CE 319 F

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Elementary Mechanics of Fluids

Resistance

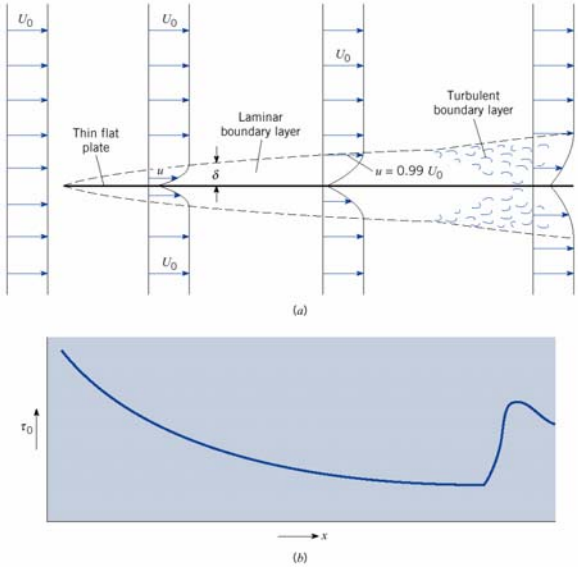
Flow Past a Flat plate

- Boundary layer: Region next to an object where fluid has its velocity changed due to shear resistance of the boundary.
- Velocity gradient exists between free stream and object, thus shear stress exists at surface which retards the flow.
- Boundary layer grows in downstream direction, until the onset of turbulence

$$\text{Pipes}\quad \mathrm{Re} = \frac{\rho V D}{\mu}, \qquad \text{Transition}\ \mathrm{Re} \approx 2{,}000$$

$$\text{Plates}\quad \mathrm{Re} = \frac{\rho V x}{\mu}, \qquad \text{Transition}\ \mathrm{Re} \approx 500{,}000$$

- Shear stress is high at leading edge and decreases until transition and then increases again



Laminar region

$$\text{Stress:}\quad \tau = 0.332\,\mu \frac{U_o}{x}\sqrt{\mathrm{Re}_x}$$

$$\text{Resistance:}\quad F = 0.664\,B\mu U_o \sqrt{\mathrm{Re}_L}$$

$$\text{Coefficient:}\qquad C_F = \frac{F_s}{BL\rho U_o^2/2} = \frac{1.33}{\sqrt{\mathrm{Re}_L}}$$

Turbulent region

$$\text{Stress:}\quad \tau = 0.5\rho U_o^2 \frac{0.058}{(\mathrm{Re}_x)^{1/5}}$$

$$\text{Resistance:}\quad F = 0.5\rho U_o^2 BL \frac{0.072}{(\mathrm{Re}_L)^{1/5}}$$

$$\text{Coefficient:}\qquad C_F = \frac{F}{A\rho U_o^2} = \frac{0.523}{\ln^2(0.06 * \mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L}$$

Shear Stress Coefficients

- Shear stress coefficient = ratio of shear stress at wall to dynamic pressure of free stream

