CE 319 F

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Elementary Mechanics of Fluids

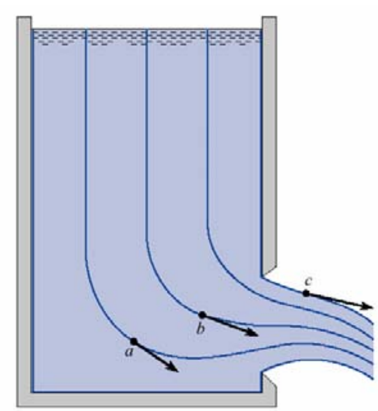
Description of Motion

Fluid Motion

- Two ways to describe fluid motion
  - Lagrangian
    - Follow particles around
  - Eularian
    - Watch fluid pass by a point or an entire region
  - Flow pattern
    - Streamlines – velocity is tangent to them

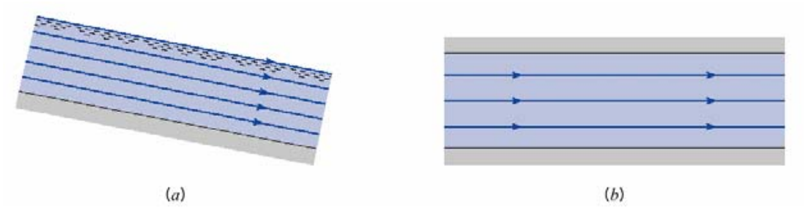
$$\boldsymbol{V} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$$

$$\boldsymbol{V} = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$$

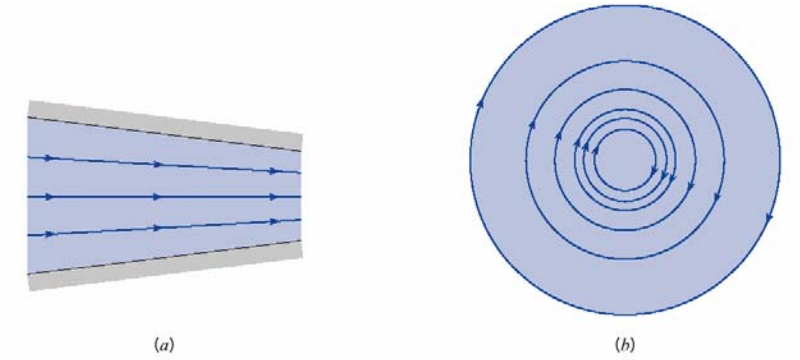


Flow Patterns

- Uniform flow $\frac{\partial \boldsymbol{V}}{\partial s} = 0$



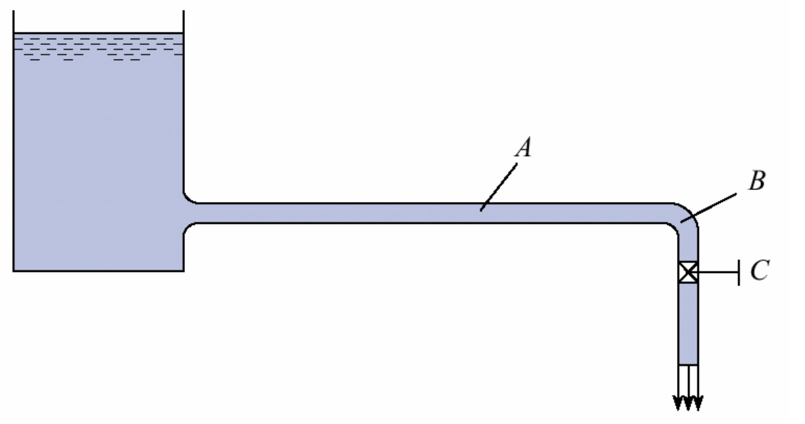
- Non-uniform flow $\frac{\partial \boldsymbol{V}}{\partial s} \neq 0$



