CE 319 F

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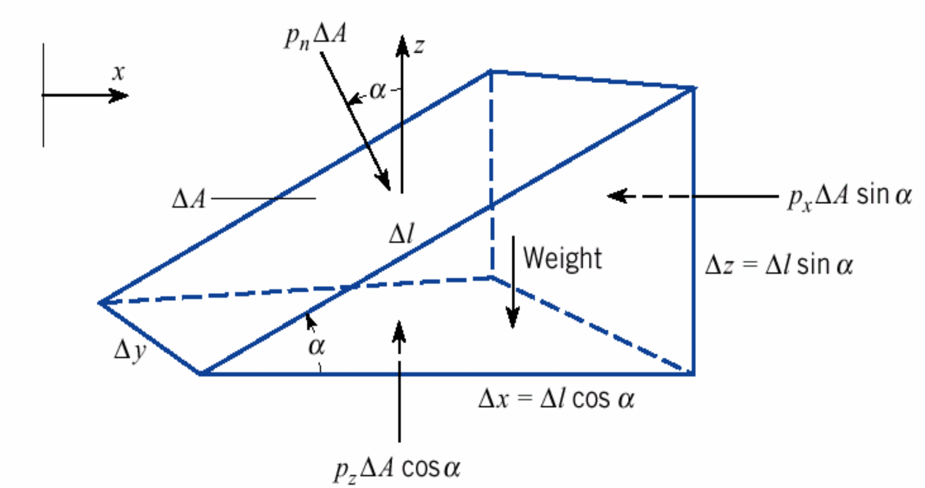
Elementary Mechanics of Fluids

Pressure

Pressure

- Fluid at rest – can not support shear stress
- Normal stress on any plane is *pressure* (+ for compression)



$$\sum F_x = 0 = p_x \Delta y \Delta l \sin\alpha - p_n \Delta y \Delta l \sin\alpha \qquad \Rightarrow \quad p_n = p_x$$

$$\sum F_z = 0 = p_z \Delta y \Delta l \cos\alpha - p_n \Delta y \Delta l \cos\alpha - \frac{1}{2}\gamma \Delta l \cos\alpha \Delta l \sin\alpha \Delta y$$

$$\Rightarrow \quad p_n = p_z$$

$$\therefore \quad p_n = p_x = p_y = p_z = p$$

Example (3.4)

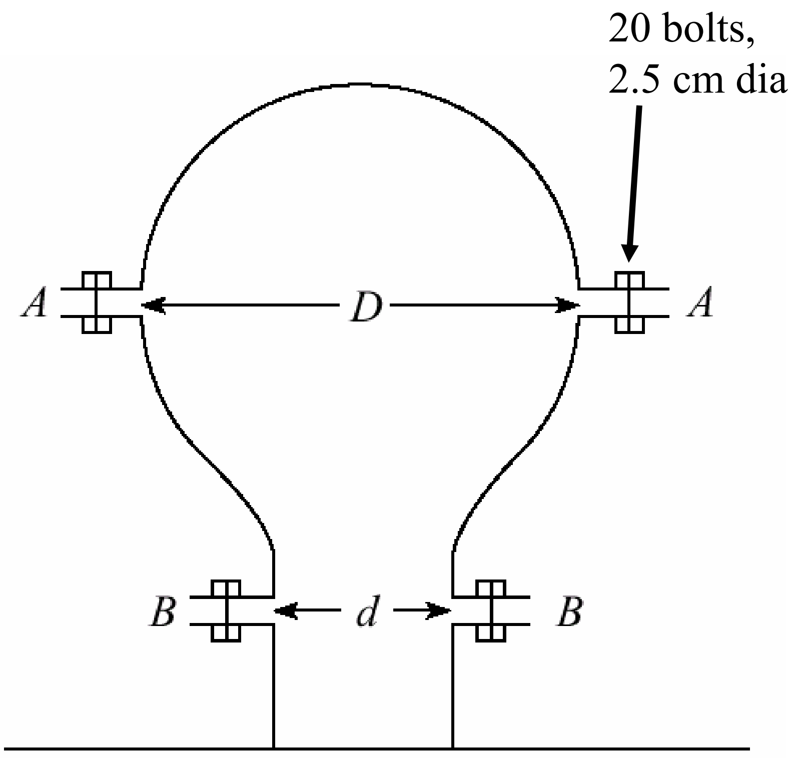
- Prob. 3.4: How many bolts are needed at B-B?