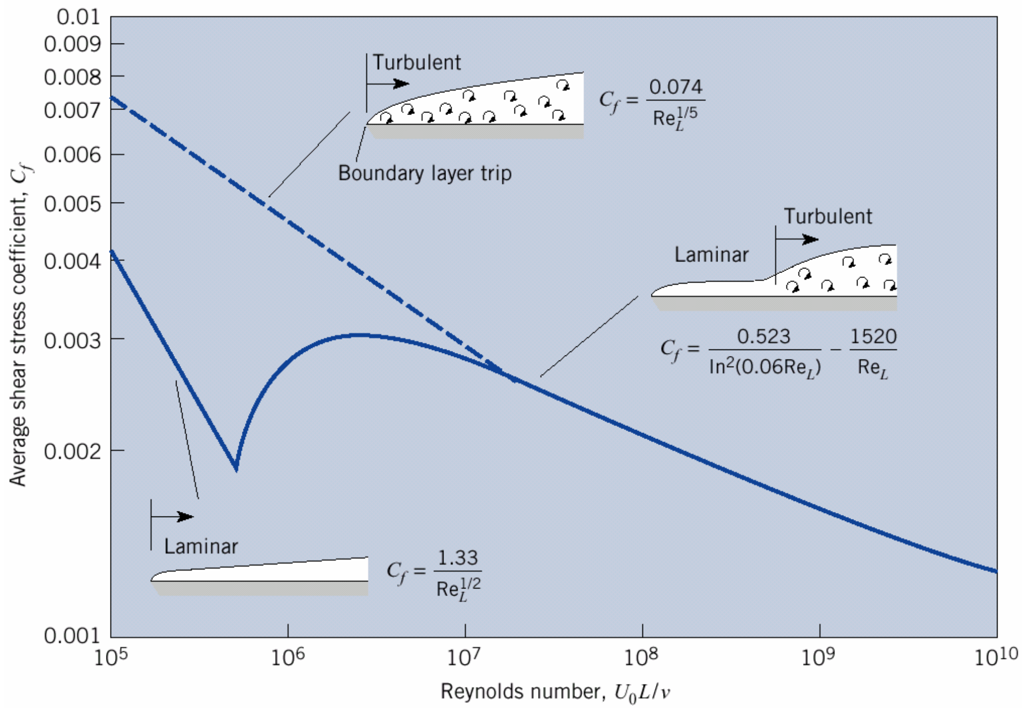
$$c_f = \frac{\tau_0}{\rho U_o^2/2}$$

- Total shearing force

$$F_s = \int_A \tau_0 dA$$

$$= \frac{\rho U_o^2}{2} \int_A c_f dA$$

$$C_f = \frac{F_s}{\rho U_o^2/2}$$



Ex (9.77)

Given: Ship prototype 500 ft long, wetted area of 25,000 ft^2, and velocity of 30 ft/s in sea water at 10^{o}C. Model is in fresh water at 60^{o}F, model:prototype scale = 1/100, and Froude numbers are matched. Drag is calculated as flat plate with model wetted area and length. A drag of 0.1 lbf is measured in the model tests.

Find: Drag on the ship.

Froude No. similarity

$$Fr_m = Fr_p$$

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\frac{1}{100}} = \frac{1}{10}$$

$$V_m = (1/10) * 30 = 3\ ft/s$$

Shear resistance on model

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{3 * 5L}{1.22x10^{-5}} = 1.23x10^6$$

$$C_f = \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} = 0.00293$$

$$F_m = C_f\left(\frac{\rho V^2 A}{2}\right) = (0.00293) * 0.5 * 1.94 * 3^2 * 2.5$$

$$F_m = 0.0639\,lbf$$

$$F_{wave} = F_{total} - F_{shear} = 0.1 - 0.0639 = 0.0361\,lbf$$

Ex (9.77)

Scale up of wave drag

$$\left(C_p\right)_m = \left(C_p\right)_p$$

$$\left(\frac{\Delta p}{\rho V^2/2}\right)_m = \left(\frac{\Delta p}{\rho V^2/2}\right)_p$$

$$\frac{\Delta p_m}{\Delta p_p} = \frac{\rho_m V_m^2}{\rho_p V_p^2}$$

Force on Prototype

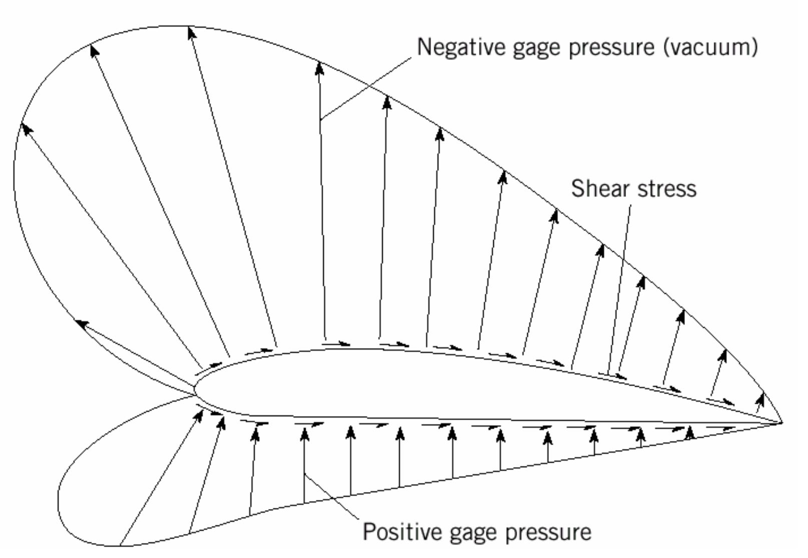
$$\frac{F_m}{F_p} = \frac{\Delta p_m}{\Delta p_p}\frac{A_m}{A_p} = \frac{\rho_m V_m^2}{\rho_p V_p^2}\frac{A_m}{A_p} = \frac{\rho_m}{\rho_p}\left(\frac{L_m}{L_p}\right)^3 = \frac{1.94}{1.99}\left(\frac{1}{100}\right)^3$$

$$F_p = F_m \frac{1.99}{1.94}(100)^3 = 0.0361 * \frac{1.99}{1.94} * 10^6$$

$$F_p = 3.7x10^4\ lbf$$

Lift of an Airfoil

- Forces acting on airfoil: Velocity over the top of foil is greater than free stream velocity, pressure there is less than freestream. Similarly, the pressure on the bottom of the foil is greater than freestream pressure. This difference in pressure contributes to the **lift** of the foil.
- Shear stress along the foil acts to **drag** on the foil.



