CE 319 F

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Elementary Mechanics of Fluids

Momentum

Equation

Momentum Equation

- Reynolds Transport Theorem

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho d\forall + \int_{CS} b\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

- b = velocity; B_{sys} = system momentum

$$\sum \vec{\mathbf{F}} = \frac{d}{dt}\int_{CV} \vec{\mathbf{V}}\rho d\forall + \int_{CS} \vec{\mathbf{V}}\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

- Vector equation -- 3 components, e.g., x

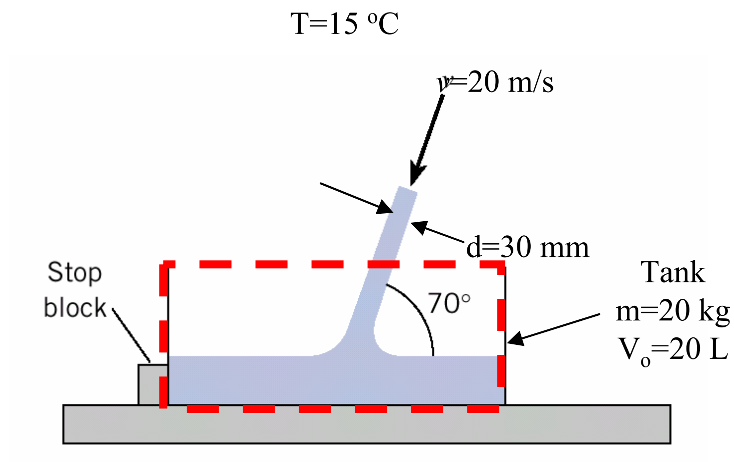
$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$\frac{dB_{sys}}{dt} = \frac{d\vec{\mathbf{M}}_{sys}}{dt} = \sum \vec{\mathbf{F}}$$

$$\vec{\mathbf{V}} = u\vec{\mathbf{i}} + v\vec{\mathbf{j}} + w\vec{\mathbf{k}}$$

Ex (6.3)

- **Given:** Figure
- **Find:** (a) Force acting on bottom of the tank and (b) the force acting on the stop block.
- **Solution:**



$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$F = \sum_{CS} u\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

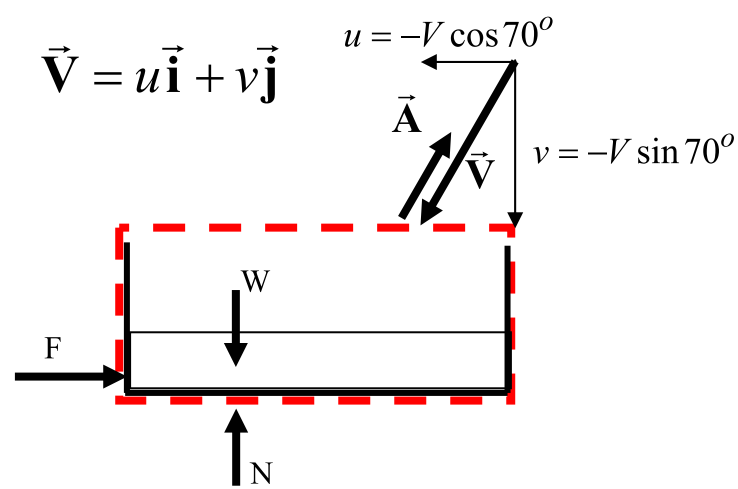
$$= -V\cos 70^o \rho(-VA)$$

$$F = \rho A V^2 \cos 70^o$$

$$= (999)*(\pi * 0.03^2/4)(20^2)\cos 70^o$$

$$= 97\,N$$

$$\vec{\mathbf{V}} = u\vec{\mathbf{i}} + v\vec{\mathbf{j}}$$



Ex (6.3)

$$\sum F_y = \frac{d}{dt}\int_{CV} v\rho d\forall + \int_{CS} v\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

