CE 319 F

Daene McKinney

Elementary Mechanics of Fluids

Flow in

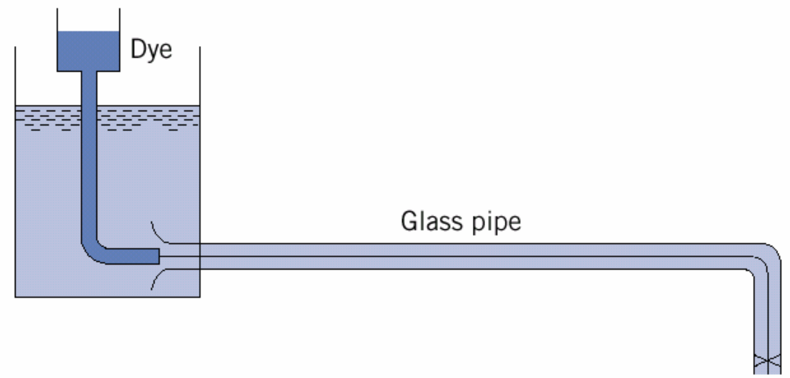
Pipes

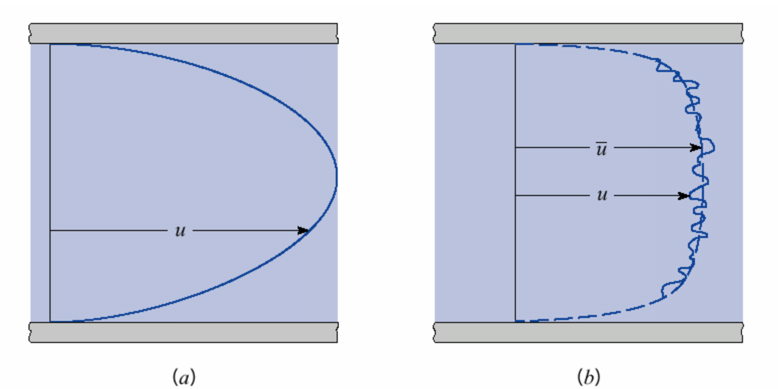
Reynolds Experiment

- Reynolds Number

$$Re = \frac{\rho V D}{\mu}\begin{cases} < 2000 & \text{Laminar flow} \quad h_f \propto V \\ 2000-4000 & \text{Transition flow} \\ > 4000 & \text{Turbulent flow} \quad h_f \propto V^2 \end{cases}$$

- Laminar flow: Fluid moves in smooth streamlines
- Turbulent flow: Violent mixing, fluid velocity at a point varies randomly with time
- Transition to turbulence in a 2 in. pipe is at V=2 ft/s, so most pipe flows are turbulent

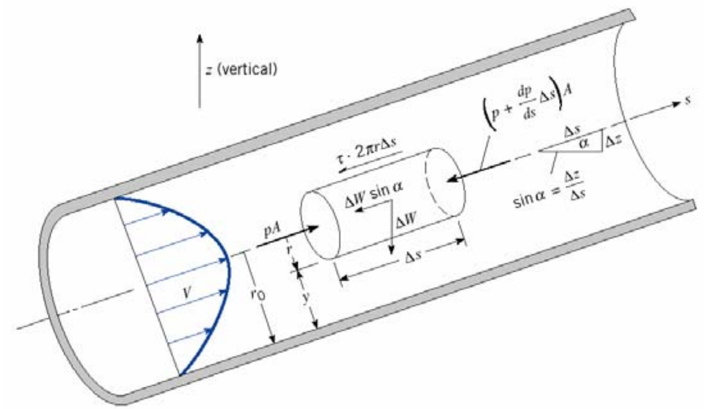




Laminar　　　　Turbulent

Shear Stress in Pipes

- Steady, uniform flow in a pipe: momentum flux is zero and pressure distribution across pipe is hydrostatic, equilibrium exists between pressure, gravity and shear forces



$$\sum F_s = 0 = pA - (p + \frac{dp}{ds}\Delta s)A - \Delta W \sin\alpha - \tau_0(\pi D)\Delta s$$

$$0 = -\frac{dp}{ds}\Delta s A - \gamma A \Delta s \frac{dz}{ds} - \tau_0(\pi D)\Delta s$$

$$\tau_0 = \frac{D}{4}[-\frac{d}{ds}\gamma(\frac{p}{\gamma} + z)]$$

$$\tau_0 = -\frac{D\gamma}{4}\frac{dh}{ds}$$

$$h_1 - h_2 = h_f = \frac{4L\tau_0}{\gamma D}$$

- Since h is constant across the cross-section of the pipe (hydrostatic), and $-dh/ds>0$, then the shear stress will be zero at the center ($r = 0$) and increase linearly to a maximum at the wall.
- Head loss is due to the shear stress.

- Applicable to either laminar or turbulent flow
- Now we need a relationship for the shear stress in terms of the Re and pipe roughness

Darcy-Weisbach Equation

τ_0	ρ	V	μ	D	e
$ML^{-1}T^{-2}$	ML^{-3}	LT^{-1}	$ML^{-1}T^{-1}$	L	L

$$\tau_0 = F(\rho, V, \mu, D, e)$$

$$\pi_4 = F(\pi_1, \pi_2)$$

Repeating variables : ρ, V, D

$$\pi_1 = \text{Re}; \quad \pi_2 = \frac{e}{D}; \quad \pi_3 = \frac{\tau_0}{\rho V^2}$$

$$\frac{\tau_0}{\rho V^2} = F(\text{Re}, \frac{e}{D})$$

$$\tau_0 = \rho V^2 F(\text{Re}, \frac{e}{D})$$

$$h_f = \frac{4L}{\gamma D}\tau_0$$

$$= \frac{4L}{\gamma D}\rho V^2 F(\text{Re}, \frac{e}{D})$$

$$= \frac{L}{D}\frac{V^2}{2g}\left[8F(\text{Re}, \frac{e}{D})\right]$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Darcy-Weisbach Eq.

