CE 319 F

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Elementary Mechanics of Fluids

Energy

Equation

Energy

- First Law of Thermodynamics
  - Where
    - E = energy of a system (extensive property)
    - Q = Heat <u>added to</u> the system
    - W = Work <u>done by</u> the system

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Energy of a system changes as heat is added to the system or work is done on the system

$$E = E_u + E_k + E_p$$

  - Where
    - E_u = Internal energy of the system
    - E_k = Kinetic energy of the system
    - E_p = Potential energy of the system

  - Intensive property (energy per unit mass)

$$e = u + e_k + e_p$$

Energy

- Kinetic energy per unit mass

$$e_k = \frac{kinetic\ energy\ of\ mass}{mass} = \frac{\Delta M V^2 / 2}{\Delta M} = \frac{V^2}{2}$$

- Potential energy per unit mass

$$e_p = \frac{potential\ energy\ of\ mass}{mass} = \frac{\gamma \Delta \forall z}{\gamma \Delta \forall} = gz$$

Energy Equation

- Reynolds Transport Theorem

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho d\forall + \int_{CS} b\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

- $b = e$; $B_{sys} = E$

$$\frac{dE}{dt} = \frac{d}{dt}\int_{CV} e\rho d\forall + \int_{CS} e\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

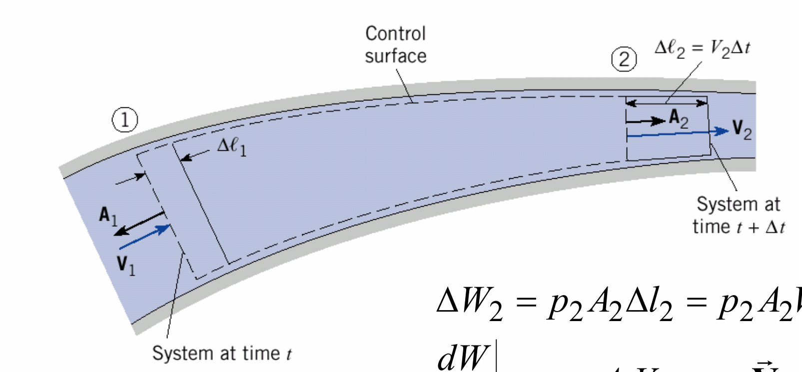
$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt}\int_{CV} (e_k + e_p + u)\rho d\forall + \int_{CS} (e_k + e_p + u)\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

Work

- Rate of Work $$\frac{dW}{dt} = \left.\frac{dW}{dt}\right|_{shaft} + \left.\frac{dW}{dt}\right|_{flow} + \left.\frac{dW}{dt}\right|_{viscous}$$
- Shaft work $\left.\frac{dW}{dt}\right|_{shaft}$ work done by a mechanical device which crosses the CS
- Flow work $\left.\frac{dW}{dt}\right|_{flow}$ work done by pressure forces on the CS
- Viscous work $\left.\frac{dW}{dt}\right|_{viscous}$ work done by viscous stresses at the CS

Flow Work

- Work occurs at the CS when a force associated with the normal stress of the fluid acts over a distance. The normal stress equals the negative of the fluid pressure.



$$\Delta W_2 = p_2 A_2 \Delta l_2 = p_2 A_2 V_2 \Delta t$$

$$\left.\frac{dW}{dt}\right|_2 = p_2 A_2 V_2 = p_2 \vec{\mathbf{V}}_2 \cdot \vec{\mathbf{A}}_2$$

$$\left.\frac{dW}{dt}\right|_1 = p_1 \vec{\mathbf{V}}_1 \cdot \vec{\mathbf{A}}_1$$

$$\left.\frac{dW}{dt}\right|_{flow} = \sum_{CS} p\vec{\mathbf{V}} \cdot \vec{\mathbf{A}} = \int_{CS} \rho \frac{p}{\rho} \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

Energy Equation

$$\boxed{\frac{dQ}{dt} - \left.\frac{dW}{dt}\right|_{shaft} = \frac{d}{dt}\int_{CV} \left(\frac{V^2}{2} + gz + u\right)\rho d\forall + \int_{CS} \left(\frac{p}{\rho} + \frac{V^2}{2} + gz + u\right)\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}}$$