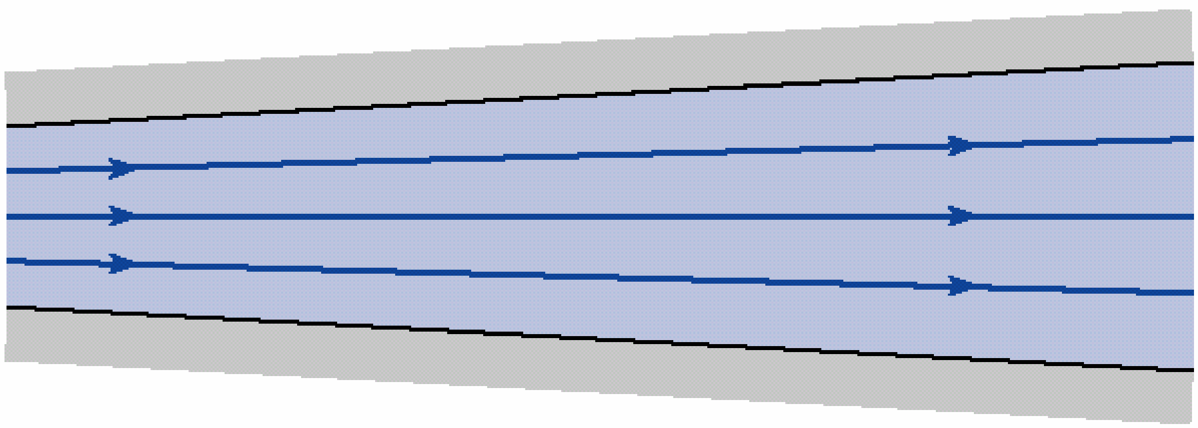
- Steady flow $\frac{\partial \boldsymbol{V}}{\partial t} = 0$
- Unsteady flow $\frac{\partial \boldsymbol{V}}{\partial t} \neq 0$

Example (4.1)

- Valve at C is opened slowly
- Classify the flow at B while valve is opened
- Classify the flow at A