$$f = 8F(\text{Re}, \frac{e}{D})$$

Friction factor

Laminar Flow in Pipes

- Laminar flow -- Newton's law of viscosity is valid:

$$\tau = \mu \frac{dV}{dy} = -\frac{r\gamma}{2}\frac{dh}{ds}$$

$$\frac{dV}{dy} = -\frac{dV}{dr}$$

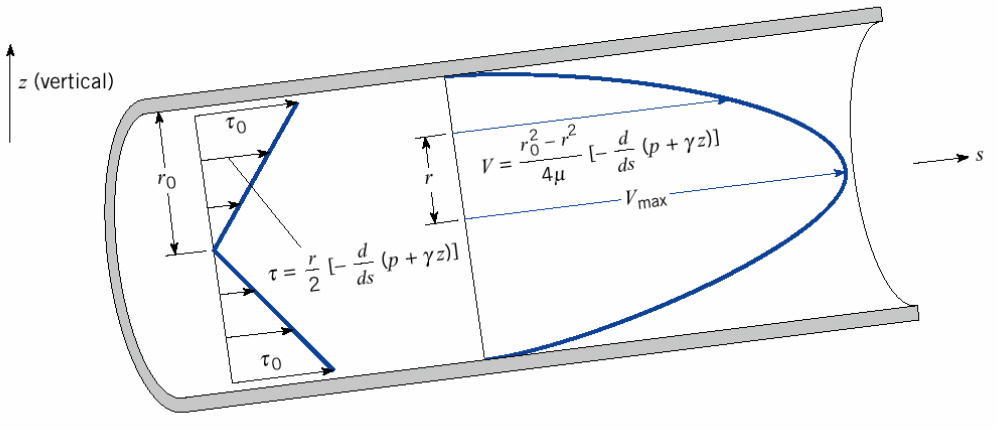
$$\frac{dV}{dr} = \frac{r\gamma}{2\mu}\frac{dh}{ds}$$

$$dV = \frac{r\gamma}{2\mu}\frac{dh}{ds}dr$$

$$V = \frac{r^2\gamma}{4\mu}\frac{dh}{ds} + C \qquad C = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}$$

$$V = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}\left[1-\left(\frac{r}{r_0}\right)^2\right]$$

$$V = V_{\max}\left[1-\left(\frac{r}{r_0}\right)^2\right]$$



- Velocity distribution in a pipe (laminar flow) is parabolic with maximum at center.

Discharge in Laminar Flow

$$V = -\frac{\gamma}{4\mu}\frac{dh}{ds}(r_0^2 - r^2)$$

$$Q = \int V dA = \int_0^{r_0} -\frac{\gamma}{4\mu}\frac{dh}{ds}(r_0^2 - r^2)(2\pi r dr)$$

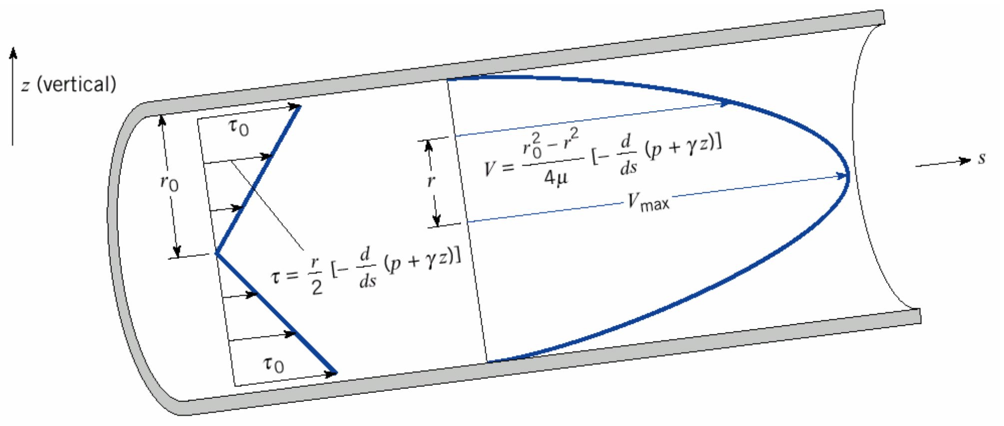
$$= \frac{\pi\gamma}{4\mu}\frac{dh}{ds}\left.\frac{(r^2 - r_0^2)^2}{2}\right|_0^{r_0}$$

