CE 319 F
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Elementary Mechanics of Fluids

Hydrostatic Forces on Plane Surfaces

Pressure on Plane Surface

Surfaces exposed to fluids experience a force due to the pressure distribution in the fluid

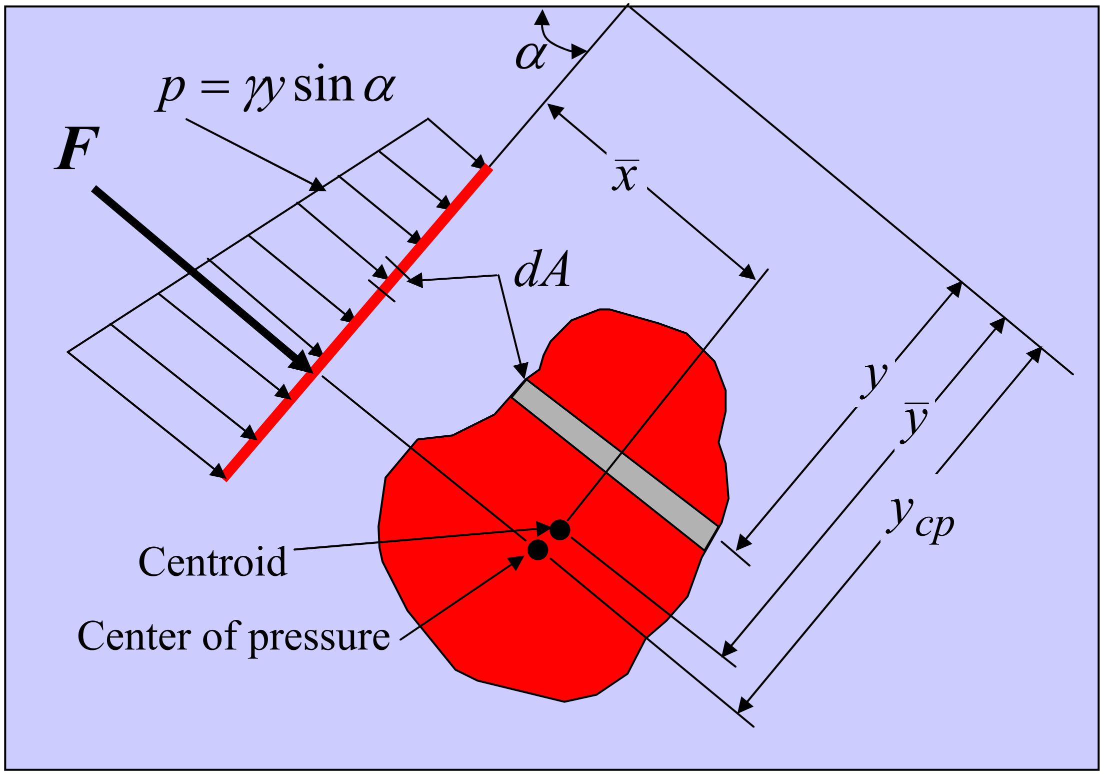
$$F = \int_A p\,dA$$

$$= \int_A \gamma y \sin\alpha\, dA$$

$$= \gamma \sin\alpha \int_A y\, dA$$

$$F = (\gamma \bar{y} \sin\alpha) A$$

$$F = \bar{p} A$$



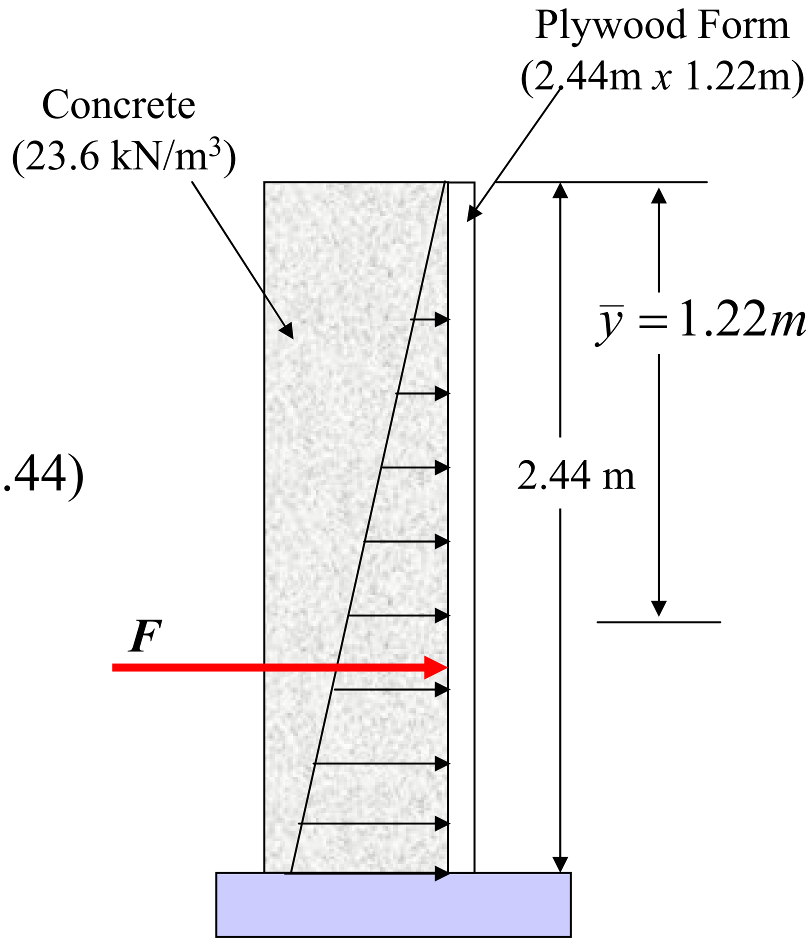