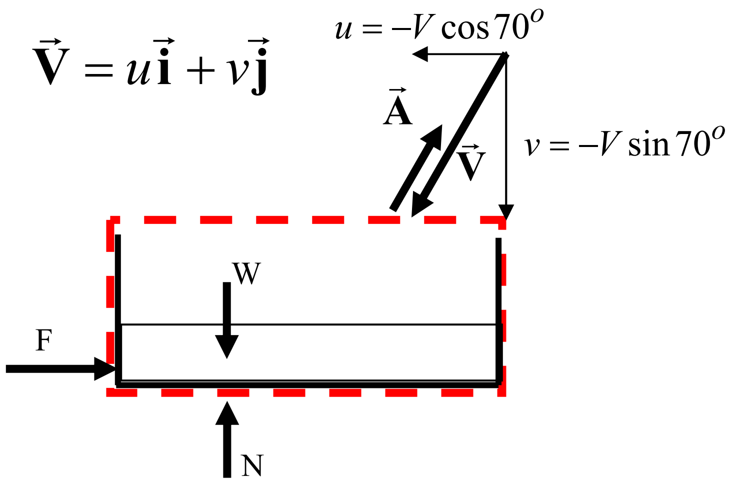
$$N - W = \sum_{CS} v\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$= -V\sin 70^o \rho(-VA)$$

$$N = W + \rho A V^2 \sin 70^o$$

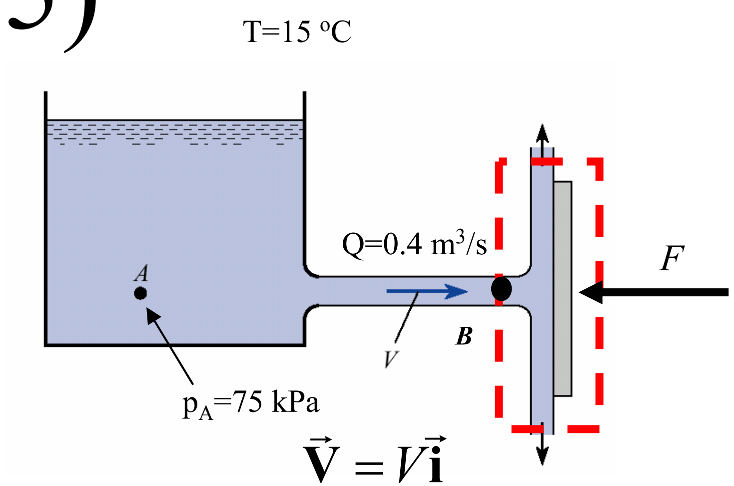
$$= (20 + 0.02 * 999)9.81 + 282.5\sin 70^o$$

$$= 658\,N$$



EX (6.5)

- **Given:** Figure
- **Find:** Horizontal force required to hold plate in position
- **Solution:**



$$\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + z_B + \frac{V_B^2}{2g}$$

$$\frac{p_A}{\gamma} = \frac{V_B^2}{2g}$$

$$V_B = \sqrt{2\frac{p_A}{\rho}} = \sqrt{2*75000/999} = 12.3\,m/s$$

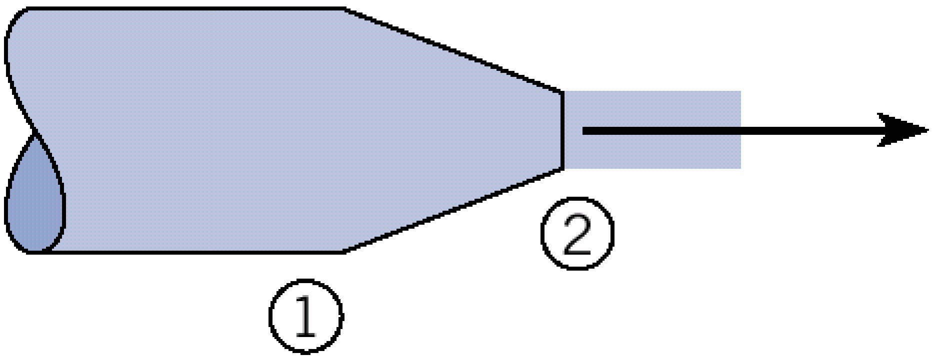
$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$-F = \sum_{CS} u\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$= V\rho(-VA)$$