$$Q = -\frac{\pi\gamma r_0^4}{8\mu}\frac{dh}{ds}$$

$$= -\frac{\pi\gamma D^4}{128\mu}\frac{dh}{ds}$$

$$\overline{V} = \frac{Q}{A}$$

$$\overline{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$



Head Loss in Laminar Flow

$$\overline{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$

$$\frac{dh}{ds} = -\overline{V}\,\frac{32\mu}{\gamma D^2}$$

$$dh = -\overline{V}\,\frac{32\mu}{\gamma D^2}\,ds$$

$$h_2 - h_1 = -\overline{V}\,\frac{32\mu}{\gamma D^2}(s_2 - s_1)$$

$$h_1 = h_2 + h_f$$

$$h_f = \frac{32\mu L\overline{V}}{\gamma D^2}$$

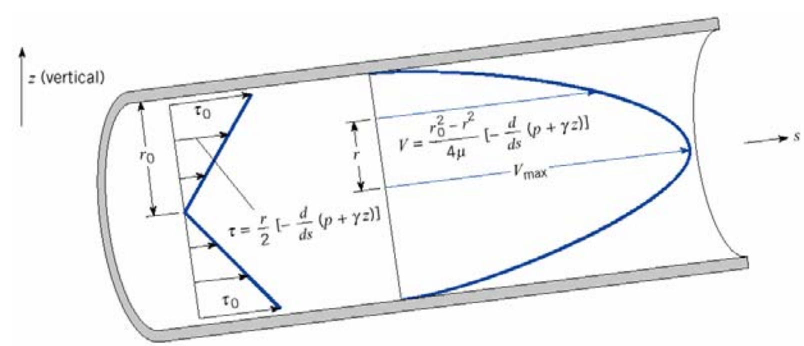
$$h_f = \frac{32\mu L\overline{V}}{\gamma D^2}$$

$$= \frac{32\mu L\overline{V}}{\gamma D^2}\,\frac{\rho\overline{V}^2/2}{\rho\overline{V}^2/2}$$

$$= 64\left(\frac{\mu}{\rho\overline{V}D}\right)\left(\frac{L}{D}\right)\rho\overline{V}^2/2$$

$$= \frac{64}{\mathrm{Re}}\left(\frac{L}{D}\right)\rho\overline{V}^2/2$$

$$h_f = f\,\frac{L}{D}\,\frac{\rho\overline{V}^2}{2} \qquad f = \frac{64}{\mathrm{Re}}$$



Nikuradse's Experiments

- In general, friction factor

  $$f = F(\mathrm{Re}, \frac{e}{D})$$

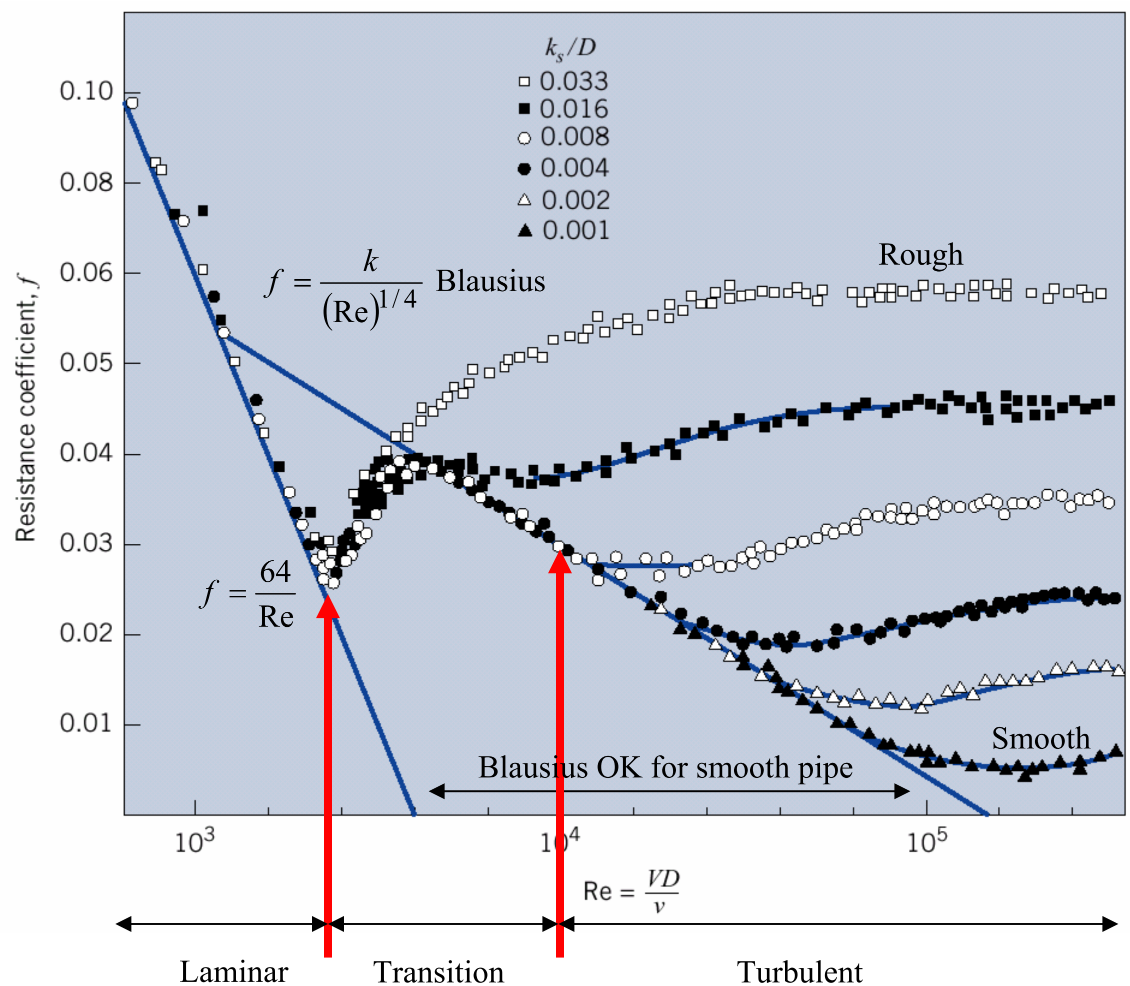
  – Function of *Re* and *roughness*
- Laminar region