Example

$$F = \bar{p}A$$

$$= (\gamma \bar{y} \sin \alpha) A$$

$$= (23{,}600 * 1.22 * 1) * (1.22 * 2.44)$$

$$F = 85.8\, kN$$



Line of Action of Force

- Lies below centroid, since pressure increases with depth

$$y_{cp}F = \int y\,dF$$

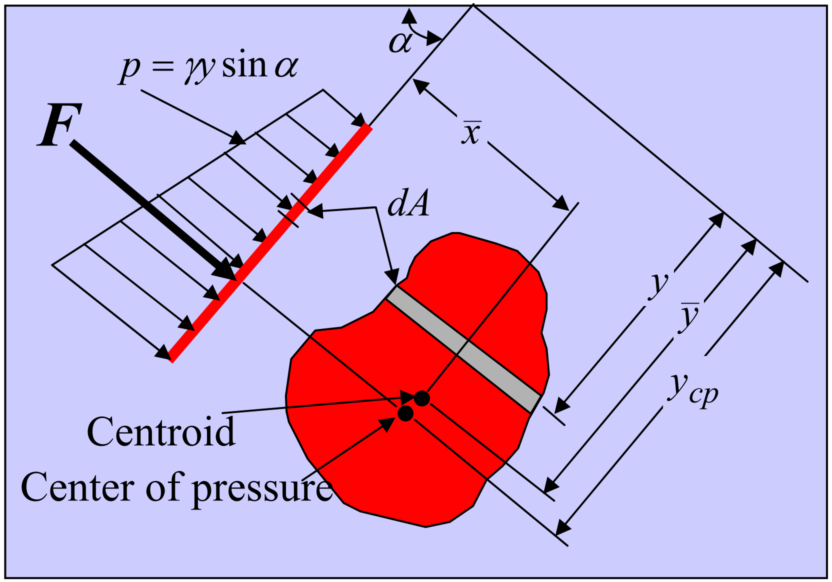
$$= \int_A y(p\,dA)$$

$$= \int_A y(\gamma y \sin\alpha)\,dA$$

$$y_{cp}(\gamma \bar{y} \sin\alpha A) = \gamma \sin\alpha I_0$$

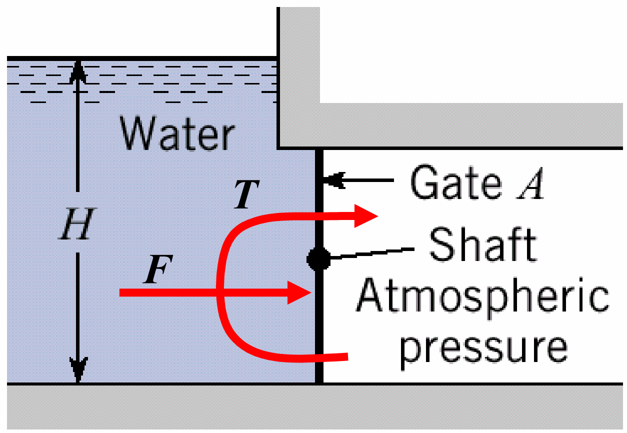
$$y_{cp}(\bar{y}A) = \bar{I} + \bar{y}^2 A$$