$$F_{bolt,A-A} = \frac{p\frac{\pi}{4}D^2}{20}$$

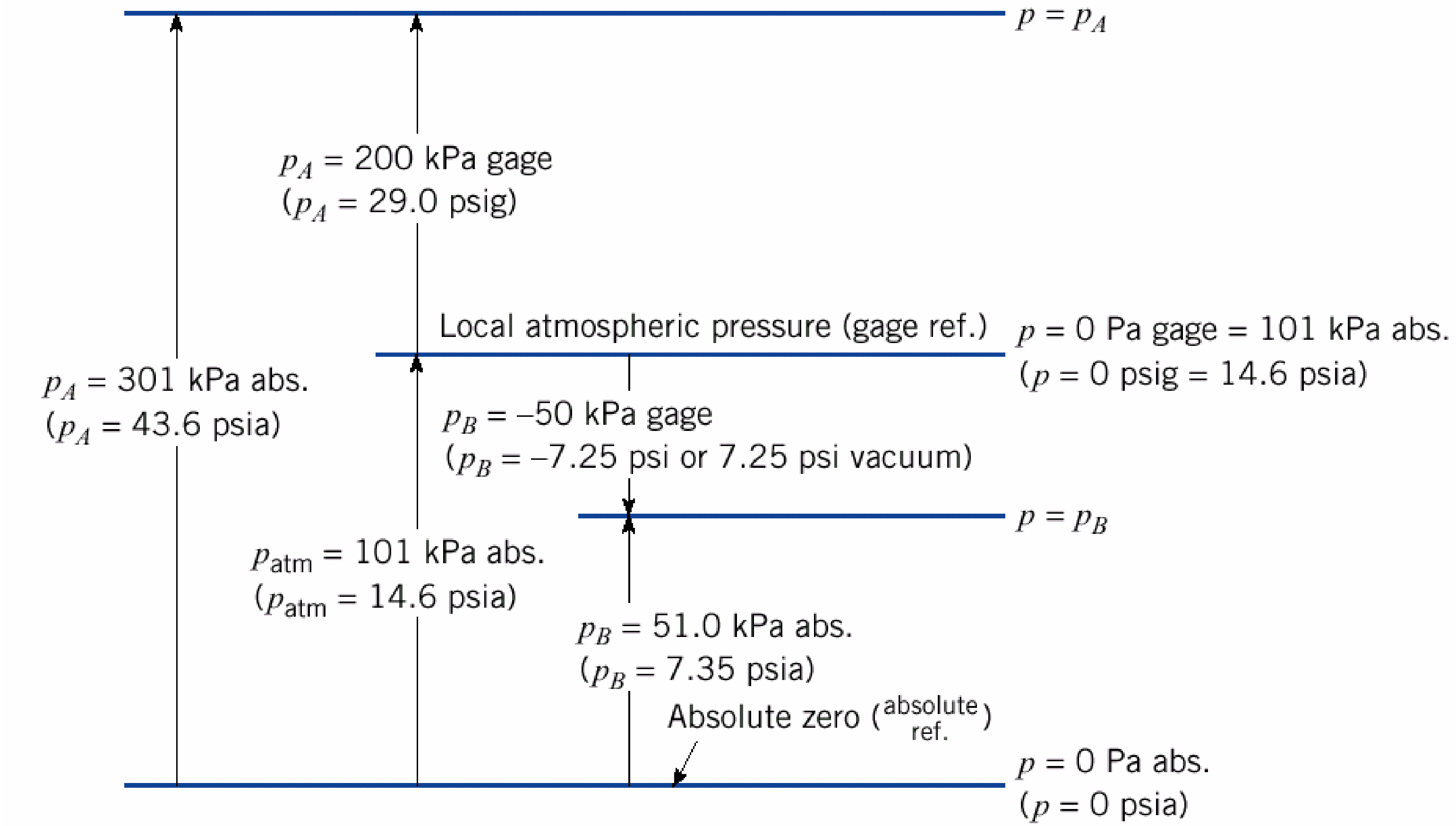
$$= \frac{p\frac{\pi}{4}d^2}{n} = F_{bolt,B-B}$$