  $$f = \frac{64}{\mathrm{Re}}$$

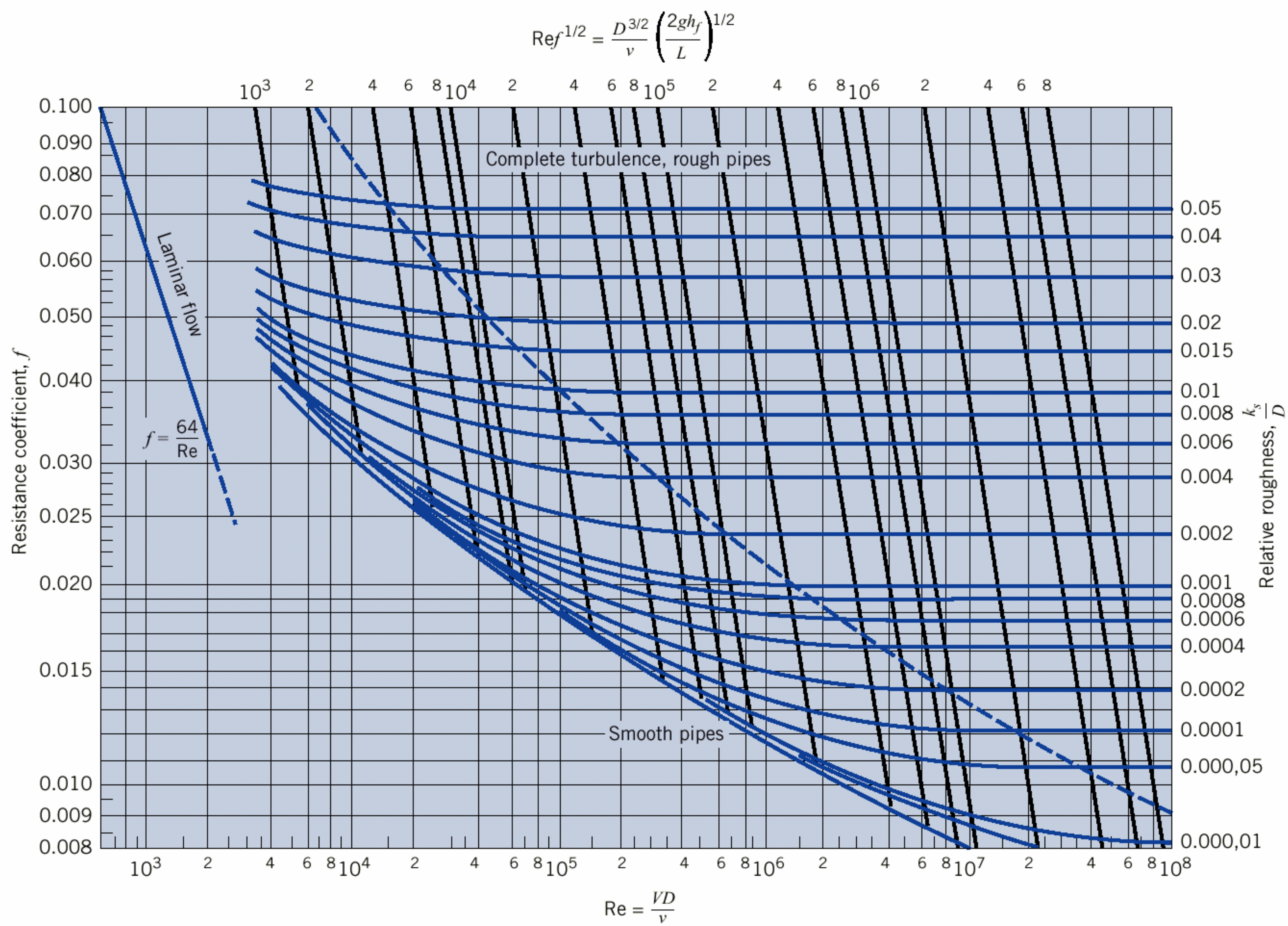
  – Independent of roughness
- Turbulent region
  – Smooth pipe curve
    - All curves coincide @ ~Re=2300

  – Rough pipe zone
    - All rough pipe curves flatten out and become independent of Re

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{e}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$



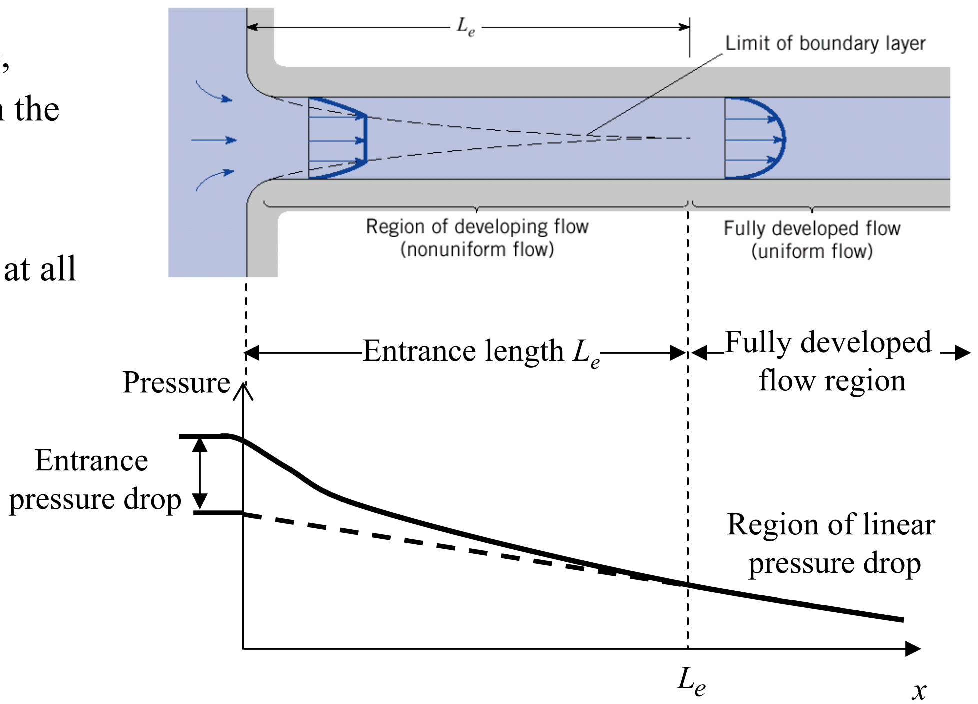
Moody Diagram



Pipe Entrance

- Developing flow
  - Includes boundary layer and core,
  - viscous effects grow inward from the wall
- Fully developed flow
  - Shape of velocity profile is same at all points along pipe