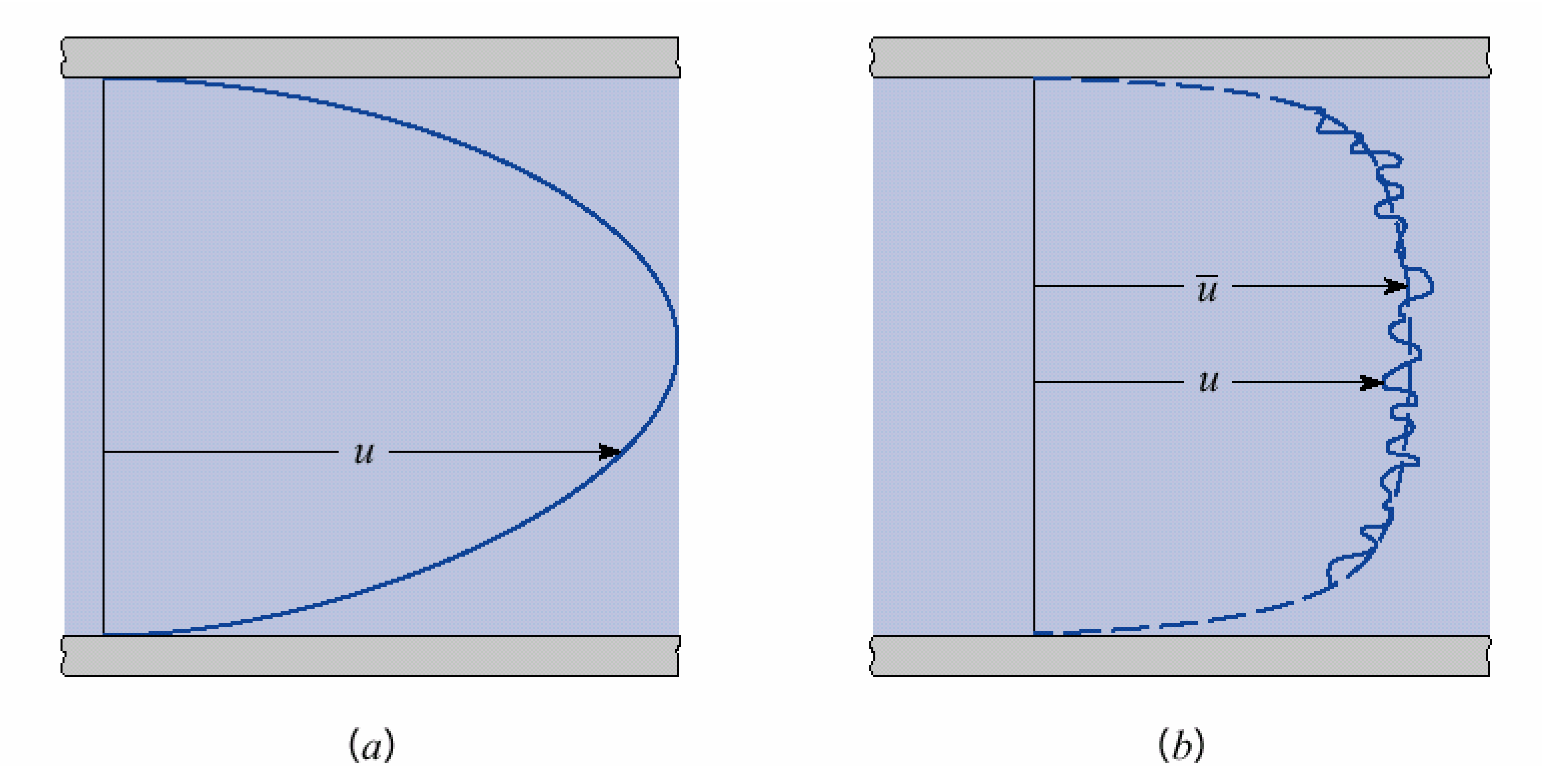


HW (4.2)



Laminar vs Turbulent Flow

- Laminar
- Turbulent



Flow Rate

- Volume rate of flow
  - Constant velocity over cross-section

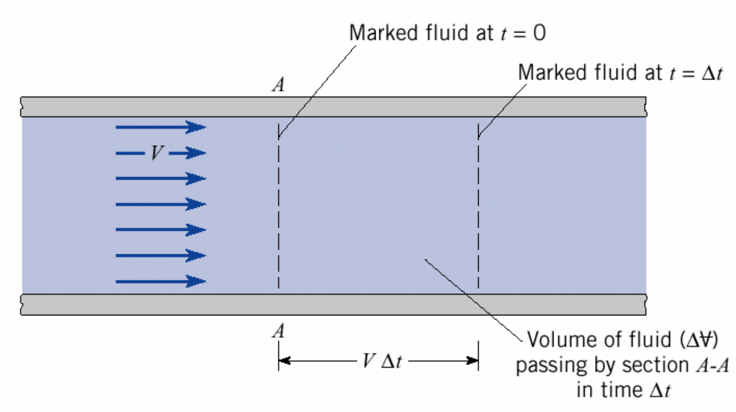
$$Q = VA$$

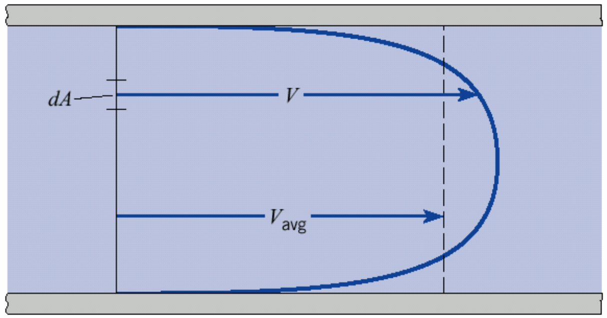
  - Variable velocity

$$Q = \int_A V dA$$

- Mass flow rate

$$\dot{m} = \int_A \rho V dA = \rho \int_A V dA = \rho Q$$





Examples

- Prob. 4.17
  Discharge in a 2-cm pipe is 0.03 m^3/s. What is the average velocity?

$$Q = VA$$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.03}{\frac{\pi}{4}(0.25)^2} = 0.611\,m/s$$

- Prob. 4.20
  A pipe whose diameter is 8 cm transports air with a temp. of 20^{o}C and pressure of 200 kPa abs. At 20 m/s. What is the mass flow rate?

$$\rho = \frac{p}{RT} = \frac{200000}{287*293} = 2.378\,kg/m^3$$

$$\dot{m} = \rho VA$$

$$= 2.378*20*\frac{\pi}{4}(0.08)^2 = 0.239\,kg/s$$

Flow Rate

- Only x-direction component of velocity (u) contributes to flow through cross-section