$$n = 20\left(\frac{d}{D}\right)^2$$

$$\boxed{n = 5}$$

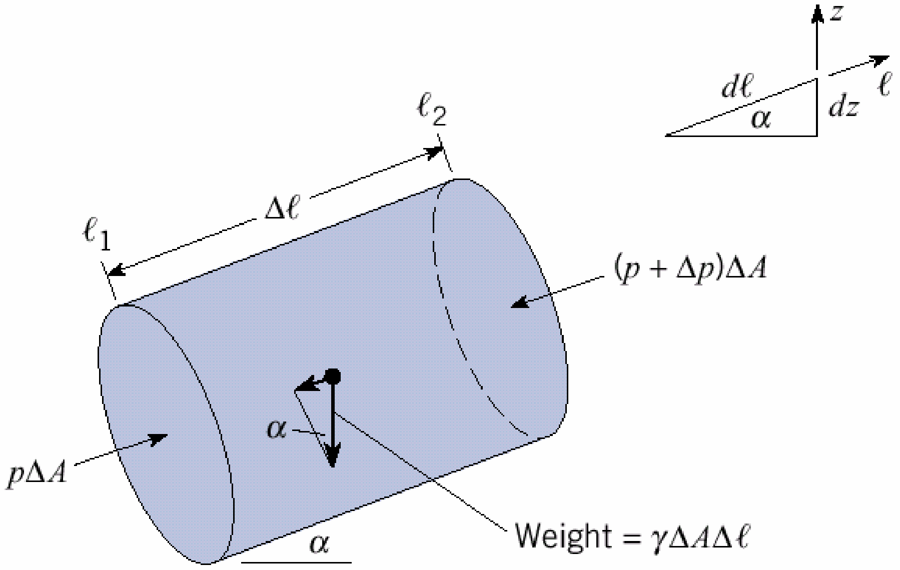


Pressure, Absolute & Gage



Pressure Variation with Elevation

- **Static fluid** – pressure varies ***only*** with elevation in the fluid.



$$\sum F_l = 0$$
$$= F_{left} - F_{right} - F_{weight}$$
$$= p\Delta A - (p + \Delta p)\Delta A - \gamma \Delta A \Delta l \sin\alpha$$

$$\frac{\Delta p}{\Delta l} = -\gamma \sin\alpha = -\gamma \frac{\Delta z}{\Delta l} \qquad \text{or} \qquad \frac{dp}{dl} = -\gamma \frac{dz}{dl}$$