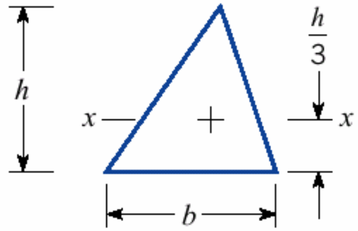
$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$$



Example (3.59)

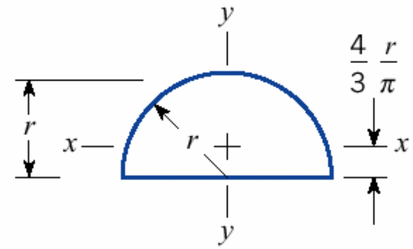
- F ↑ as H ↑?
- $y_{cp} - \bar{y}$ ↓ as H ↑?
- $y_{cp} - \bar{y}$ is constant as H ↑?
- T ↑ as H ↑?
- T is constant as H ↑?





$$A = \frac{bh}{2}$$

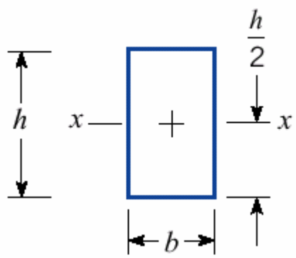
$$\bar{I}_{xx} = \frac{bh^3}{36}$$



$$A = \frac{\pi r^2}{2}$$

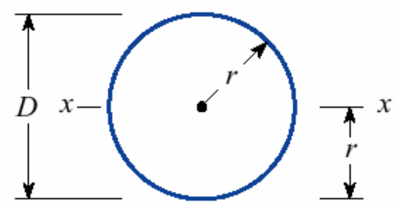
$$\bar{I}_{xx} = 0.110r^4$$

$$\bar{I}_{xx} = \frac{\pi r^4}{8}$$



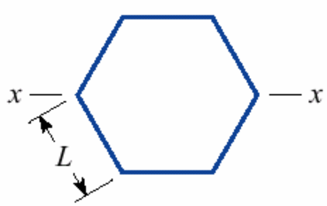
$$A = bh$$

$$\bar{I}_{xx} = \frac{bh^3}{12}$$



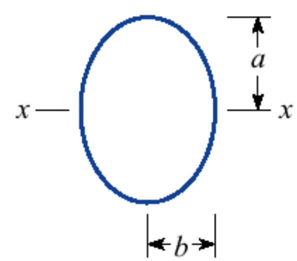