Kinetic Energy Correction Factor

$$\left.\frac{dQ}{dt}-\frac{dW}{dt}\right|_{shaft}=\frac{d}{dt}\int_{CV}\left(\frac{V^2}{2}+gz+u\right)\rho d\forall+\int_{CS}\left(\frac{p}{\rho}+\frac{V^2}{2}+gz+u\right)\rho\vec{\mathbf{V}}\cdot d\vec{\mathbf{A}}$$

$$=\sum_{CS}\left(\frac{p}{\rho}+gz+u\right)\rho\vec{\mathbf{V}}\cdot\vec{\mathbf{A}}+\int_{CS}\frac{V^2}{2}\rho\vec{\mathbf{V}}\cdot d\vec{\mathbf{A}}$$

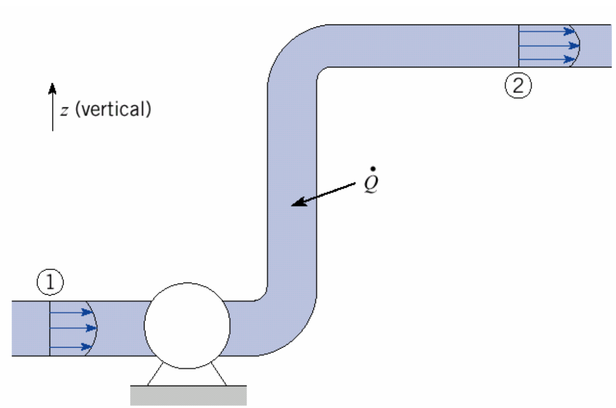
For nonuniform flows, this term requires special attention

We can modify this kinetic energy term by a dimensionless factor, α, so that the integral is proportional to the square of the average velocity

$$\int_{CS}\frac{V^2}{2}\rho\vec{\mathbf{V}}\cdot d\vec{\mathbf{A}}=\alpha\,\dot{m}\frac{\bar{V}^2}{2}$$

where $\bar{V}=\frac{1}{A}\int_A\vec{\mathbf{V}}\cdot d\vec{\mathbf{A}}$ $\qquad \dot{m}=\rho\bar{V}A=\rho\bar{V}_1A_1=\rho\bar{V}_2A_2$

and $\alpha=\frac{1}{A}\int_A\left(\frac{V}{\bar{V}}\right)^3 dA$ $\qquad$ Kinetic energy correction factor



so
$$\boxed{q-w_{shaft}+\alpha_1\frac{V_1^2}{2}+\frac{p_1}{\rho}+gz_1+u_1=\alpha_2\frac{V_2^2}{2}+\frac{p_2}{\rho}+gz_2+u_2}$$

$$q=\frac{1}{\dot{m}}\frac{dQ}{dt};\qquad w_{shaft}=\left.\frac{1}{\dot{m}}\frac{dW}{dt}\right|_{shaft}$$

Energy Equation for Pipe Flow

$$q - w_{shaft} + \alpha_1 \frac{V_1^2}{2} + \frac{p_1}{\rho} + gz_1 + u_1 = \alpha_2 \frac{V_2^2}{2} + \frac{p_2}{\rho} + gz_2 + u_2$$

$$w_{shaft} = w_{turbine} - w_{pump}$$

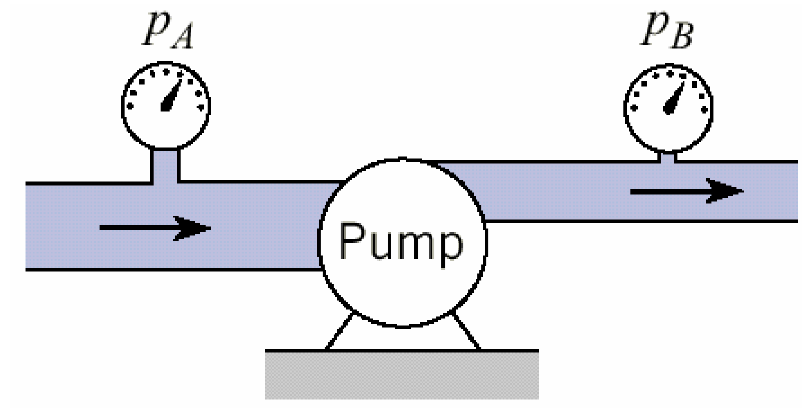
$$\alpha_1 \frac{V_1^2}{2} + \frac{p_1}{\rho} + gz_1 + w_{pump} = \alpha_2 \frac{V_2^2}{2} + \frac{p_2}{\rho} + gz_2 + w_{turbine} + (u_2 - u_1 - q)$$

$$\boxed{\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + h_{pump} - h_{turbine} - h_{loss} = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2}$$

$$h_{pump} = \frac{w_{pump}}{g}; \quad h_{turbine} = \frac{w_{turbine}}{g}; \qquad h_{loss} = \frac{u_2 - u_1 - q}{g}$$