$$\boxed{\frac{dp}{dz} = -\gamma}$$

Pressure Variation with Elevation

- If γ is a constant $\frac{dp}{dz} = -\gamma$

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$

$$p_2 - p_1 = -\gamma (z_2 - z_1)$$

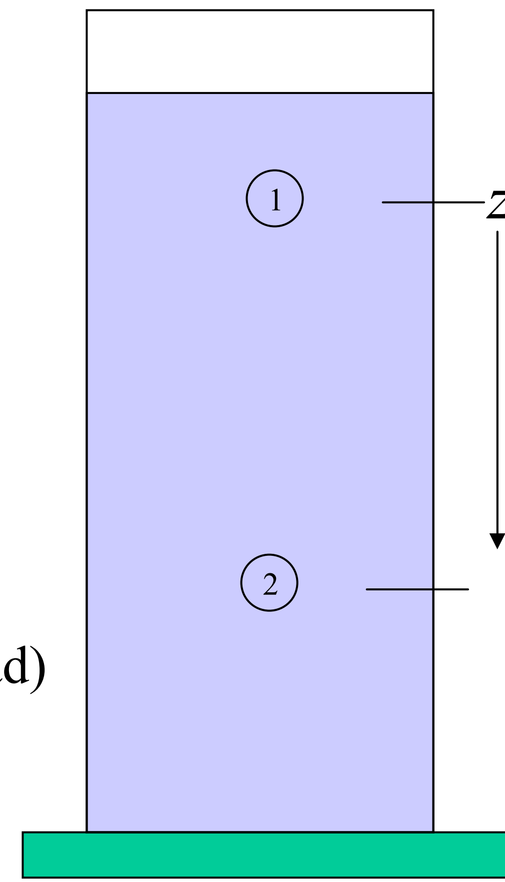
$$\Delta p = -\gamma \Delta z$$

$$h = \frac{p}{\gamma} + z = \text{constant (piezometric head)}$$

Elevation head

Pressure head

Piezometric head



$z = z_1, p = p_1$

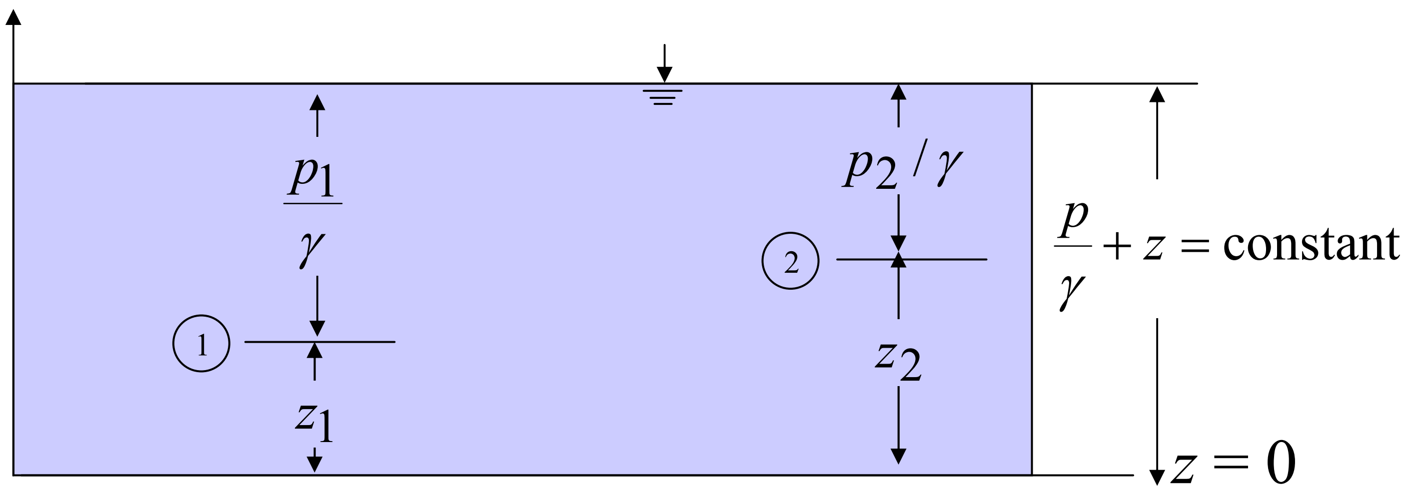
$\Delta p = -\gamma \Delta z$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$z = z_2, p = p_2$