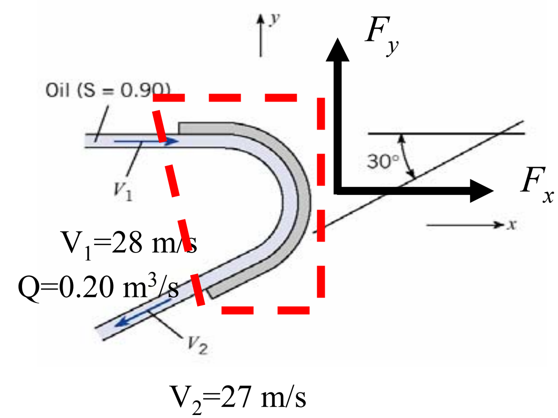
$$F = \rho Q V = 999*0.4*12.3 = 4.9\,kN$$

HW (6.12)



Ex (6.17)

- **Given:** Figure
- **Find:** External reactions in x and y directions needed to hold fixed vane.
- **Solution:**



$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$F = \sum_{CS} u\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$= V_1\rho(-V_1 A) + (-V_2\cos 30^o)\rho(V_2 A_2)$$

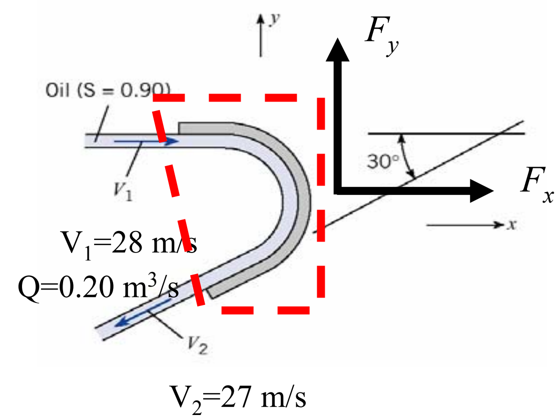
$$F = -V_1\ \rho(V_1\ A_1) - V_2\ \cos 30^o\ \rho(V_2\ A_2)$$

$$= -\rho Q(V_1\ + V_2\ \cos 30^o)$$

$$= -0.9 * 1000 * 0.2(28 + 27\cos 30^o)$$

$$F_x = -9.25\,kN\ (\text{to the left})$$

Ex (6.17)



$$\sum F_y = \frac{d}{dt}\int_{CV} v\rho \, d\forall + \int_{CS} v\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$F_y = \sum_{CS} v\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$= (-V_2 \sin 30^o)\rho(V_2 A_2)$$

$$F = -V_2 \sin 30^o \rho(V_2 A_2)$$

$$= -\rho Q(V_2 \sin 30^o)$$

$$= -0.9 * 1000 * 0.2(27 \sin 30^o)$$

$$F_y = -2.43 kN \, (\mathrm{d}own)$$

Ex (6.34)

- **Given:** Figure
- **Find:** Force applied to flanges to hold pipe in place
- **Solution:** Continuity equation

$$Q = V_1 A_1 = V_2 A_2$$

$$V = Q / A = 0.6/(\pi * 0.3^2 / 4) = 8.49\, m/s$$

- Momentum

$$\sum F_x = \frac{d}{dt} \int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$F_x + p_1 A_1 + p_2 A_2 = V_1 \rho(-V_1 A_1) - V_2 \rho(V_2 A_2)$$

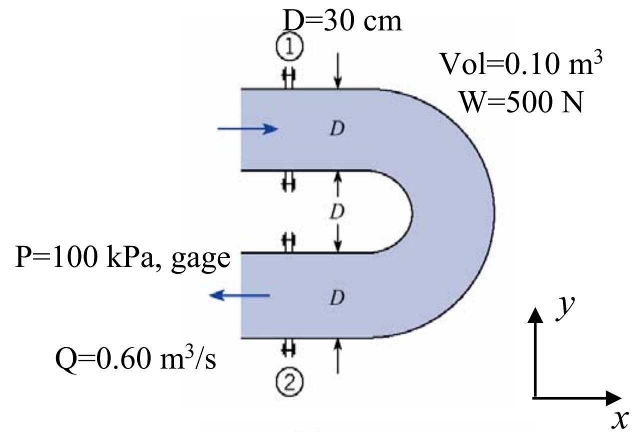
$$F_x = -2(100{,}000)(\pi * 0.3^2 / 4) - 2 * 8.49 * 1000 * 0.6$$

