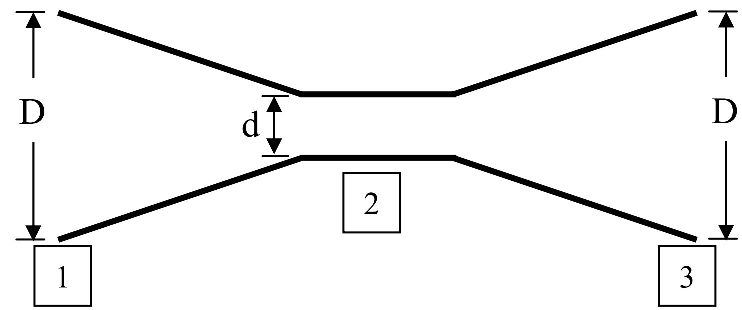
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2)$$

$$= p_1 + \frac{\rho}{2}[1-(D/d)^4]V_1^2$$

$$= 150{,}000 + \frac{1000}{2}[1-(6/3)^4]2^2 \ Pa$$

$$p_2 = 120\,kPa$$



Nozzle: velocity increases, pressure decreases

Diffuser: velocity decreases, pressure increases

Similarly for 2 → 3, or 1 → 3

$$p_3 = 150\,kPa$$

Pressure drop is fully recovered, since we assumed no frictional losses

Knowing the pressure drop 1 → 2 and d/D, we can calculate the velocity and flow rate

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(d/D)^4]}}$$

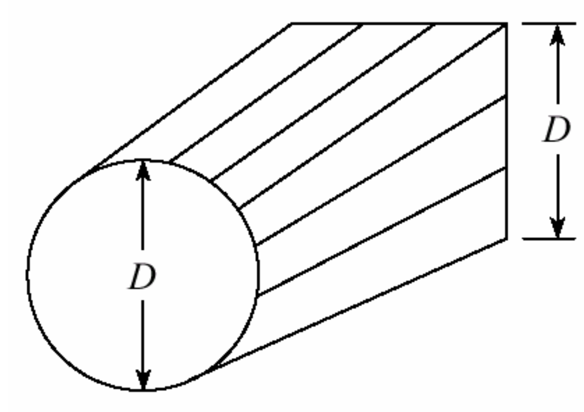
Ex (5.48)

- **Given:** Velocity in circular duct = 100 ft/s, air density = 0.075 lbm/ft3.
- **Find:** Pressure change between circular and square section.
- **Solution:** Continuity equation

$$V_c A_c = V_s A_s$$

$$100(\frac{\pi}{4} D^2) = V_s D^2$$

$$V_s = 100(\frac{\pi}{4}) = 78.54\ ft/s$$



Air conditioning (~ 60 °F)

- Bernoulli equation

$$\frac{p_c}{\gamma} + z_c + \frac{V_c^2}{2g} = \frac{p_s}{\gamma} + z_s + \frac{V_s^2}{2g}$$

$$p_c - p_s = \frac{\rho}{2}(V_s^2 - V_c^2)$$

$$p_c - p_s = \frac{0.075\,lbm/ft^3}{2*32.2\,lbm/slug}(78.54^2 - 100^2)$$

$$= -4.46\,lbf/ft^2$$

Ex (5.49)

- **Given:** ρ = 0.0644 lbm/ft^3 V_1= 100 ft/s, and A_2/A_1=0.5, γ_m=120 lbf/ft^3
- **Find:** Δh
- **Solution:** Continuity equation

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \frac{A_1}{A_2} = 100 * 2 = 200\ ft/s$$

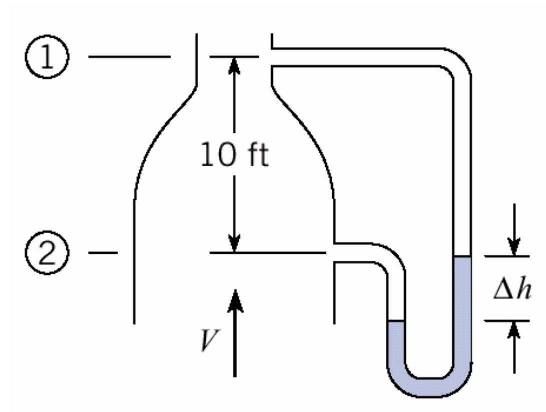
- Bernoulli equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$p_1 - p_2 = \frac{0.0644\, lbm/ft^3}{2 * 32.2\, lbm/slug}(200^2 - 100^2)$$

$$= 30\, lbf/ft^2$$



Heating (~ 170 ^{o}F)