$$A = \pi r^2$$

$$\bar{I}_{xx} = \frac{\pi r^4}{4}$$



$$A = 2.5981L^2$$

$$\bar{I}_x = 0.5127L^4$$

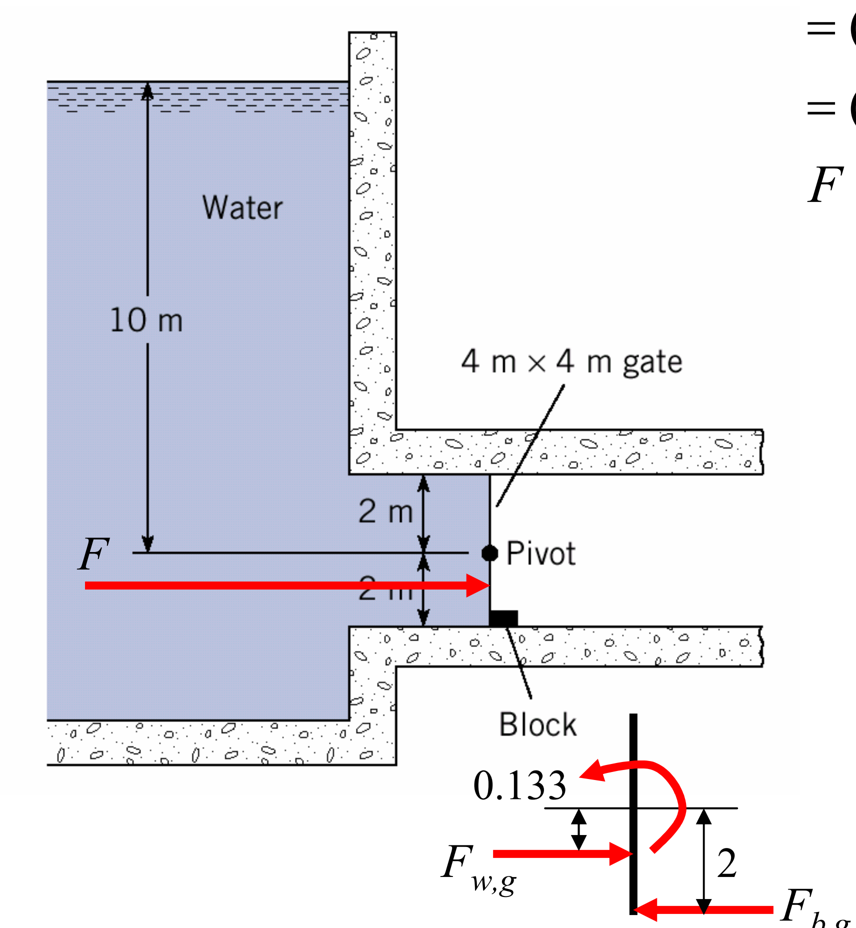


$$A = \pi ab$$

$$\bar{I}_{xx} = \frac{\pi a^4 b}{4}$$

Example (3.73)

Find: Force of block on gate

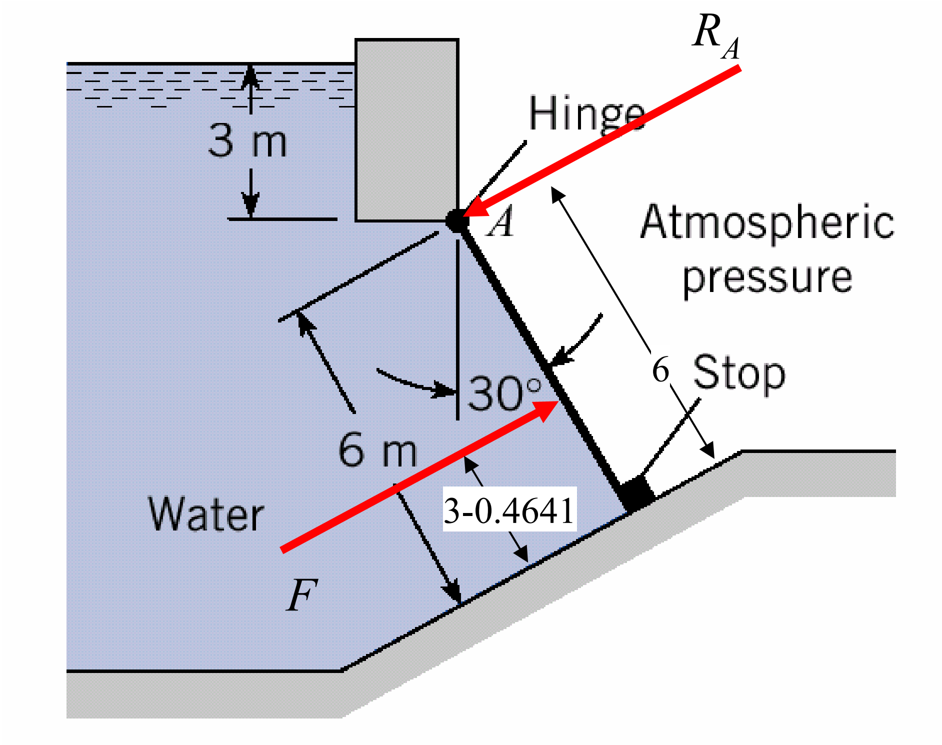


$$F = \bar{p}A$$
$$= (\gamma \bar{y} \sin\alpha)A$$
$$= (9810*10*1)*(4*4)$$
$$F = 1569.6\,kN$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$
$$= \frac{4*4^3/12}{(10*4*4)}$$
$$= 0.133\,m$$

$$\sum M = 0$$
$$= 0.133F_{w,g} - 2F_{b,g}$$
$$F_{b,g} = \frac{0.133}{2}F_{w,g}$$
$$= \frac{0.133}{2}1569.6\,kN$$
$$F_{b,g} = 104.378\,kN$$