Ex (7.35)

- Given: P_A= 10 psi (12 in dia. pipe), P_B= 40 psi (6 in dia. pipe), Q=3.92 cfs
- Find: Horsepower of pump
- Solution:



$$\alpha_A \frac{V_A^2}{2g} + \frac{p_A}{\gamma} + z_A + h_{pump} - h_{turbine} - h_{loss} = \alpha_B \frac{V_B^2}{2g} + \frac{p_B}{\gamma} + z_B$$

$$V_A = \frac{Q}{A_A} = \frac{3.92}{\pi(1)^2/4} = 4.99\ ft/s;$$

$$V_B = \frac{Q}{A_B} = \frac{3.92}{\pi(0.5)^2/4} = 19.96\ ft/s$$

$$h_{pump} = \frac{V_B^2 - V_A^2}{2g} + \frac{p_B - p_A}{\gamma}$$

$$= \frac{(4.99)^2 - (6.19)^2}{2g} + \frac{(10-40)*144}{62.4}$$

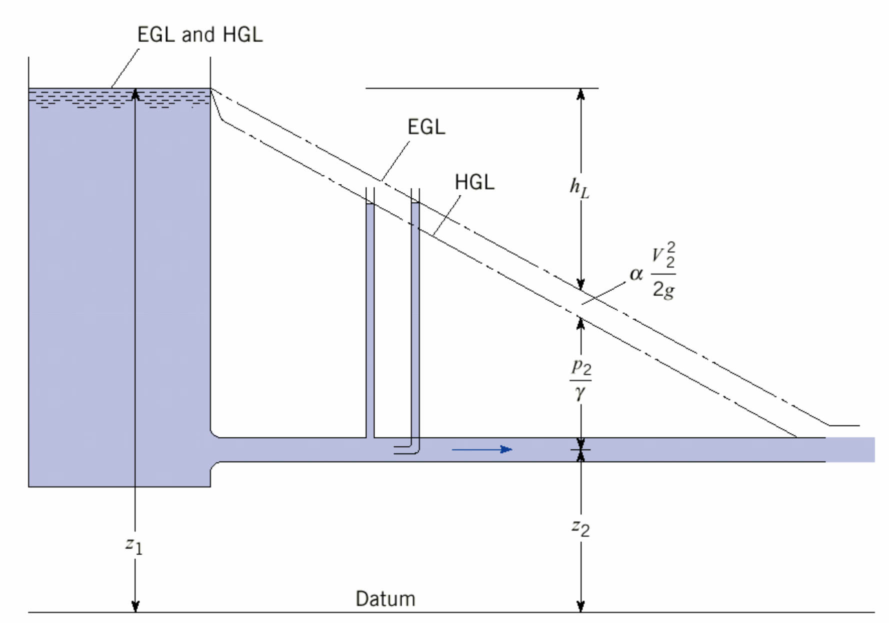
$$= 75.04\ ft$$

$$P = \frac{Q\gamma h}{550} = \frac{3.92*62.4*75.04}{550} = 33.4\,hp$$

EGL & HGL for a Pipe System

- Energy equation

$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$



- All terms are in dimension of length (head energy per unit weight)
- HGL – Hydraulic Grade Line