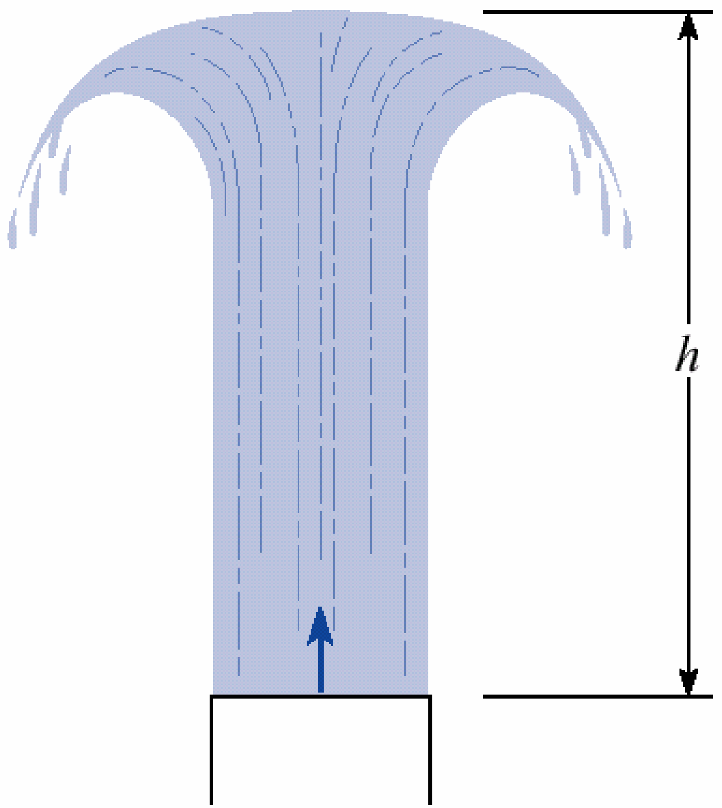
- Manometer equation

$$p_1 - p_2 = \Delta h(\gamma_m - \gamma_{air})$$

$$30 = \Delta h \frac{(120 - 0.0644) lbm/ft^3}{32.2\, lbm/slug} * 32.2\ ft/s^2$$

$$\Delta h = 0.25\ ft$$

HW (5.51)



Stagnation Tube

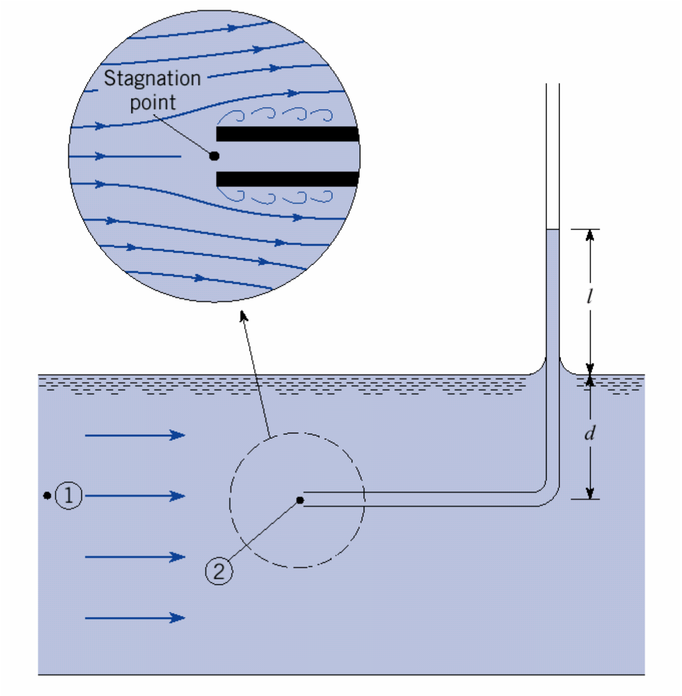
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma}$$