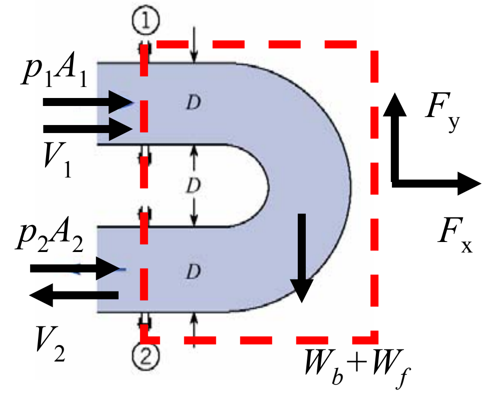
$$F_x = -24{,}325\ N$$

$$\sum F_y = \frac{d}{dt} \int_{CV} v\rho d\forall + \int_{CS} v\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

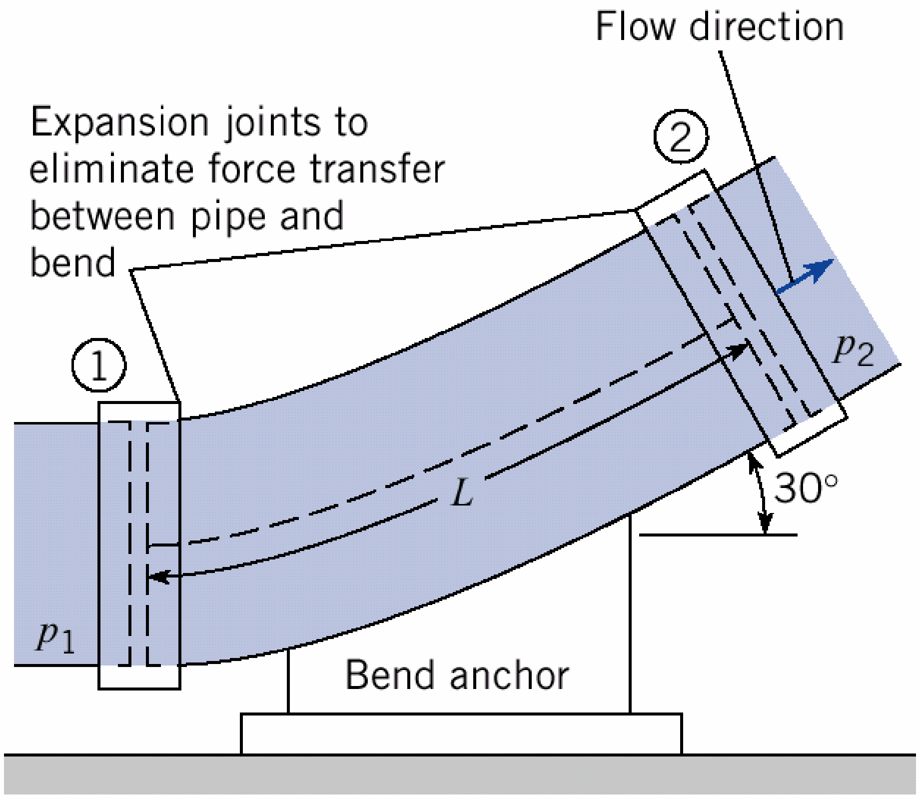
$$F_y - W_b - W_f = 0$$

$$F_y = 500 + 0.1 * 9810 = 1481\, N$$



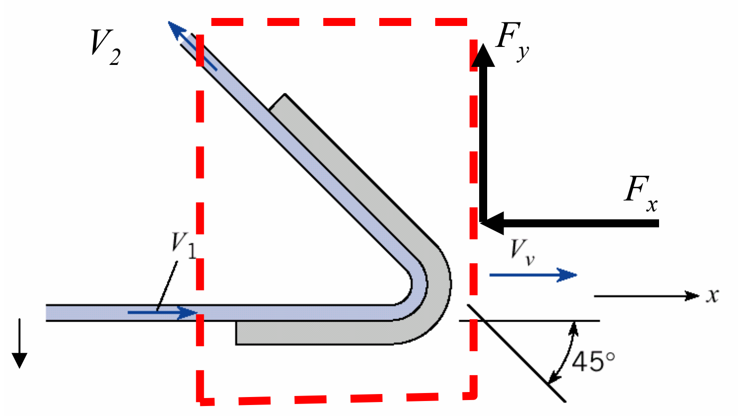


HW (6.37)


Flow direction
Expansion joints to eliminate force transfer between pipe and bend
2
1
L
p_2
p_1
30°
Bend anchor

Ex (6.72)

- **Given:** Water jet, 6 cm diameter, with velocity 20 m/s hits vane moving at 7 m/s.
- **Find:** Find force on vane by water.