$$\frac{L_e}{D} \approx \begin{cases} 0.06\,\mathrm{Re} & \text{Laminar flow} \\ 4.4\mathrm{Re}^{1/6} & \text{Turbulent flow} \end{cases}$$

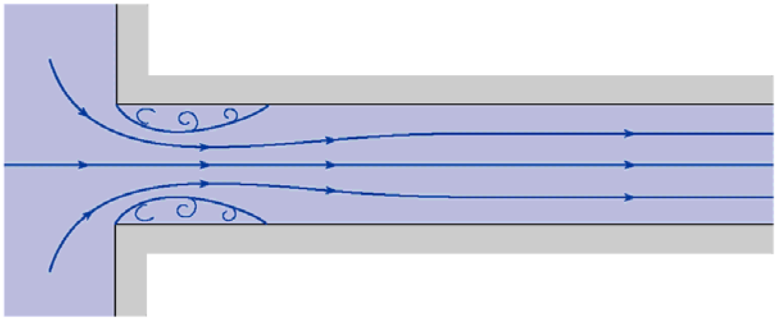


Entrance Loss in a Pipe

- In addition to frictional losses, there are minor losses due to
  - Entrances or exits
  - Expansions or contractions
  - Bends, elbows, tees, and other fittings
  - Valves
- Losses generally determined by experiment and then corellated with pipe flow characteristics
- Loss coefficients are generally given as the ratio of head loss to velocity head

$$K = \frac{h_L}{\frac{V^2}{2g}} \qquad \text{or} \qquad h_L = K\frac{V^2}{2g}$$

- K – loss coefficent
  - $K \sim 0.1$ for well-rounded inlet (high Re)
  - $K \sim 1.0$ abrupt pipe outlet
  - $K \sim 0.5$ abrupt pipe inlet



Abrupt inlet, $K \sim 0.5$

Elbow Loss in a Pipe

- A piping system may have many minor losses which are all correlated to $V^2/2g$
- Sum them up to a total system loss for pipes of the same diameter

$$h_L = h_f + \sum_m h_m = \frac{V^2}{2g}\left[f\frac{L}{D} + \sum_m K_m \right]$$

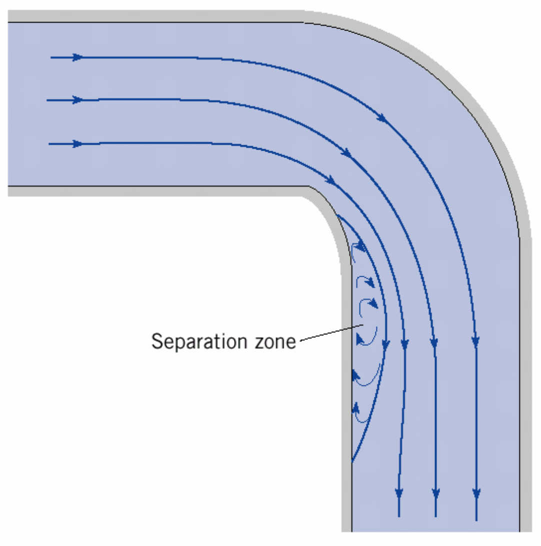
- Where,

h_L = Total head loss

h_f = Frictional head loss

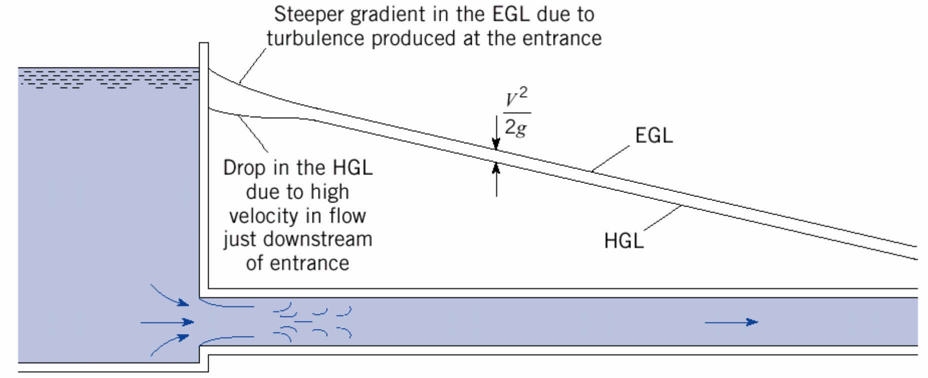
h_m = Minor head loss for fitting m

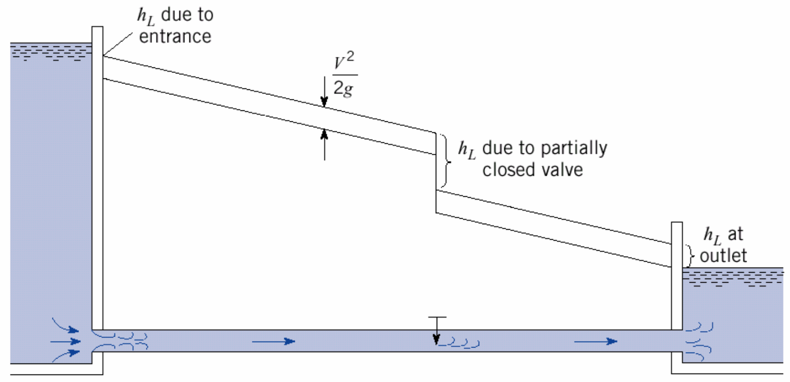
K_m = Minor head loss coefficient for fitting m