$$HGL = \frac{p}{\gamma} + z$$

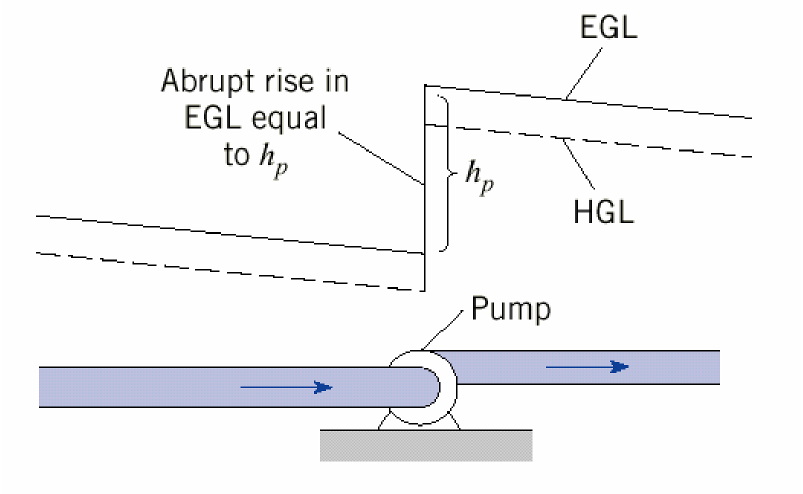
- EGL – Energy Grade Line

$$EGL = \alpha \frac{V^2}{2g} + \frac{p}{\gamma} + z = HGL + \alpha \frac{V^2}{2g}$$

- EGL=HGL when V=0 (reservoir surface, etc.)
- EGL slopes in the direction of flow

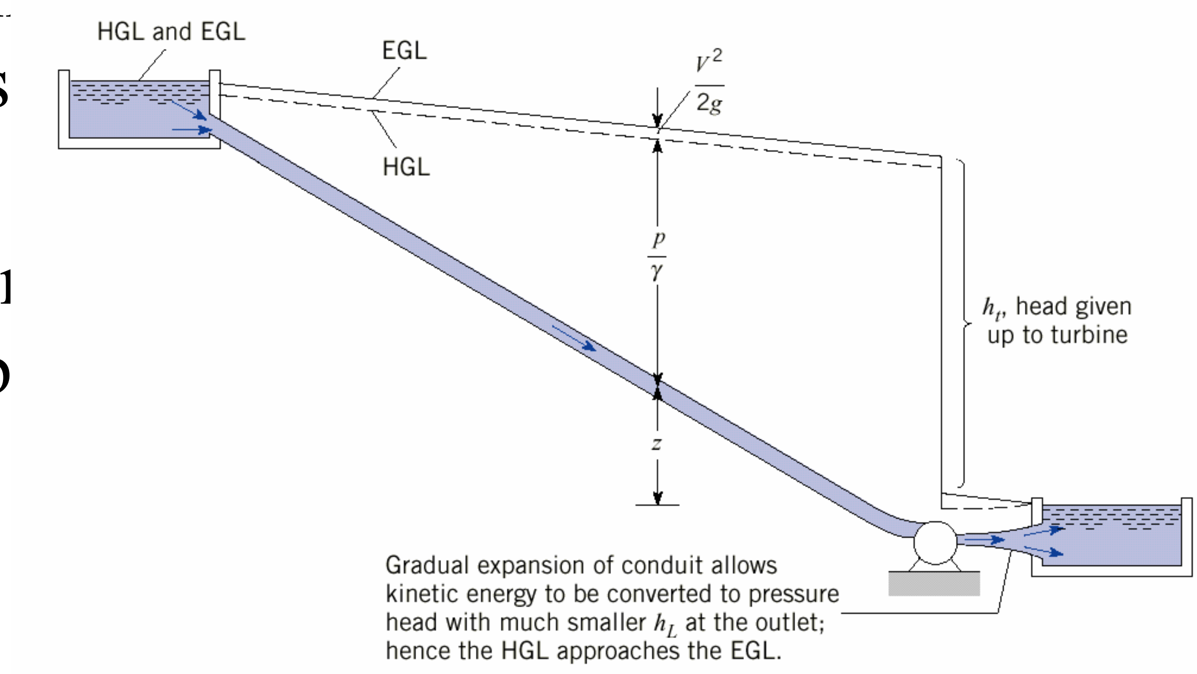
EGL & HGL for a Pipe System

- A pump causes an abrupt rise in EGL (and HGL) since energy is introduced here



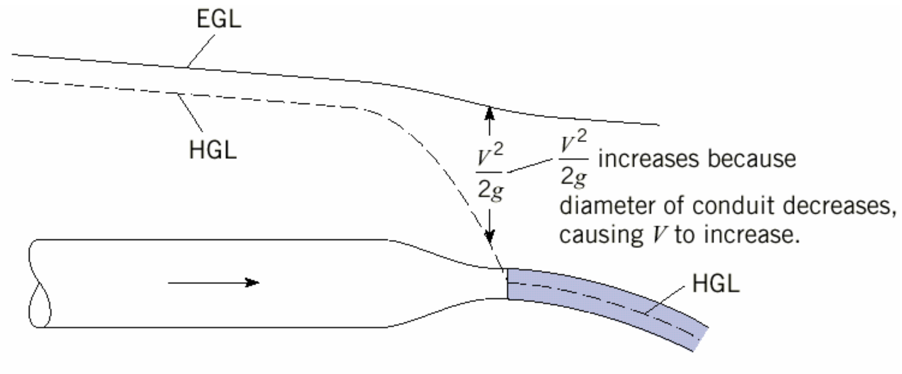
EGL & HGL for a Pipe System

- A turbine causes an abrupt drop in EGL (and HGL) as is taken out
- Gradual expa increases turb efficiency