- **Solution:** Select CV moving with the vane at constant velocity. The magnitude of the velocity along the vane is constant



$$V_1 = V_2 = V - V_v$$

$$V_1 A_1 = V_2 A_2 = (V - V_v) A$$

$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

$$-F_x = (V - V_v)\rho[-(V - V_v)A_1] - (V - V_v)\cos 45^o \rho[(V - V_v)A_2]$$

$$= (V - V_v)^2 \rho A(1 + \cos 45^o)$$

$$= (20 - 7)^2 (1000)(\pi * 0.06^2 / 4)(1 + \cos 45^o)$$

$$F_x = 815.7\, N$$

$$\sum F_y = \frac{d}{dt}\int_{CV} v\rho d\forall + \int_{CS} v\rho \vec{\mathbf{V}} \cdot \vec{\mathbf{A}}$$

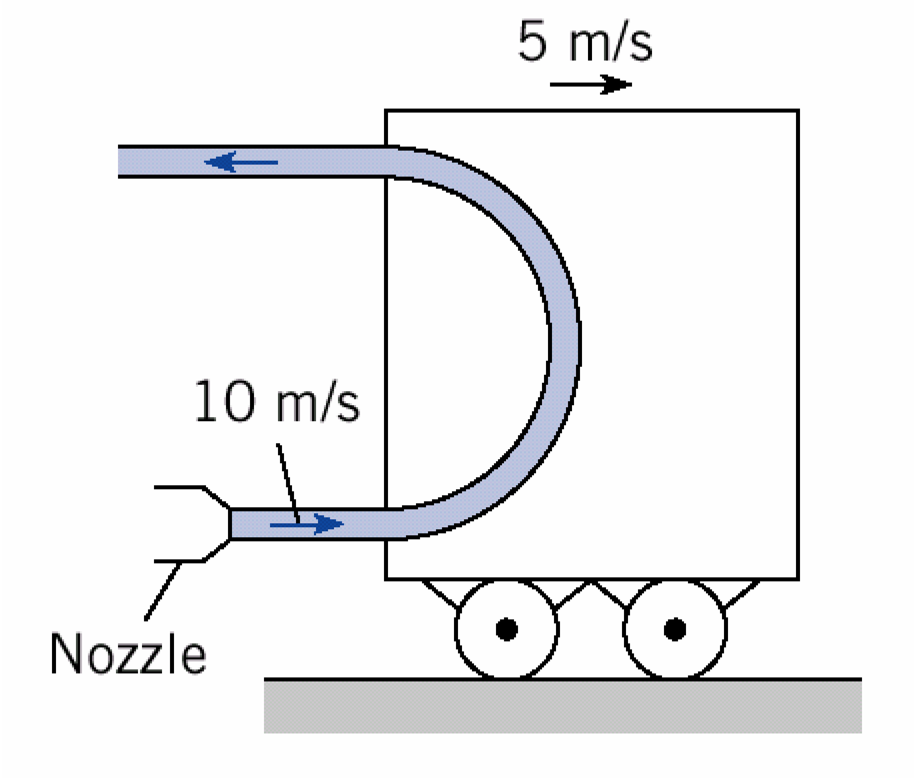
$$F_y = (V - V_v)\sin 45^o \rho[(V - V_v)A_2]$$