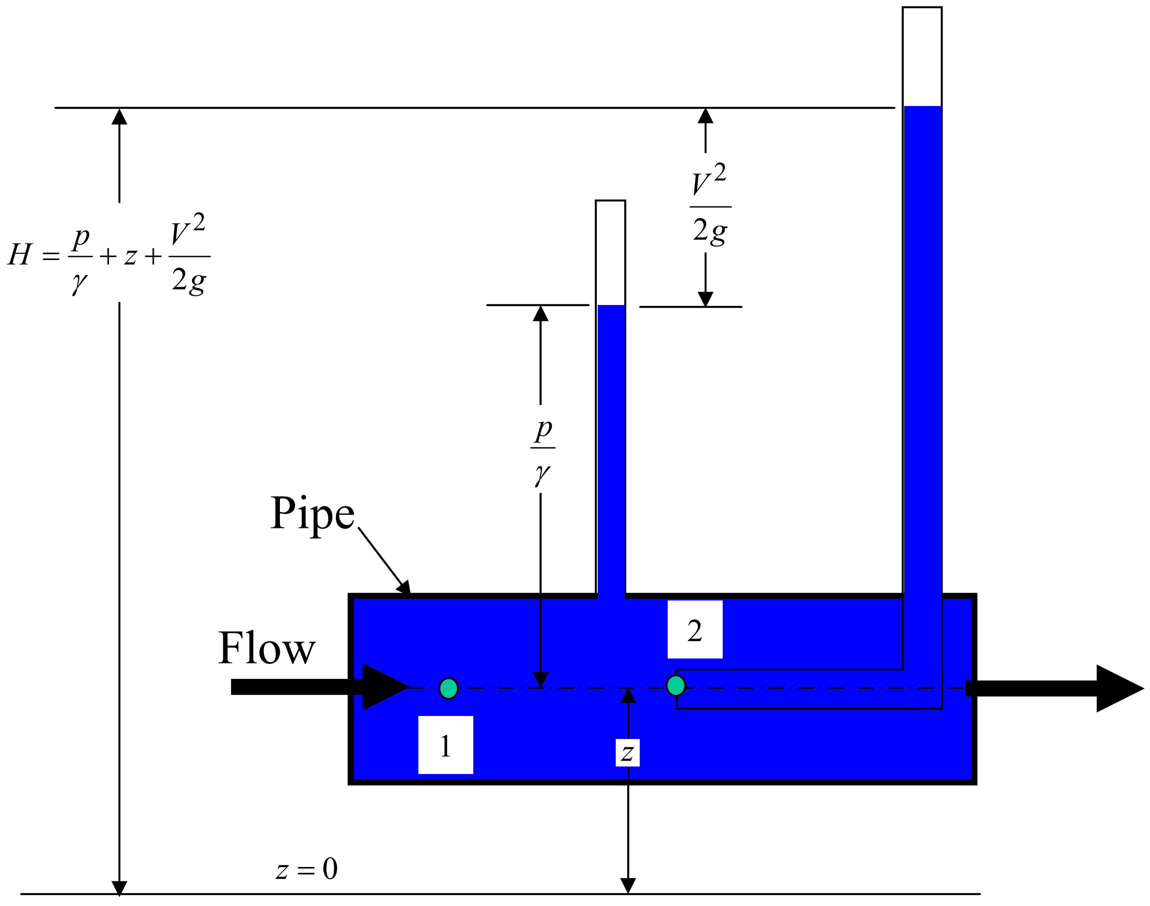
$$V_1^2 = \frac{2}{\rho}(p_2 - p_1)$$

$$= \frac{2}{\rho}(\gamma(l + d) - \gamma d)$$

$$V_1 = \sqrt{2gl}$$



Stagnation Tube in a Pipe



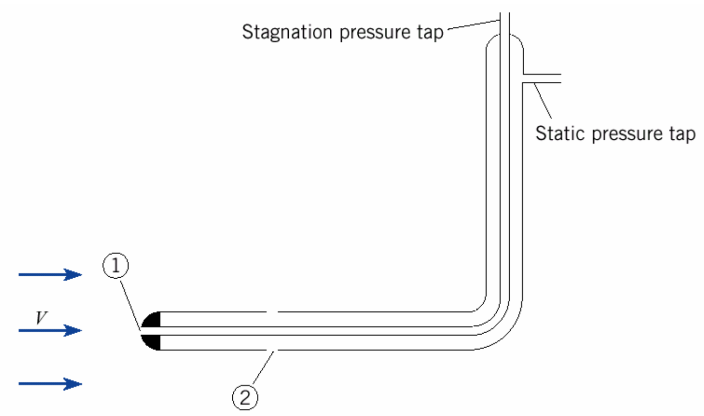
Pitot Tube

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

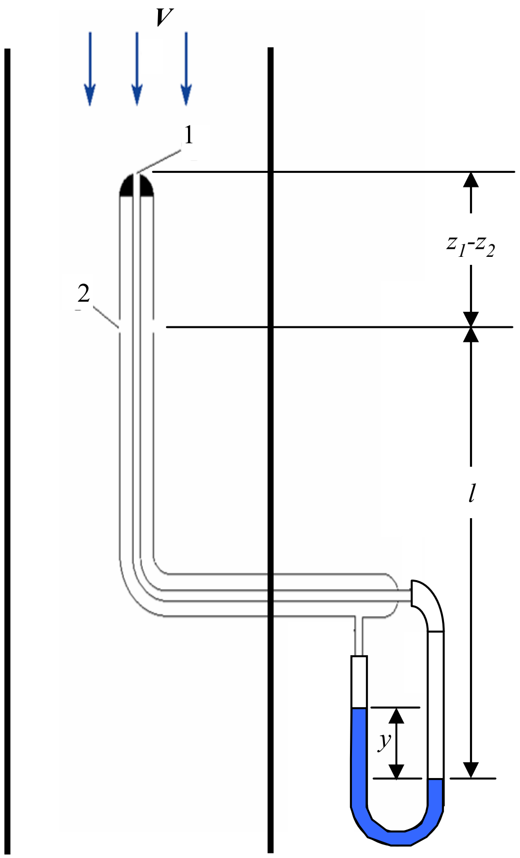
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2g[(\frac{p_1}{\gamma} + z_1) - (\frac{p_1}{\gamma} + z_1)}$$