EGL & HGL for Losses in a Pipe

- Entrances, bends, and other flow transitions cause the EGL to drop an amount equal to the head loss produced by the transition.
- EGL is steeper at entrance than it is downstream of there where the slope is equal the frictional head loss in the pipe.
- The HGL also drops sharply downstream of an entrance



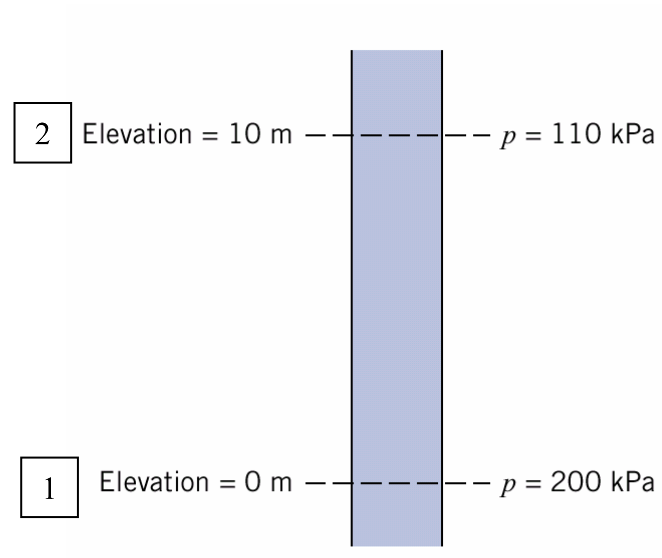


Ex(10.2)

Given: Liquid in pipe has $\gamma = 8$ kN/m^3. Acceleration = 0.
$D = 1$ cm, $\mu = 3\text{x}10^{-3}$ N-m/s^2.

Find: Is fluid stationary, moving up, or moving down?
What is the mean velocity?

Solution: Energy eq. from z = 0 to z = 10 m



$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{200{,}000}{8000} - h_L = \frac{110{,}000}{8000} + 10$$

$$h_L = \frac{90}{8} - 10$$

$$h_L = 1.25\,m \text{ (moving upward)}$$

$$h_L = \frac{32\mu L V}{\gamma D^2}$$

$$V = h_L \frac{\gamma D^2}{32\mu L}$$

$$V = 1.25 \frac{8000*(0.01)^2}{32*3x10^{-3}*10}$$

$$V = 1.04\,m/s$$

Ex (10.4)

- Given: Oil (S = 0.97, m = 10^{-2} lbf-s/ft^2) in 2 in pipe, Q = 0.25 cfs.
- Find: Pressure drop per 100 ft of horizontal pipe.
- Solution:

$$V = \frac{Q}{A} = \frac{0.25}{\pi(2/12)^2/4} = 11.46\ ft/s$$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{0.97*1/94*11.46*(2/12)}{10^{-2}} = 360\ \text{(laminar)}$$

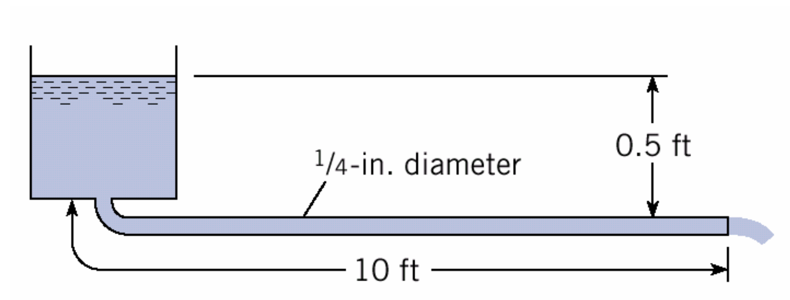
$$\Delta p = \frac{32\mu LV}{D^2} = \frac{32*10\text{-}2*100*11.46}{(2/12)^2} = 91.7\ \text{psi/100 ft}$$

Ex. (10.8)

Given: Kerosene (S=0.94, μ=0.048 N-s/m^2). Horizontal 5-cm pipe. Q=2x10^{-3} m^3/s.

Find: Pressure drop per 10 m of pipe.

Solution:



$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$h_L = \frac{32\mu LV}{\gamma D^2}$$

$$0 + 0 + 0.5 - \frac{32\mu LV}{\gamma D^2} = \alpha_2 \frac{V_2^2}{2g} + 0 + 0$$

$$\frac{\alpha_2}{2g} V_2^2 + \frac{32\mu L}{\gamma D^2} V - 0.5 = 0$$

$$\frac{2}{2g} V_2^2 + \frac{32 * 4 * 10^{-5} * 10}{0.8 * 62.4 * (1/32)^2} V - 0.5 = 0$$

$$V_2^2 + 8.45V - 16.1 = 0$$

$$V = 1.60\ ft/s$$

$$\mathrm{Re} = \frac{0.8 * 1.94 * 1.6 * (0.25/12)}{4 * 10^{-5}} = 1293\,(\text{laminar})$$