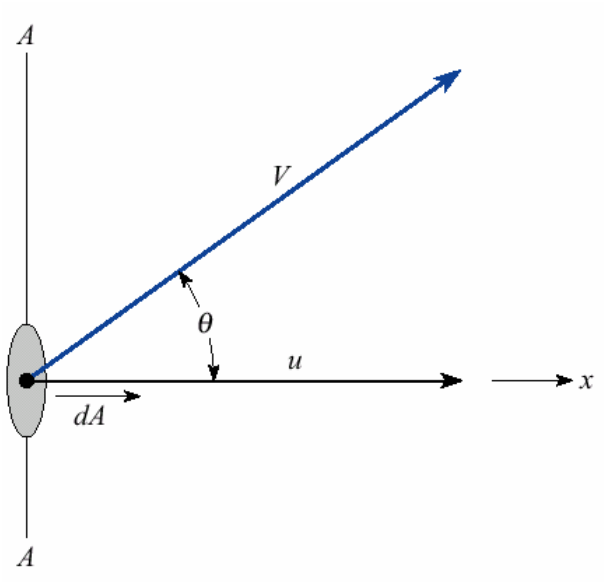
$$Q = \int_A V dA = \int_A u dA = \int_A V \cos\theta dA$$

or

$$Q = \int_A \boldsymbol{V} \cdot d\boldsymbol{A}$$

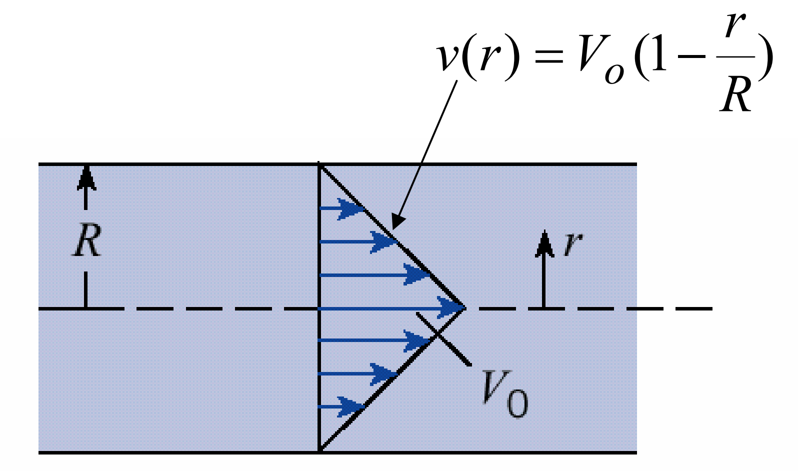
or

$$Q = \boldsymbol{V} \cdot \boldsymbol{A}$$



Example (4.24)

- Find: $\dfrac{\bar{V}}{V_o}$



$$Q = \int_A V dA = \int_0^R V_o(1 - r/R) 2\pi r dr$$

$$= 2\pi V_o\left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Bigg|_0^R = 2\pi V_o\left(\frac{R^2}{2} - \frac{R^2}{3}\right)$$

$$= \frac{1}{3}\pi V_o R^2$$

$$\frac{\bar{V}}{V_o} = \frac{Q}{A}V_o = \frac{\frac{1}{3}\pi V_o R^2}{\pi R^2 V_o} = \frac{1}{3}$$

Example (4.28)