$$V = \sqrt{2g(h_1 - h_2)}$$



Pitot Tube Application (p.170)



$$p_1 + (z_1 - z_2)\gamma_k + l\gamma_k - y\gamma_{Hg} - (l - y)\gamma_k = p_2$$

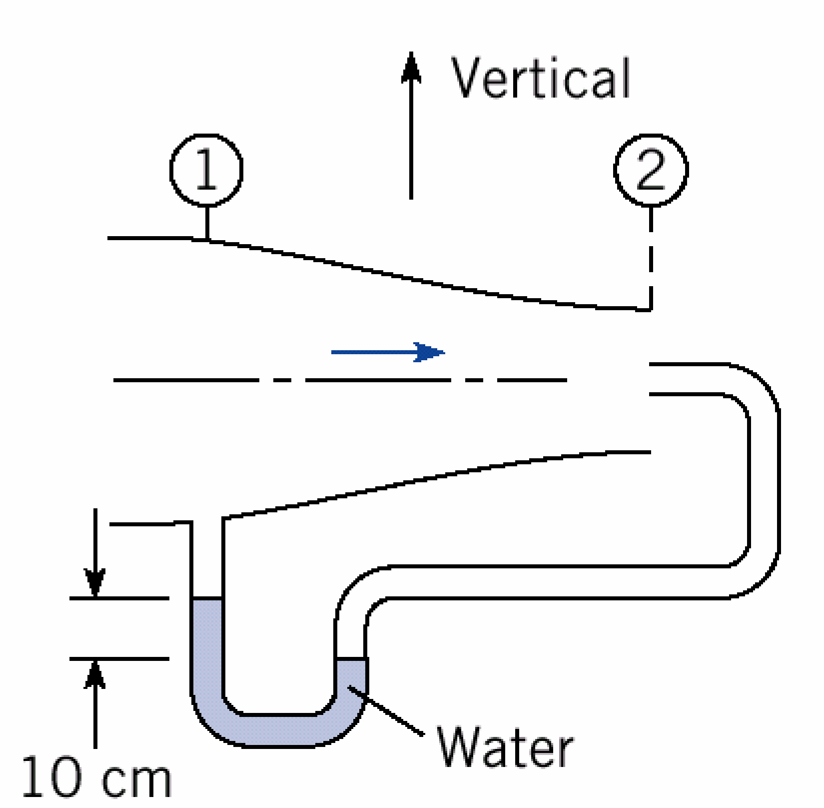
$$p_1 - p_2 = y(\gamma_{Hg} - \gamma_k) + (z_1 - z_2)\gamma_k$$

$$\frac{p_1 - p_2}{\gamma_k} + z_1 - z_2 = \frac{y(\gamma_{Hg} - \gamma_k)}{\gamma_k}$$