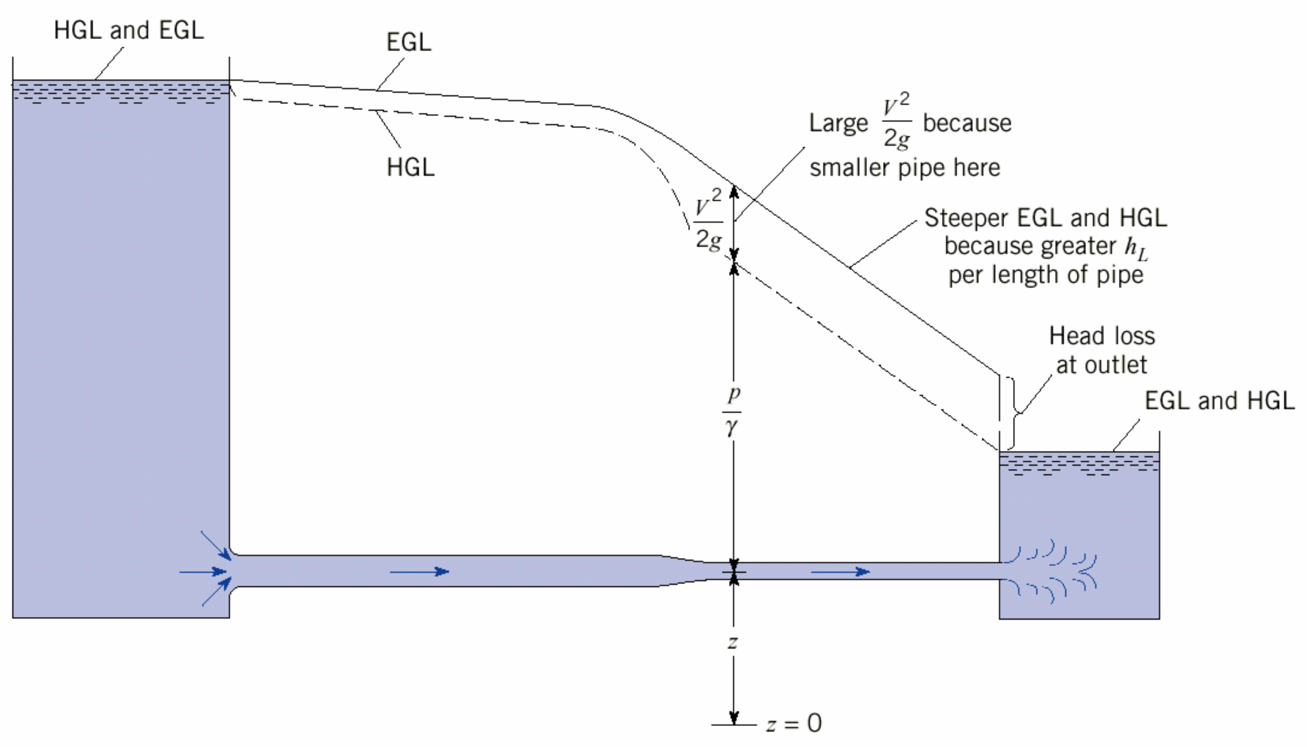
EGL & HGL for a Pipe System

- When the flow passage changes diameter, the velocity changes so that the distance between the EGL and HGL changes
- When the pressure becomes 0, the HGL coincides with the system