Example (3.78)

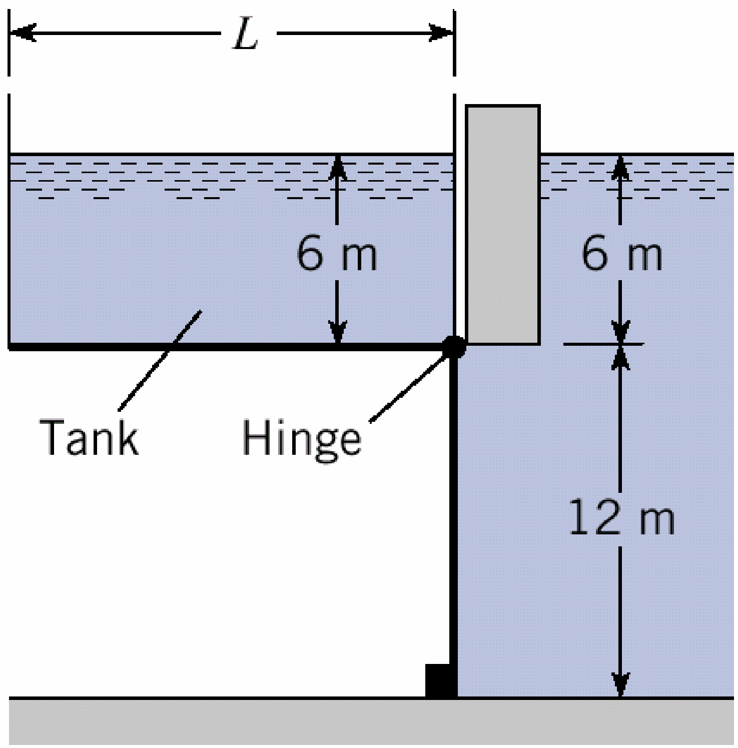


$$F = \bar{p}A = (\gamma \bar{y} \sin\alpha)A$$
$$= 9810 * (3 + 3\cos 30) * (4 * 6)$$
$$= 1{,}318{,}000\, N$$

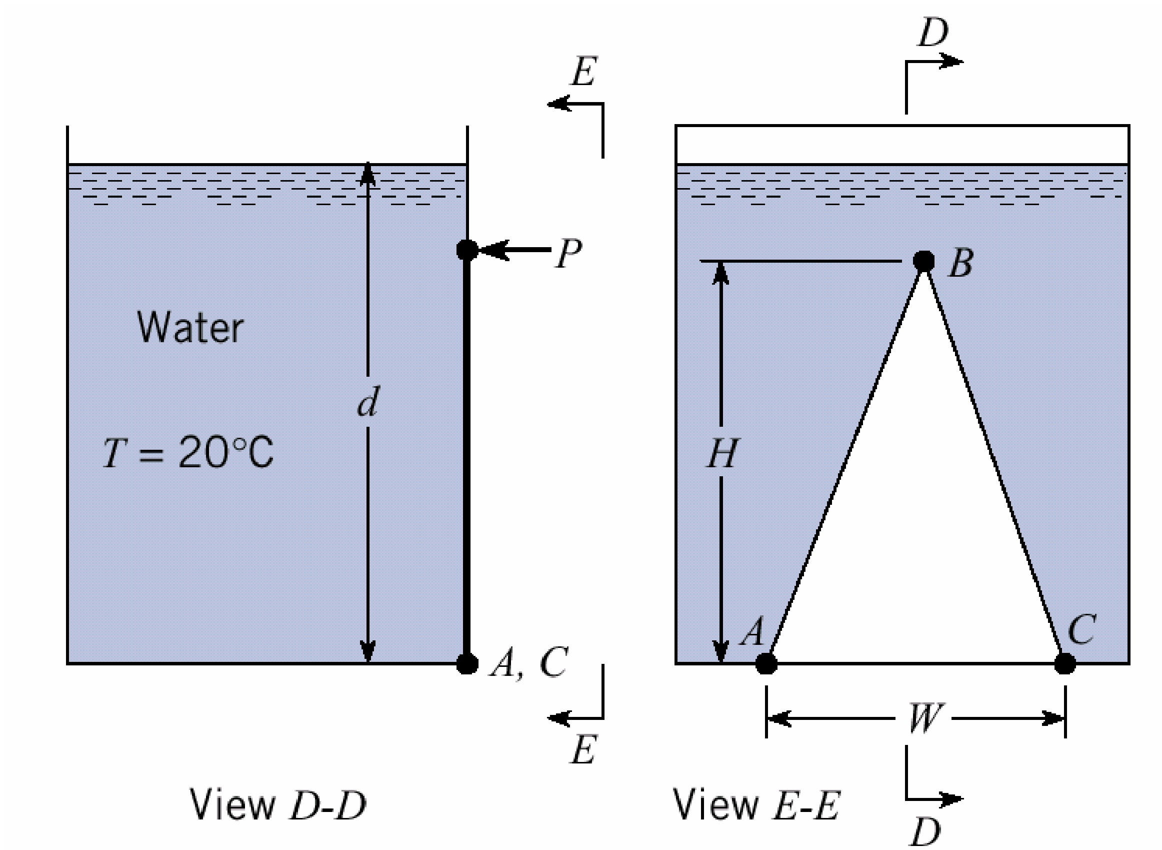
$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{4 * 6^3 / 12}{(6.464 * 24)}$$
$$= 0.4641\, m$$