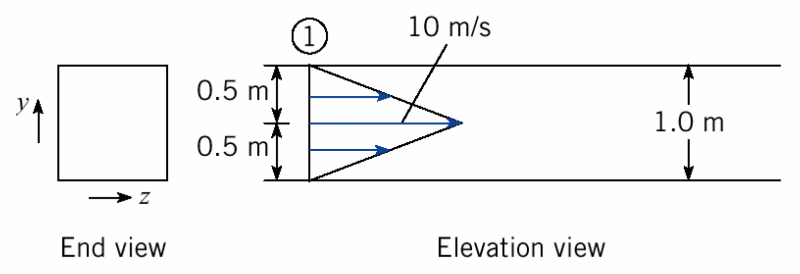
- Find: $Q, \overline{V}, \dot{m}$

$$Q = 2\int_0^{0.5} V dA = 2\int_0^{0.5} 20 y dy$$

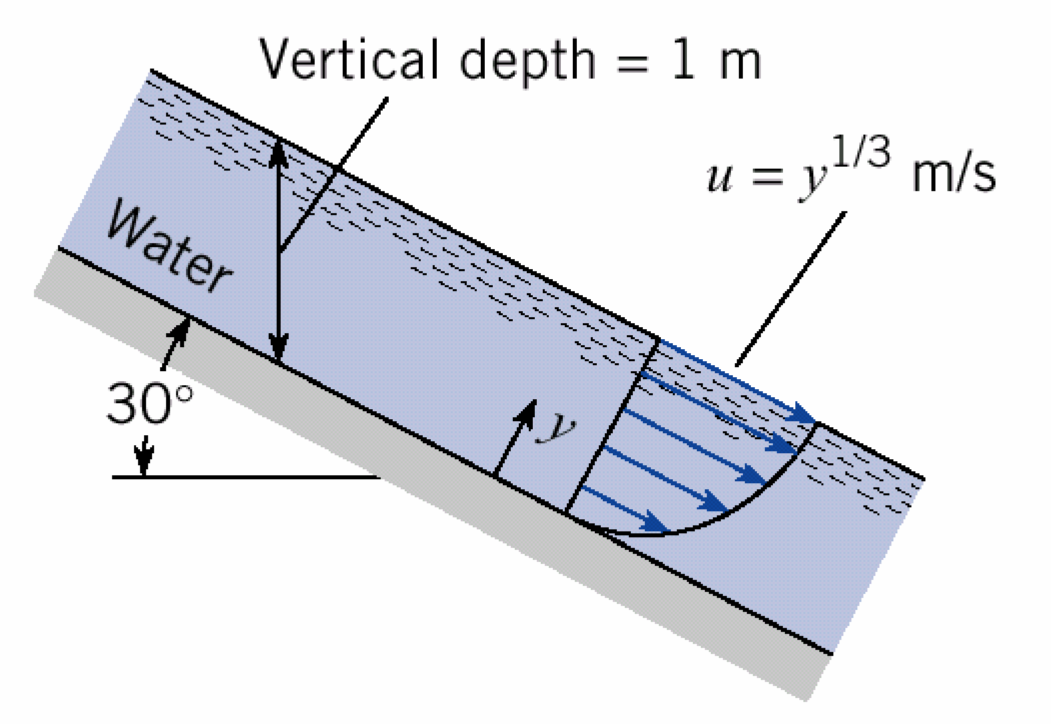
$$= 40\left.\frac{y^2}{2}\right|_0^{0.5} = 5\, m^3/s$$

$$\overline{V} = \frac{Q}{A} = \frac{5}{1} = 5\, m/s$$

$$\dot{m} = \rho Q = 1.2 * 5 = 6\, kg/s$$

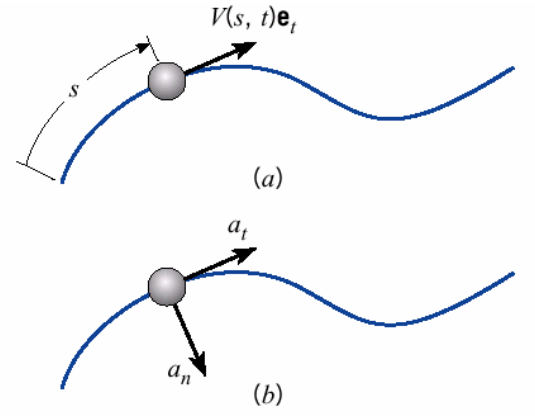


HW (4.30)



Acceleration

- Acceleration = rate of change of velocity
- Components:
  - Normal – changing direction
  - Tangential – changing speed



$$\vec{V} = V(s,t)\vec{e}_t$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{dV}{dt}\vec{e}_t + V\frac{d\vec{e}_t}{dt}$$

$$\frac{dV}{dt} = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$$

$$\frac{d\vec{e}_t}{dt} = \frac{V}{r}\vec{e}_n$$

$$\vec{a} = (V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t})\vec{e}_t + \frac{V^2}{r}\vec{e}_n$$

Acceleration

- Cartesian coordinates $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$ $\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

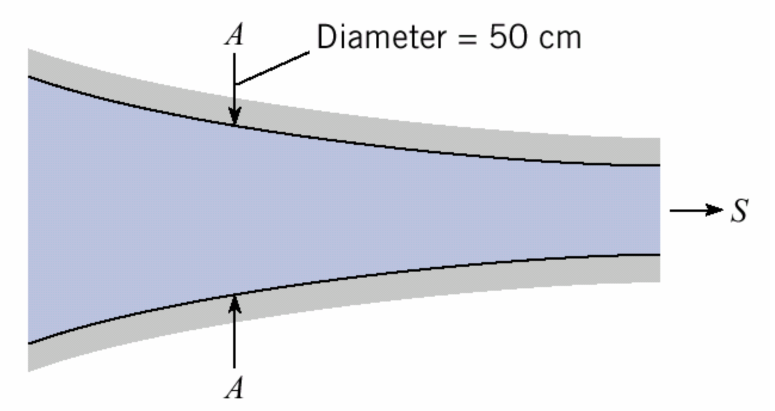
$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t} = .\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial v}{\partial z}\frac{dz}{dt} + \frac{\partial v}{\partial t} = .\frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} + \frac{\partial w}{\partial t} = .\underbrace{\frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w}_{\text{Convective}} + \underbrace{\frac{\partial w}{\partial t}}_{\text{Local}}$$