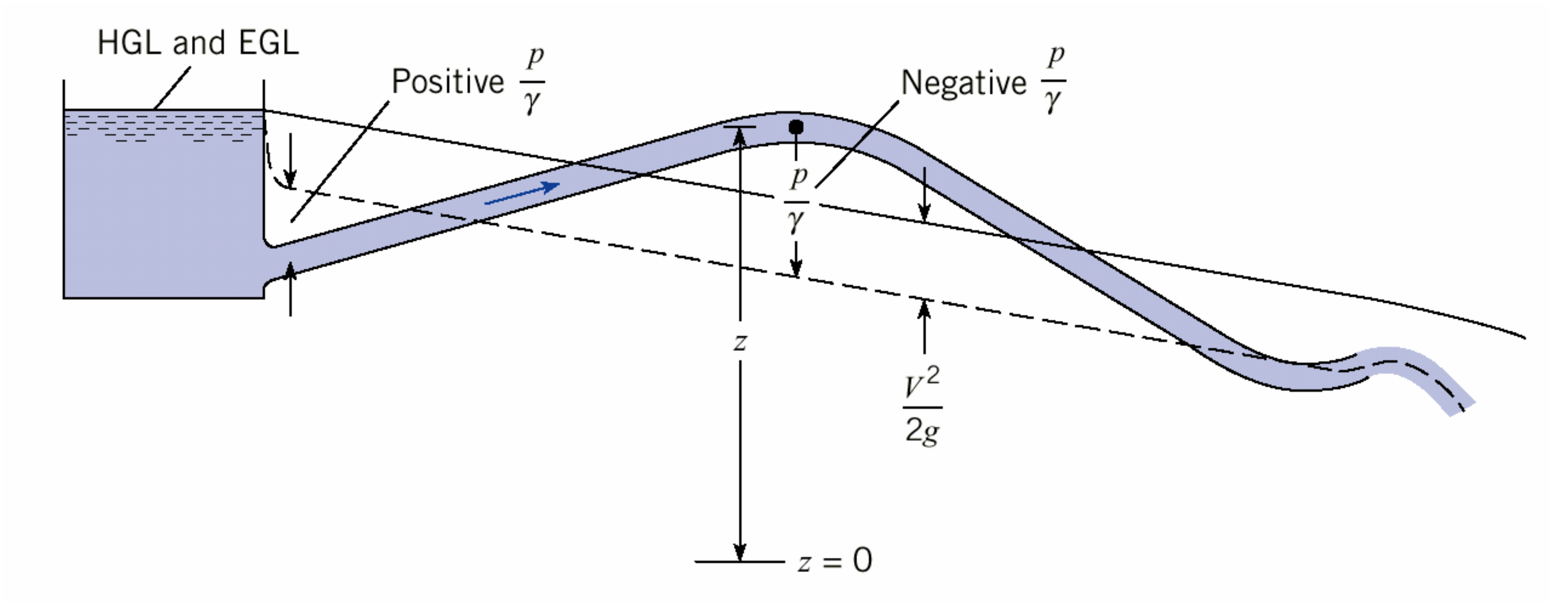
EGL & HGL for a Pipe System

- Abrupt expansion into reservoir causes a complete loss of kinetic energy there