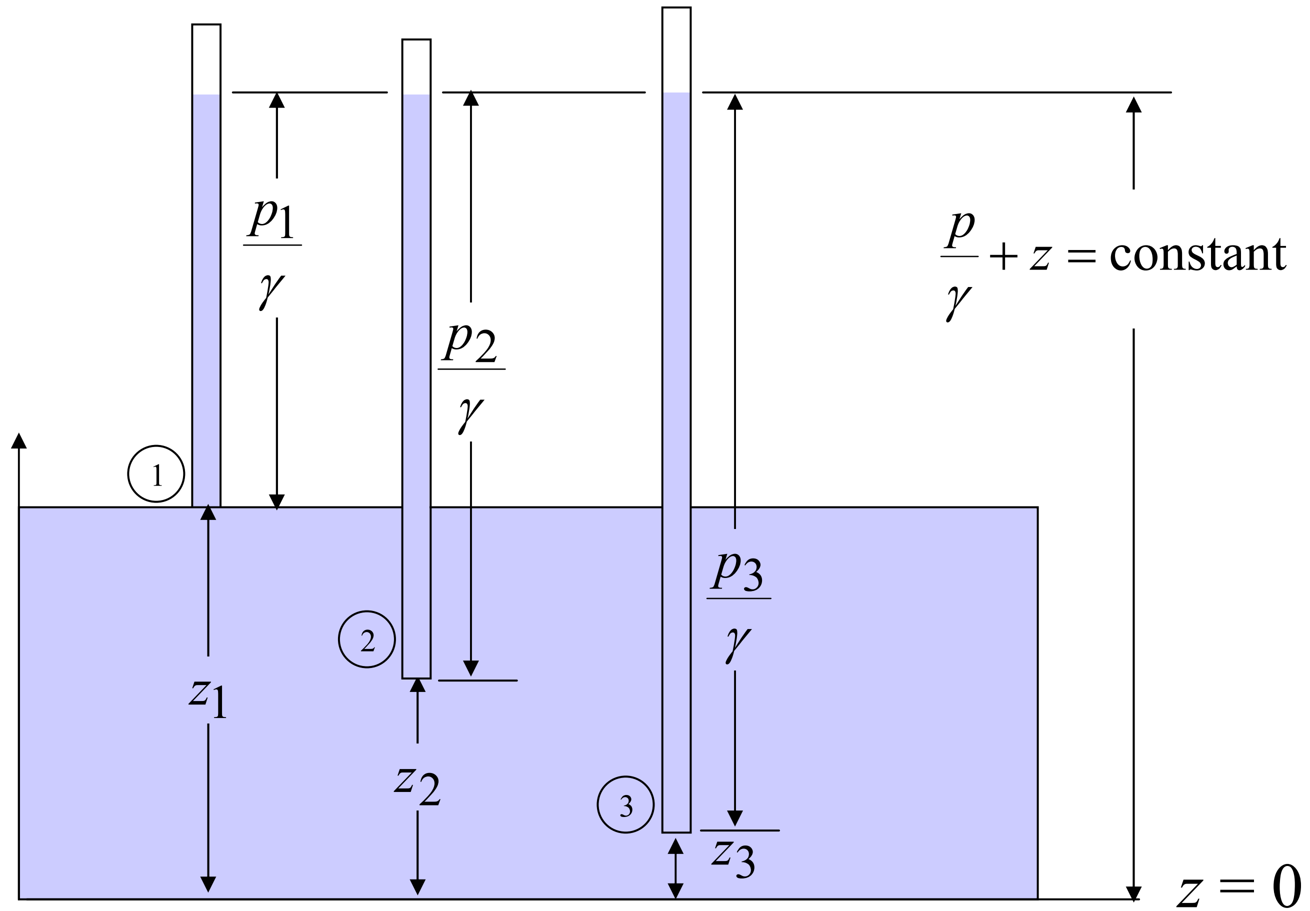
$z = 0$

Piezometric Head



Open Tank

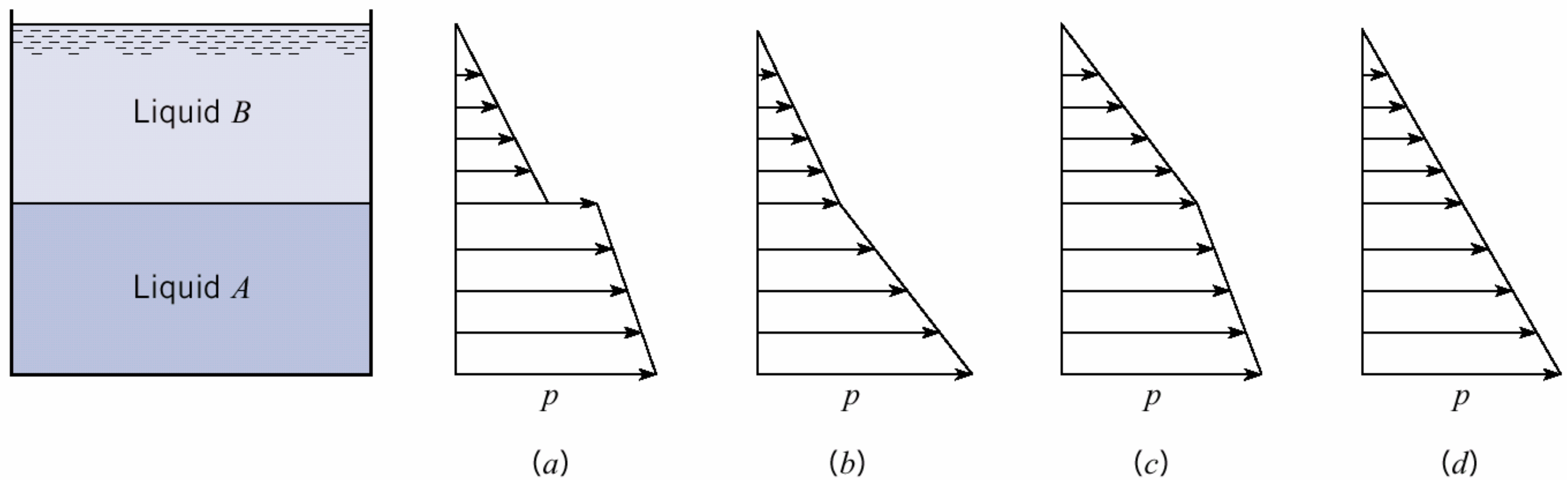
Piezometric Head



Pressurized Tank

Example (3.6)

- Tank contins fluid with $\gamma_A > \gamma_B$
- Which graph depicts the correct pressure distribution?



Leaking Storage Tank

- Water has seeped in to the depth shown