$$\sum M = 0$$
$$= 6R_A - (3 - 0.4641)F$$
$$R_A = \frac{3 - 0.4641}{6} F$$
$$= (0.42265)1318\, kN$$
$$R_A = 557.05\, kN$$

HW (3.87)

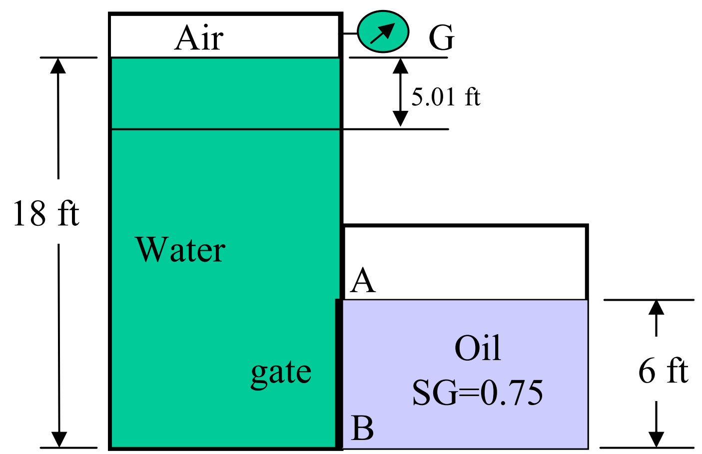


HW (3.92)



Example

Given: Gate AB is 4 ft wide, hinged at A.
Gage G reads -2.17 psi
Find: Horizontal force at B to hold gate.
Solution:



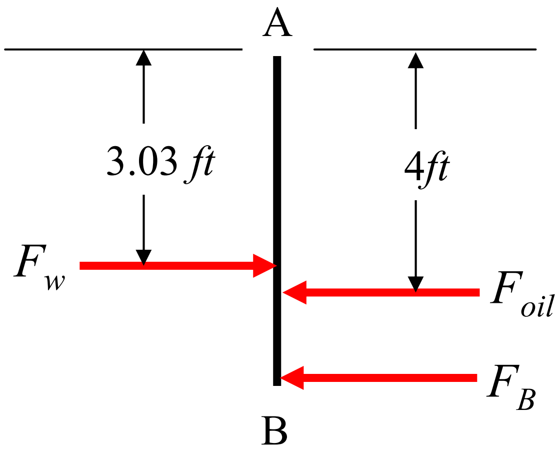
$$F_{oil} = \bar{p}A$$
$$= 0.75 * 62.4 * 3 * (4 * 6)$$
$$= 3{,}370\, lbf$$