$$= (V - V_v)^2 \rho A \sin 45^o$$

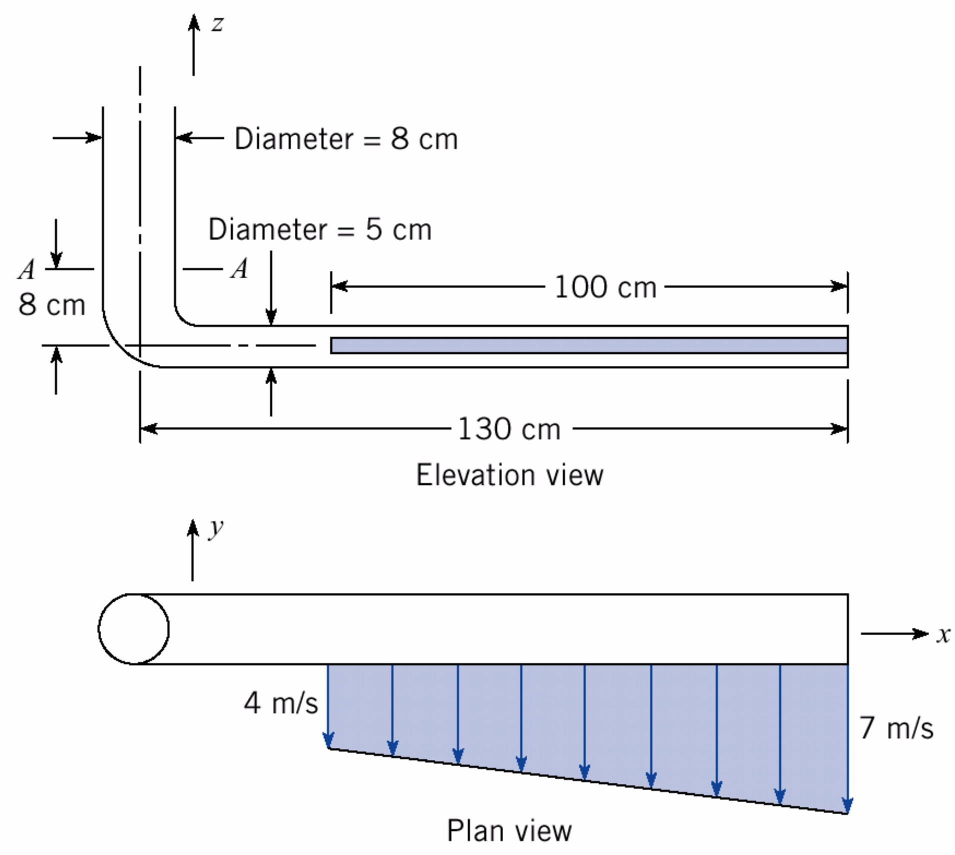
$$= (20 - 7)^2 (1000)(\pi * 0.06^2 / 4)(\sin 45^o)$$