- HW (4.43)

Example (4.49)



$$Q = Q_o - Q_1 \frac{t}{t_o} = 0.985 - 0.5t$$

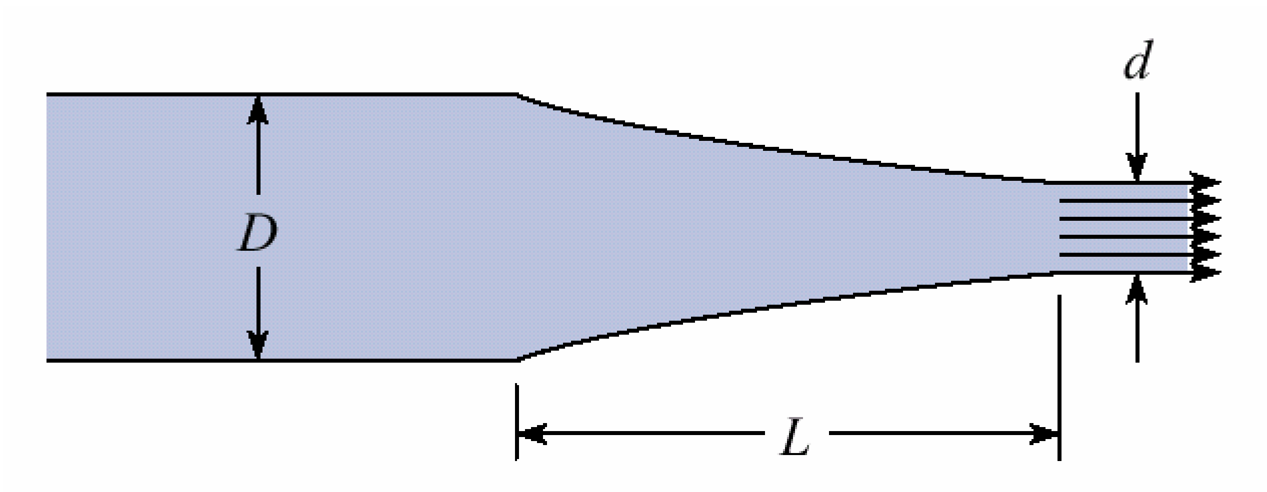
$$\frac{\partial V}{\partial s} = 2\,m/s$$

$$V = \frac{Q}{A} = \frac{Q_o - Q_1 \frac{t}{t_o}}{\frac{\pi}{4} d^2} = \frac{0.985 - 0.5(0.5)}{\frac{\pi}{4}(0.5)^2} = 3.4743\,m/s$$

$$a_L = \frac{\partial V}{\partial t} = \frac{\partial (Q/A)}{\partial t} = \frac{-Q_1}{\frac{\pi}{4} d^2 t_o} = \frac{-0.5}{\frac{\pi}{4}(0.5)^2(1)} = -2.55\,m/s^2$$

$$a_C = V \frac{\partial V}{\partial s} = 3.743 * 2 = 7.49\,m/s^2$$

HW (4.50 & 4.51)



Example

Given : $\vec{\boldsymbol{V}} = 3t\vec{\boldsymbol{i}} + xz\vec{\boldsymbol{j}} + ty^2\vec{\boldsymbol{k}}$

Find : Acceleration, $\vec{\boldsymbol{a}}$

$$u = 3t; \quad v = xz; \quad w = ty^2$$

$$a_x = \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t} = 0(3t) + 0(xz) + 0(ty^2) + 3 = 3$$

$$a_y = \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w + \frac{\partial v}{\partial t} = z(3t) + 0(xz) + x(ty^2) + 0 = 3zt + xy^2t$$

$$a_z = \frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w + \frac{\partial w}{\partial t} = 0(3t) + 2ty(xz) + 0(ty^2) + y^2 = 2xyzt + y^2$$

$$\vec{\boldsymbol{a}} = a_x\vec{\boldsymbol{i}} + a_y\vec{\boldsymbol{j}} + a_z\vec{\boldsymbol{k}} = 3\vec{\boldsymbol{i}} + (3tz + txy^2)\vec{\boldsymbol{j}} + (2xyzt + y^2)\vec{\boldsymbol{k}}$$