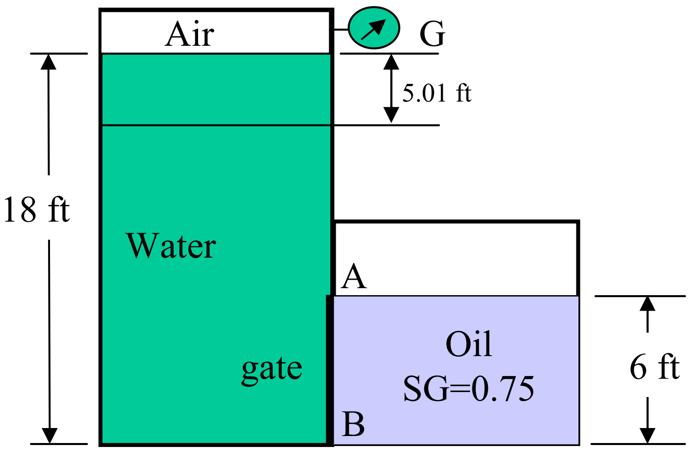
Convert negative pressure in tank to ft of water

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$
$$= \frac{4 * 6^3 / 12}{(3 * 24)} = 1\, ft$$

$$h = \frac{p}{\gamma} = \frac{-2.17 * 144}{62.4} = -5.01\, ft$$

Example



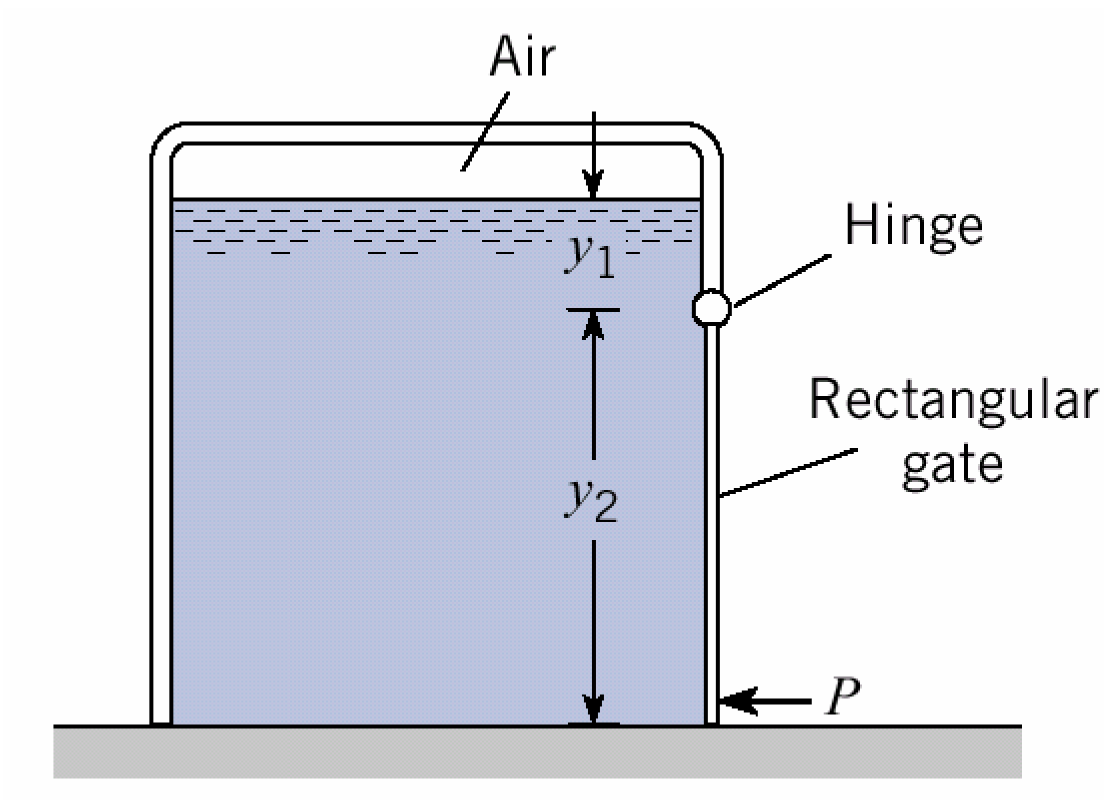


$$F_w = \bar{p}A$$
$$= 62.4 * (15 - 5.01) * (4 * 6)$$
$$= 15{,}000\, lbf$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$
$$= \frac{4 * 6^3 / 12}{(15 - 5.01) * 24} = 0.3\ ft$$

$$\sum M_A = 0$$
$$= F_w * 3.3 - F_{oil} * 4 - F_B * 6$$
$$= 15000 * 3.3 - 3700 * 4 - F_B * 6$$
$$F_B = 6000\, lbf$$

HW (3.96)



HW (3.102)