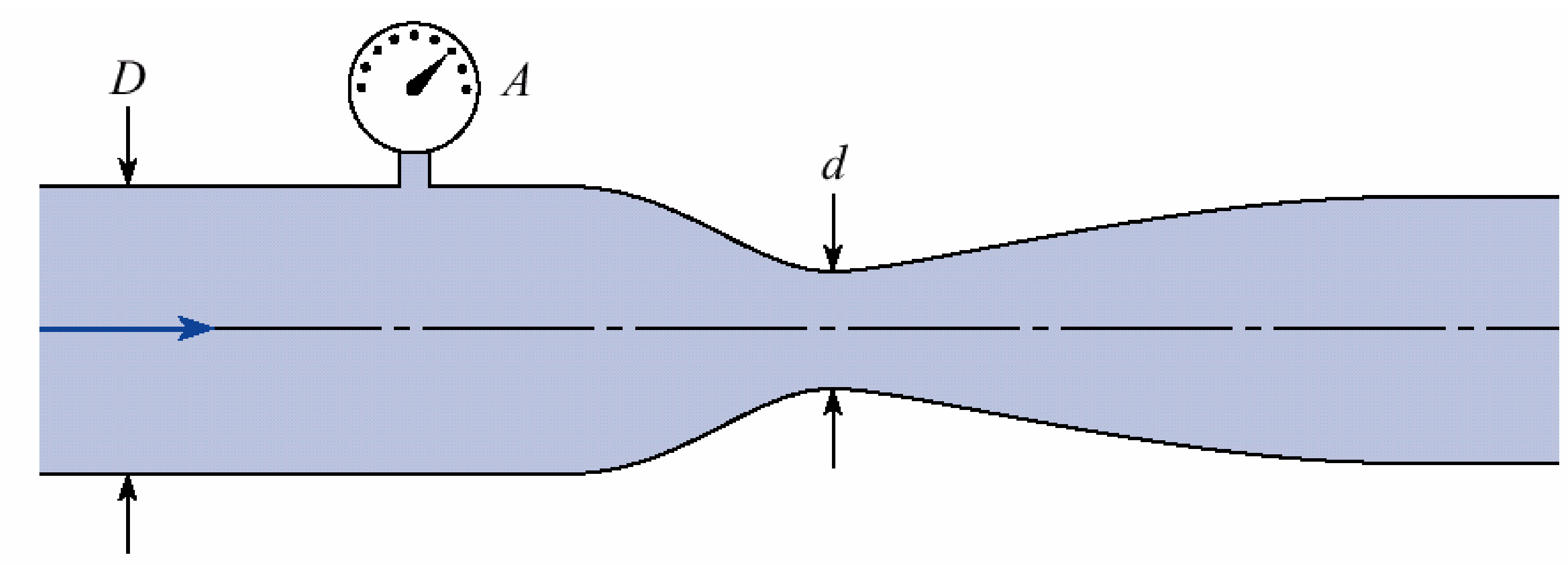
$$h_1 - h_2 = y(\gamma_{Hg} / \gamma_k - 1)$$

$$V = \sqrt{2gy(\gamma_{Hg} / \gamma_k - 1)} = 24.3\ ft/s$$

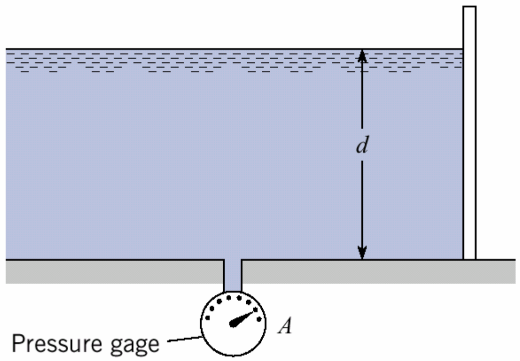
HW (5.69)



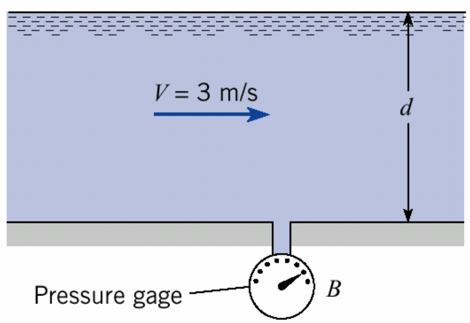
HW (5.75)



HW (5.84)



(a)



(b)

HW (5.93)