- Find: Pressure at gas-water interface, pressure at bottom of tank
- Assume: Fluids are at rest

$$\frac{p_a}{\gamma} + z_a = \frac{p_b}{\gamma} + z_b$$

$$p_0 - p_1 = -SG\gamma_w(z_0 - z_1)$$

$$0 - p_1 = -(0.68)(62.4)(20 - 3)$$

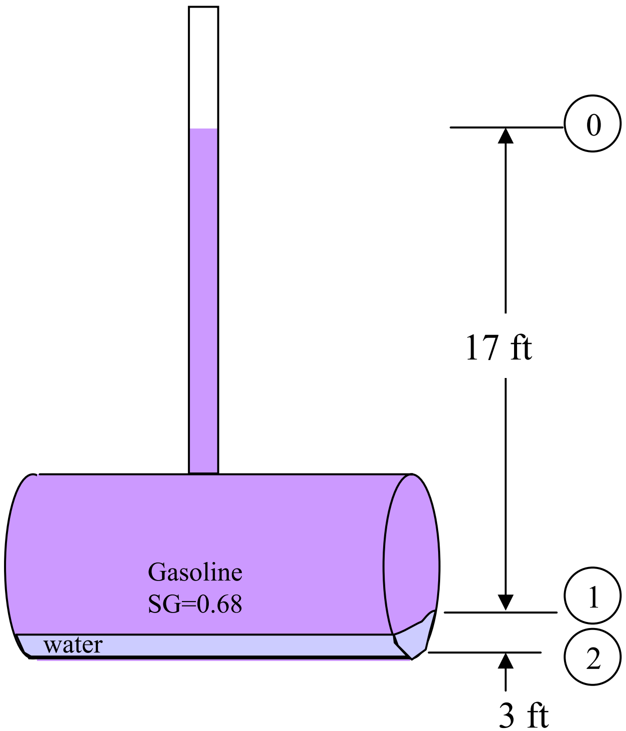
$$p_1 = 721\, lbf / ft^2 \div 144 = 5.01\, psi$$