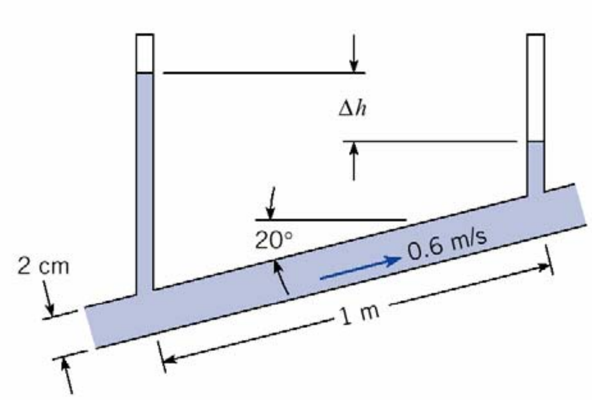
$$\mathrm{Q} = \mathrm{V} * \mathrm{A} = 1.6 * \pi * (0.25/12)^2 / 4 = 1.23 * 10^{-3}\ cfs$$

Ex. (10.34)

Given: Glycerin@ 20°C flows commercial steel pipe.

Find: Δh

Solution: $\gamma = 12{,}300\, N/m, \mu = 0.62\, Ns/m^2$



$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{p_1}{\gamma} + z_1 - h_L = \frac{p_2}{\gamma} + z_2$$

$$\Delta h = \frac{p_1}{\gamma} + z_1 - (\frac{p_2}{\gamma} + z_2) = h_L$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu} = \frac{0.6 * 0.02}{5.1 * 10^{-4}} = 23.5\,(\text{laminar})$$

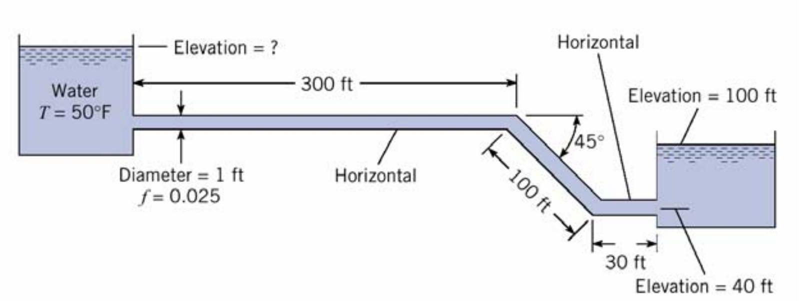
$$\Delta h = h_L = \frac{32\mu LV}{\gamma D^2} = \frac{32(0.62)(1)(0.6)}{12{,}300 * (0.02)^2} = 2.42\, m$$

Ex. (10.43)

Given: Figure

Find: Estimate the elevation required in the upper reservoir to produce a water discharge of 10 cfs in the system. What is the minimum pressure in the pipeline and what is the pressure there?

Solution:



$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \sum h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$0 + 0 + z_1 - \sum h_L = 0 + 0 + z_2$$

$$\sum h_L = \left(K_e + 2K_b + K_E + f \frac{L}{D} \right) \frac{V^2}{2g}$$

$$K_e = 0.5;\; K_b = 0.4\,(\text{assumed});\; K_E = 1.0;\; f \frac{L}{D} = 0.025 * \frac{430}{1} = 10.75$$

$$V = \frac{Q}{A} = \frac{10}{\pi / 4 * 1^2} = 12.73\ ft/s$$

$$z_1 = 100 + (0.5 + 2*0.4 + 1.0 + 10.75) \frac{12.73^2}{2*32.2} = 133\ ft$$

$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \sum h_L = \alpha_b \frac{V_b^2}{2g} + \frac{p_b}{\gamma} + z_b$$

$$0 + 0 + z_1 - \sum h_L = 1 * \frac{V_b^2}{2g} + \frac{p_b}{\gamma} + z_b$$

$$\frac{p_b}{\gamma} = z_1 - z_b - \frac{V_b^2}{2g} - \left(K_e + K_b + f \frac{L}{D} \right) \frac{V^2}{2g}$$

$$= 133 - 110.7 - \left(1.0 + 0.5 + 0.4 + 0.025 \frac{300}{1} \right) \frac{12.73^2}{2*32.2}$$

$$= -1.35\ ft$$

$$p_b = 62.4 * (-1.53) = -0.59\ psig$$

$$\text{Re} = \frac{VD}{\nu} = \frac{12.73*1}{1.14*10^{-5}} = 9*10^5$$

Ex. (10.68)

Given: Commercial steel pipe to carry 300 cfs of water at 60oF with a head loss of 1 ft per 1000 ft of pipe. Assume pipe sizes are available in even sizes when the diameters are expressed in inches (i.e., 10 in, 12 in, etc.).

Find: Diameter.

Solution: $\nu = 1.22x10^{-5}\ ft^2/s;\ k_s = 1.5x10^{-4}\ ft$

Assume f = 0.015

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$1 = 0.015 * \frac{1000}{D}\frac{(Q/(\pi/4)D^2)^2}{2g}$$

$$1 = \frac{33{,}984}{D^5}$$

$$D = 8.06\ ft$$

Relative roughness: $\frac{k_s}{D} = \frac{1.5x10^{-4}}{8.06} = 0.00002$

Get better estimate of f

$$\text{Re} = \frac{VD}{\nu}$$

$$= \frac{\frac{Q}{(\pi/4)D^2}D}{\nu} = \frac{Q}{(\pi/4)D\nu}$$

$$\text{Re} = \frac{300}{(\pi/4)(8.06)1.22x10^{-5}} = 3.9x10^6$$

f=0.010

$$1 = \frac{22{,}656}{D^5}$$

$$D = 7.43\ ft = 89\ in.$$

Use a 90 in pipe

Ex. (10.81)

Given: The pressure at a water main is 300 kPa gage. What size pipe is needed to carry water from the main at a rate of 0.025 m3/s to a factory that is 140 m from the main? Assume galvanized-steel pipe is to be used and that the pressure required at the factory is 60 kPa gage at a point 10 m above the main connection.