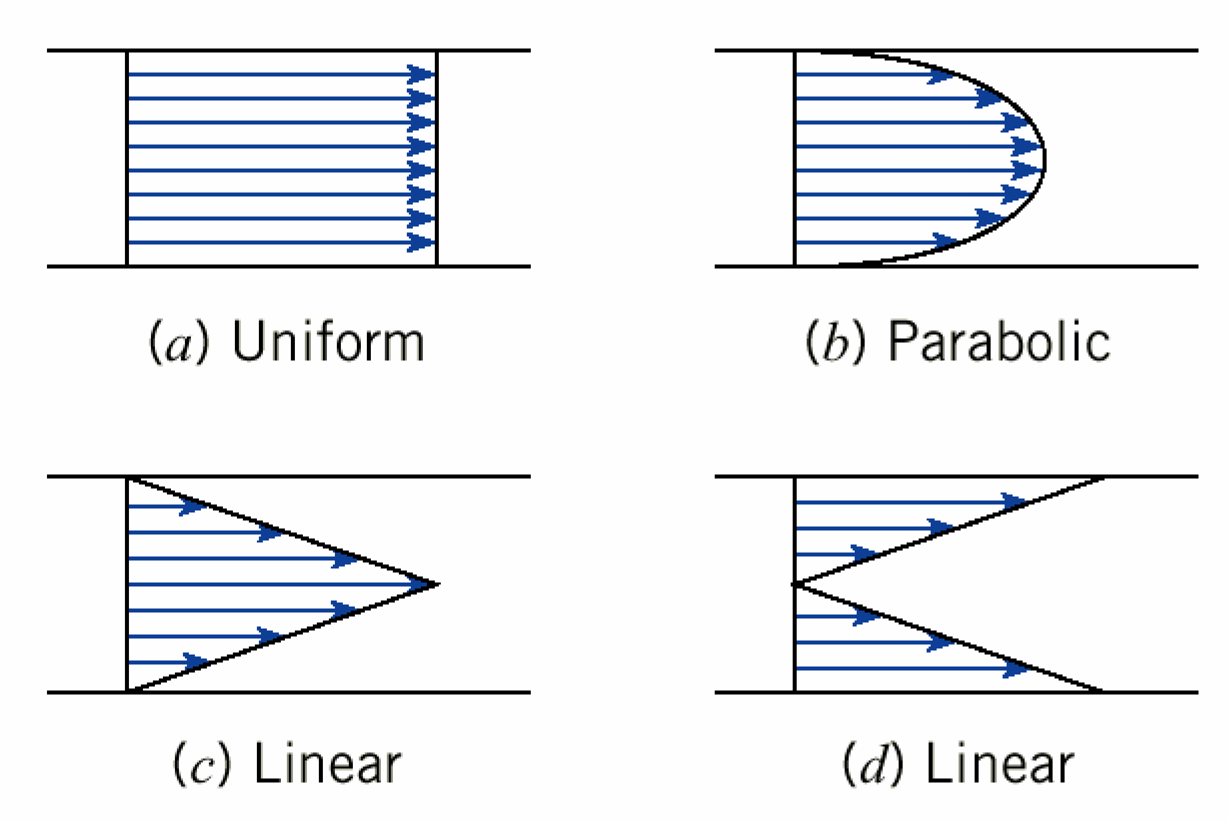


EGL & HGL for a Pipe System

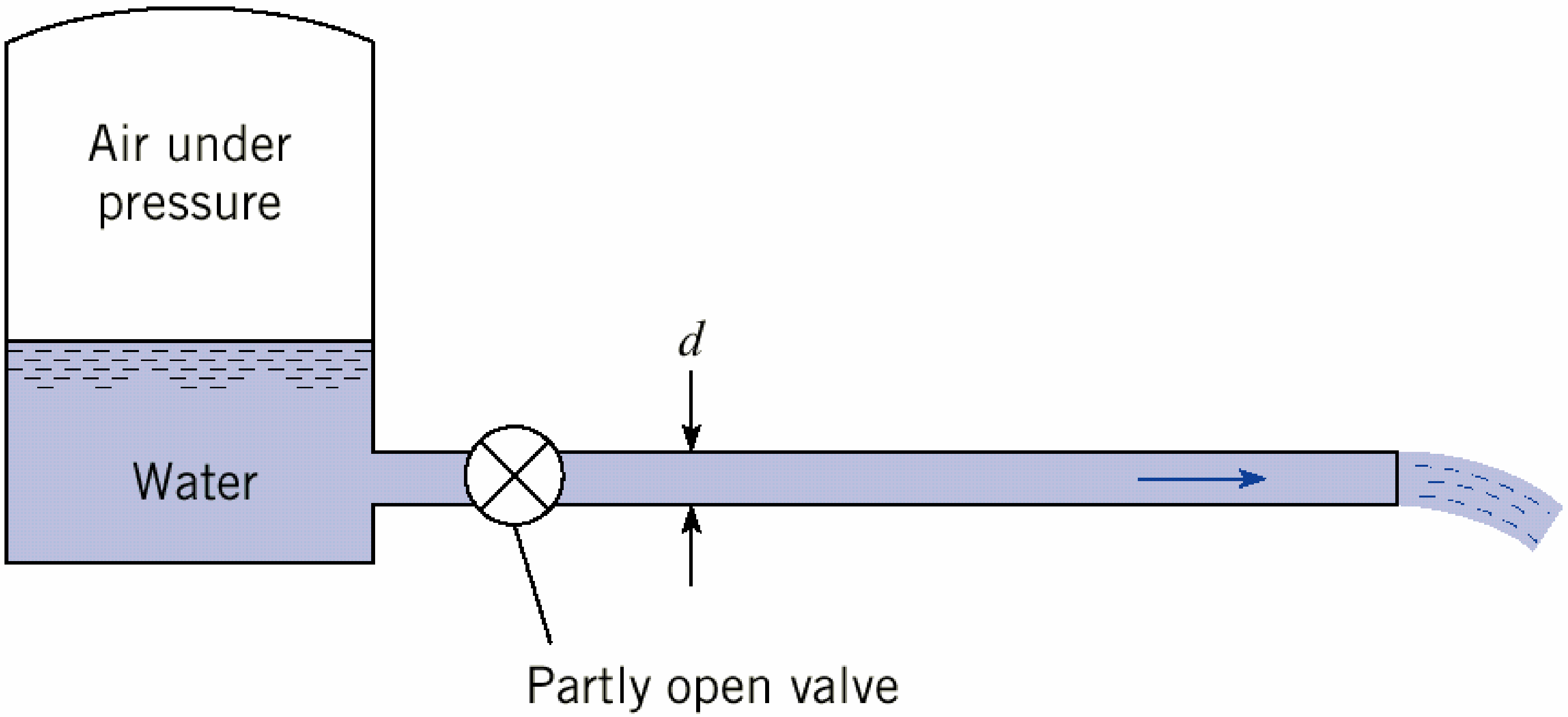
- When HGL falls below the pipe the pressure is below atmospheric pressure