$$p_1 - p_2 = -\gamma_w(z_1 - z_2)$$

$$p_2 = p_1 + \gamma_w(z_1 - z_2)$$

$$p_2 = 721 + (62.4)(3 - 0)$$

$$p_2 = 908\, lbf / ft^2 \div 144 = 6.31\, psi$$



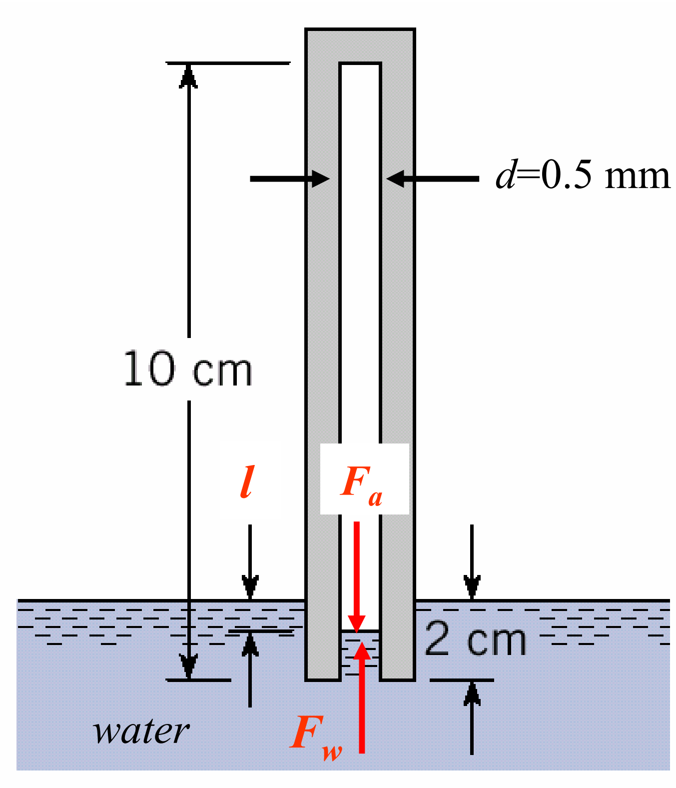
Example (3.5)

- Find l
- Forces at surface inside tube

$$\sum F_z = 0$$
$$= -F_i + F_w + F_\sigma$$
$$= -p_i A + p_w A + \sigma \pi d$$

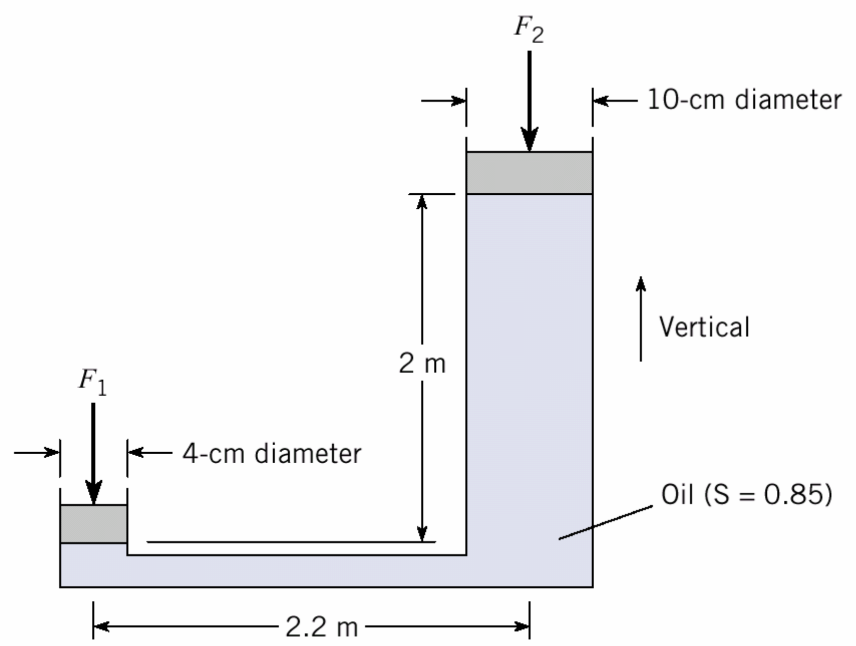
$$p_i \forall_i = p_{atm} \forall_{tube}$$
$$p_i = p_{atm} \frac{0.10 A_{tube}}{(0.08 + l) A_{tube}}$$
$$= p_{atm} \frac{0.10}{0.08 + l}$$