Drag of a Thin Plate

- For a plate parallel to the flow, shear forces are the only ones acting

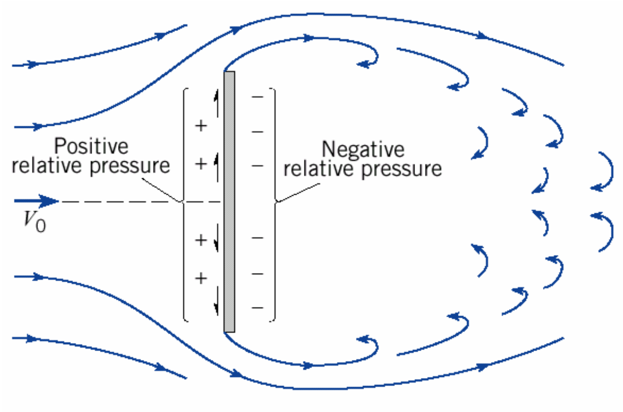
$$F_D = 2C_f BL\rho \frac{V_o^2}{2}$$

- For a plate normal to the flow, shear and pressure forces act

$$F_D = (0.8 + 1.2)BL\rho \frac{V_o^2}{2}$$

- For a more general object

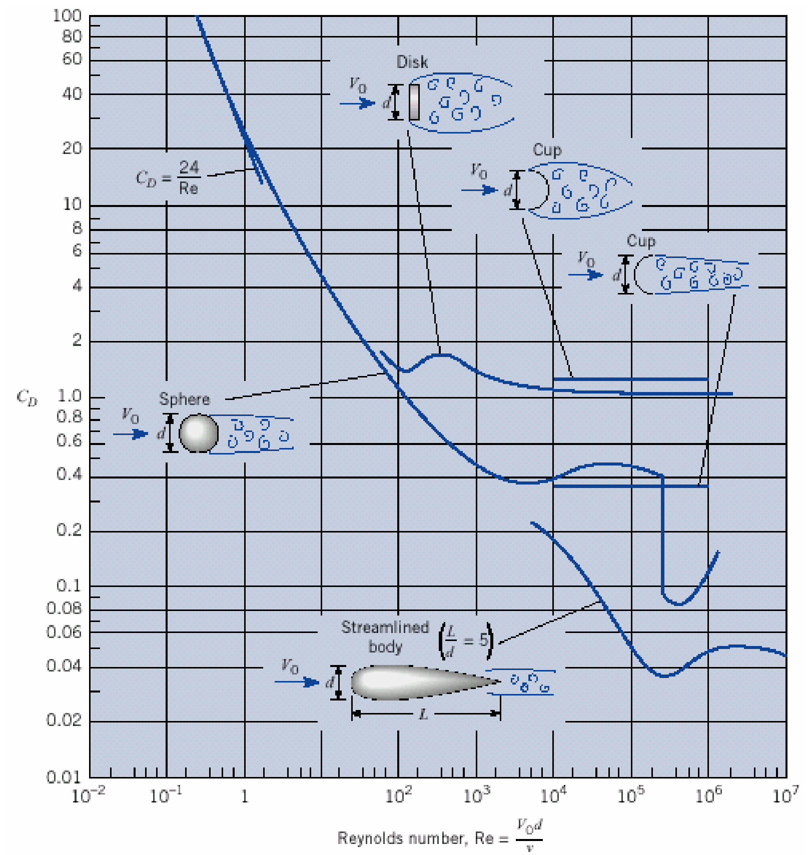
$$F_D = C_D A_p \rho \frac{V_o^2}{2}$$



Drag Coefficients

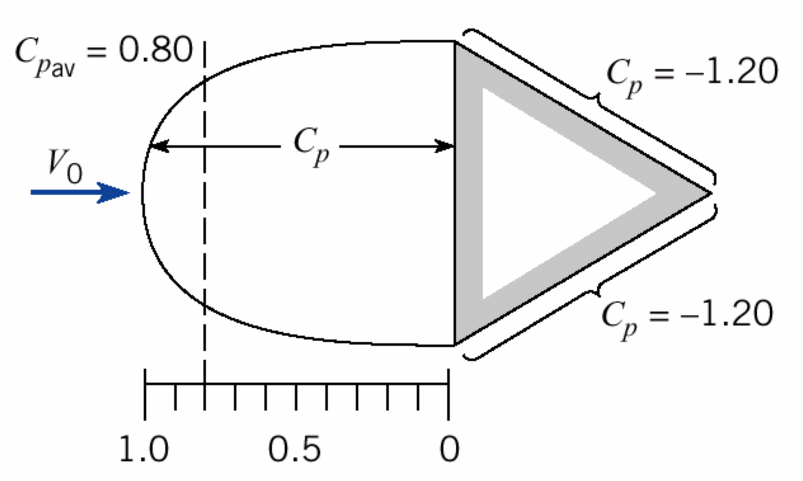
- Coefficient of Drag

$$C_D = \frac{F_D}{A_p \rho \frac{V_o^2}{2}}$$

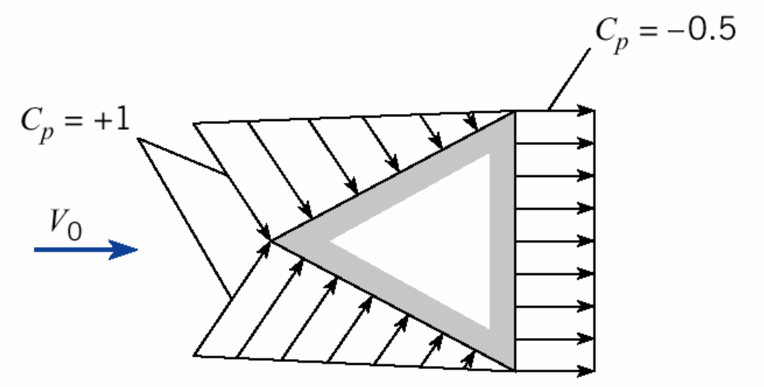


HW (11.4, 11.7, 11.56)

- 11.4



Ex (11.3)



- Given: Pressure distribution is shown, flow is left to right.
- Find: Find C_D
- Solution: C_D is based on the projected area of the block from the direction of flow. Force on downstream face is:

$$(F_D)\big|_{Drag} = C_p A_p \rho V^2 / 2 = 0.5 A_p \rho V^2 / 2$$

The total force on each side face is: $F_S = C_p A_p \rho V^2 / 2 = 0.5 A_p \rho V^2 / 2$

The drag force on one face is: $(F_S)\big|_{Drag} = F_S \sin\alpha = 0.5 A_p \rho V^2 / 2 * 0.5$

The total drag force is:

$$F_{Drag} = 2(F_S)\big|_{Drag} + (F_D)\big|_{Drag}$$

$$= 2*(0.5 A_p \rho V^2 / 2 * 0.5) + 0.5 A_p \rho V^2 / 2 = C_D A_p \rho V^2 / 2$$

Coefficient of Drag is: C_D=1

Ex (11.8)