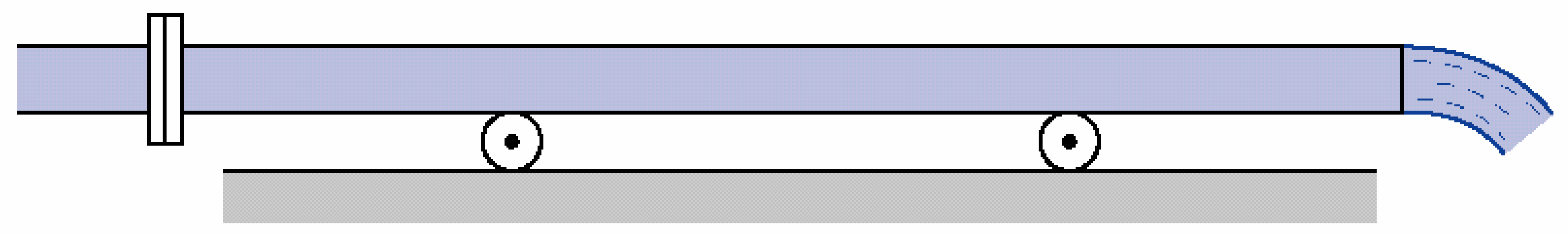
HW (7.8)



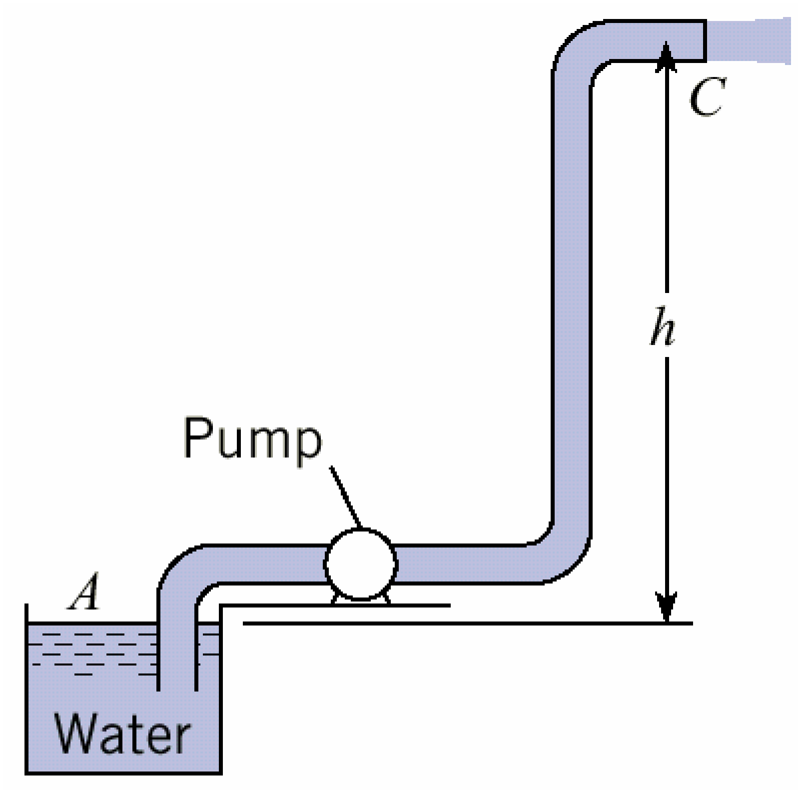
HW (7.16)



HW (7.27)



HW (7.27)



HW (7.41)