$$p_w = p_{atm} + \gamma l$$



Example (3.8)

- What is maximum force F_2 that can be supported?



$$p_1 - p_2 = -\gamma_{oil}(z_1 - z_2)$$

$$p_2 = p_1 + \gamma_{oil}(z_1 - z_2)$$

$$= \frac{F_1}{A_1} + SG\gamma_w(z_1 - z_2)$$

$$= \frac{200}{\frac{\pi}{4}(0.04)^2} + (0.85)(9810)(0 - 2)$$

$$= 142{,}500\ N/m^2$$

$$F_2 = p_2 A_2 = 142{,}500(\frac{\pi}{4})(0.1)^2$$

$$F_2 = 1{,}119\ N$$

Compressible Fluids

- Density is nearly proportional to pressure

$$\frac{dp}{dz} = -\gamma \qquad \text{but} \qquad \gamma = \rho g \quad \text{and} \qquad \rho \neq \text{constant}$$

- For perfect gasses

$$p = \rho RT \qquad \Rightarrow \quad \rho = \frac{p}{RT} \qquad \Rightarrow \quad \frac{dp}{dz} = -\frac{p}{RT} g$$

- Need to know $T = T(z)$
  - Constant (Isothermal): $T = T_0 = const.$
  - Linear: $T = T_0 - \alpha(z - z_0)$

Compressible Fluids

- Assume constant and integrate

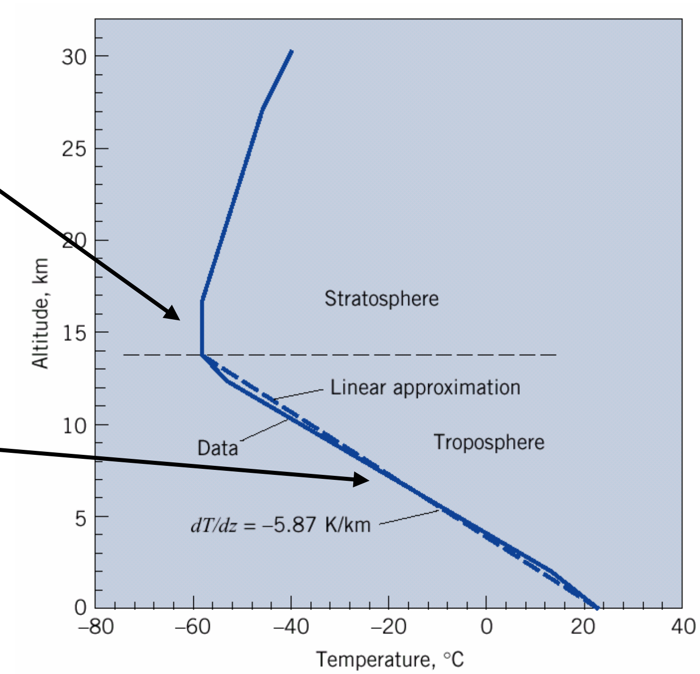
$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{RT_0}\int_{z_0}^{z} dz$$

$$p(z) = p_0 e^{-g(z-z_0)/RT_0}$$

- Assume linear and integrate

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g}{R}\int_{z_0}^{z}\frac{dz}{T_0-\alpha(z-z_0)}$$

$$p(z)=p_0\left[\frac{T_0-\alpha(z-z_0)}{T_0}\right]^{g/\alpha R}$$



Temperature variation with altitude for the U.S. standard atmosphere