Find: Size of pipe.

Solution:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{(Q/(\pi/4)D^2)^2}{2g}$$

$$D = \left(8\frac{fL}{h_f}\frac{Q^2}{\pi^2 g}\right)^{1/5}$$

$$\alpha_1\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \sum h_L = \alpha_2\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{300{,}000}{9810} - h_f = \frac{60{,}000}{9810} + 10$$

$$h_f = 14.45\,m$$

Assume $f = 0.020$

$$D = \left(8\frac{fL}{h_f}\frac{Q^2}{\pi^2 g}\right)^{1/5}$$

$$= \left(8\frac{0.02}{14.45}\frac{140}{}\frac{(0.025)^2}{\pi^2 9.81}\right)^{1/5} = 0.100\,m$$

Relative roughness: $\frac{k_s}{D} = \frac{0.15}{100} = 0.0015$

Friction factor: $f = 0.022$

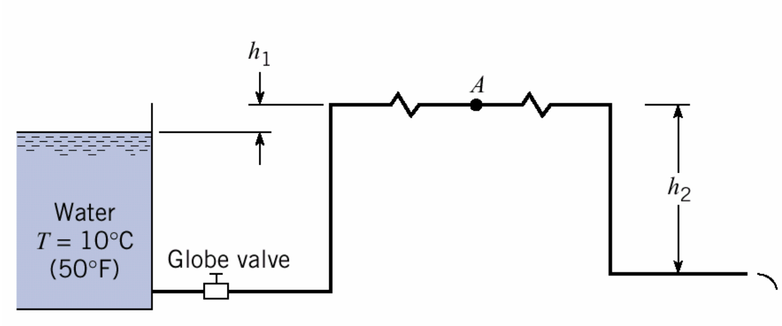
$$D = 0.100\left(\frac{0.022}{0.020}\right)^{1/5} = 0.102\,m$$

Use 12 cm pipe

Ex. (10.83)

Given: The 10-cm galvanized-steel pipe is 1000 m long and discharges water into the atmosphere. The pipeline has an open globe valve and 4 threaded elbows; h_1=3 m and h_2 = 15 m.

Find: What is the discharge, and what is the pressure at A, the midpoint of the line?



Solution:

$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \sum h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$0 + 0 + 12 = (1 + K_e + K_v + 4K_b + f\frac{L}{D})\frac{V^2}{2g} + 0 + 0$$

D = 10-cm and assume f = 0.025

$$24g = (1 + 0.5 + 10 + 4 * 0.9 + 0.025\frac{1000}{0.1})V^2$$

$$V^2 = \frac{24g}{265.1}$$

$$V = 0.942\, m/s$$

$$Q = VA = 0.942(\pi/4)(0.10)^2 = 0.0074\, m^3/s$$

$$\text{Re} = \frac{VD}{\nu} = \frac{0.942 * 0.1}{1.31x10^{-6}} = 7x10^4$$

So f = 0.025

$$\alpha_A \frac{V_A^2}{2g} + \frac{p_A}{\gamma} + z_A - \sum h_L = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{p_A}{\gamma} + 15 = (2K_b + f\frac{L}{D})\frac{V^2}{2g}$$

$$\frac{p_A}{\gamma} = (2 * 0.9 + 0.025\frac{500}{0.1})\frac{(0.942)^2}{2g} - 15 = -9.6\, m$$

$$p_A = 9810 * (-9.26) = -90.8\, kPa$$

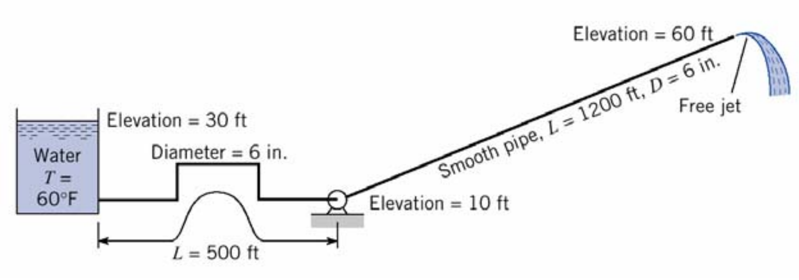
Near cavitation pressure, not good!

Ex. (10.95)

Given: If the deluge through the system shown is 2 cfs, what horsepower is the pump supplying to the water? The 4 bends have a radius of 12 in and the 6-in pipe is smooth.

Find: Horsepower

Solution:



$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + h_p = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + \sum h_L$$

$$0 + 0 + 30 + h_p = 0 + 60 + \frac{V_2^2}{2g}(1 + 0.5 + 4K_b + f\frac{L}{D})$$

$$V = \frac{Q}{A} = \frac{2}{(\pi/4)(1/2)^2} = 10.18\ ft/s$$

$$\frac{V_2^2}{2g} = 1.611\ ft$$

$$\text{Re} = \frac{VD}{\nu} = \frac{10.18*(1/2)}{1.22x10^{-5}} = 4.17x10^5$$

So $f = 0.0135$

$$h_p = 60 - 30 + 1.611(1 + 0.5 + 4*0.19 + 0.0135\frac{1700}{(1/2)})$$

$$= 107.6\ ft$$

$$p = \frac{Q\gamma h_p}{550} = 24.4\ hp$$