$$F_y = 337.9\, N$$

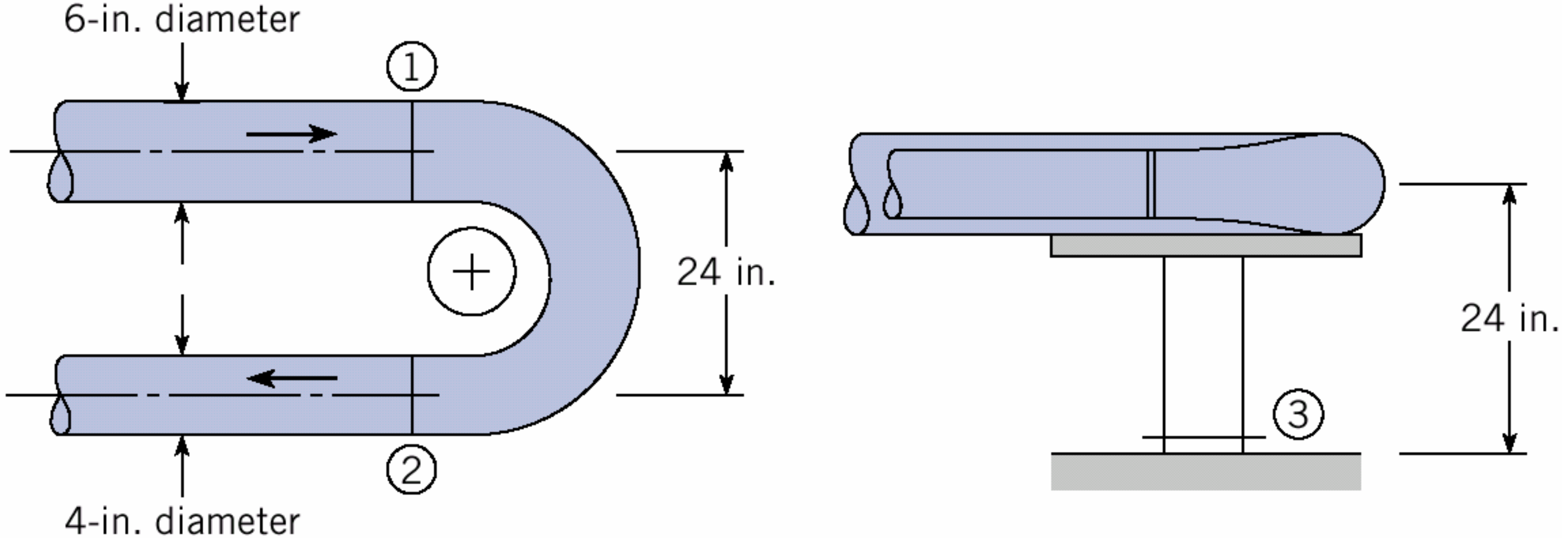
HW (6.80)



HW (6.98)


z
Diameter = 8 cm
Diameter = 5 cm
A
A
8 cm
100 cm
130 cm
Elevation view
y
x
4 m/s
7 m/s
Plan view

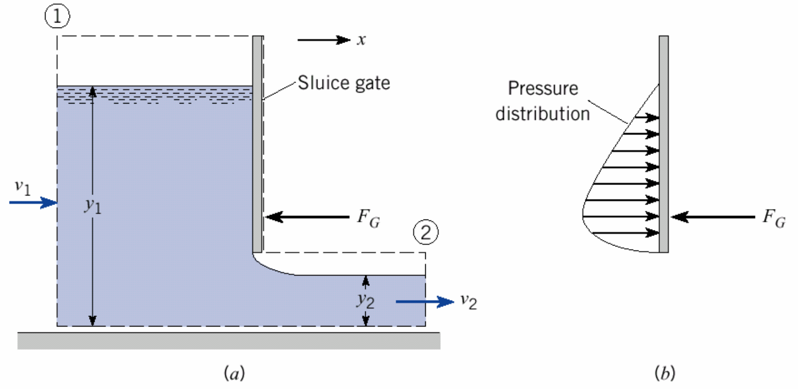
HW (6.101)



Sluice Gate

- **Find:** Force due to pressure on face of gate
- **Solution:**

  Assume: v_1 and v_2 are uniform (so pressure is hydrostatic)



$$\sum F_x = \frac{d}{dt}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{\mathbf{V}}\cdot\vec{\mathbf{A}}$$

$$\sum F_x = v_1\rho[-v_1 A_1] + v_2\rho[v_2 A_2]$$

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 - F_G = \rho Q(v_2 - v_1)$$

$$(\gamma\frac{y_1}{2}y_1 b) - (\gamma\frac{y_2}{2}y_2 b) - F_G = \rho Q(v_2 - v_1)$$

$$F_G = \rho Q(v_1 - v_2) + \frac{\gamma b}{2}(y_1^2 - y_2^2)$$