- Given: Flag pole, 35 m high, 10 cm diameter, in 25-m/s wind, P_{atm} = 100 kPa, T=20°C
- Find: Moment at bottom of flag pole
- Solution:

$$\nu = 1.51x10^{-5}\, m^2/s,\ \rho = 1.20\, kg/m^3$$

$$\text{Re} = \frac{VD}{\nu} = \frac{25*0.1}{1.51x10^{-5}} = 1.66x10^5$$

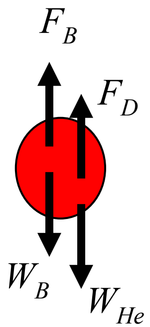
$$C_D = 0.95 \text{ (figure 11-5)}$$

$$M = F_D \frac{H}{2} = C_D A_p \rho \frac{V_o^2}{2} \frac{H}{2}$$

$$= 0.95*0.10*35*1.2*\frac{25^2}{2}*\frac{35}{2}$$

$$= 21.8\, kN$$

Ex (11.57)



- Given: Spherical balloon 2-m diameter, filled with helium @std conditions. Empty weight = 3 N.
- Find: Velocity of ascent.
- Solution:

$$\sum F_y = 0 = F_B + F_D - W_B - W_{He}$$

$$= \gamma_{air}\frac{\pi}{6}D^3 + F_D - 3 - \gamma_{He}\frac{\pi}{6}D^3$$

$$F_D = 3 - (\gamma_{air} - \gamma_{He})\frac{\pi}{6}D^3$$

$$= 3 - \gamma_{air}(1 - \frac{287}{2{,}077})\frac{\pi}{6}2^3$$

$$= -1.422\, N$$

$$F_D = C_D A_p \rho \frac{V_o^2}{2}$$

$$V_o = \sqrt{\frac{2F_D}{C_D A_p \rho}} = \sqrt{\frac{2*1.422}{C_D(\pi/4)*2^2*1.225}} = \sqrt{\frac{0.739}{C_D}}$$

Iteration: Guess C_D=0.4

$$V_o = \sqrt{\frac{0.739}{0.4}} = 1.36\, m/s$$

Check Re

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{1.36*2}{1.46x10^{-5}} = 1.86x10^5$$

Iteration: C_D=0.42

$$V_o = \sqrt{\frac{0.739}{0.42}} = 1.33\, m/s$$