CE 319 F

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Elementary Mechanics of Fluids

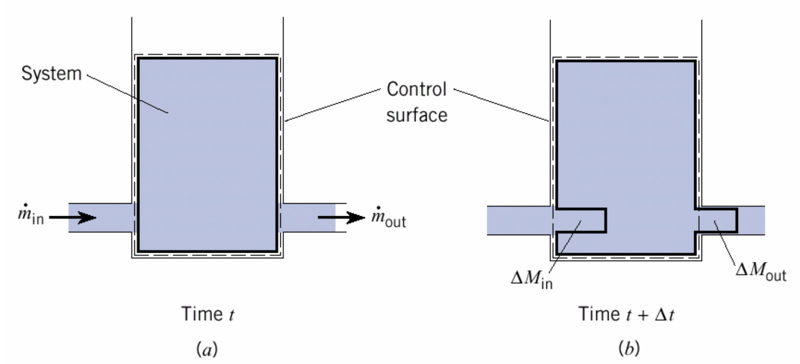
Control Volumes

Approaches to Solving Fluids Problems

- Experimental Analysis
- Differential Analysis
- Control Volume Analysis
  - Single most valuable tool available (White, Ch. 3)

Systems

- Laws of Mechanics
  - Written for systems
  - System = arbitrary quantity of mass of fixed identity
  - Fixed quantity of mass, *m*



- Conservation of Mass
  - Mass is conserved and does not change

$$\frac{dm}{dt} = 0$$

- Momentum
  - If surroundings exert force on system, mass will accelerate

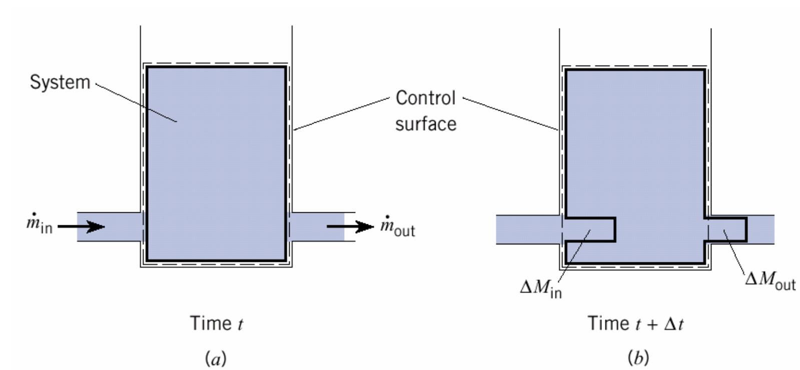
$$\vec{\boldsymbol{F}} = \frac{d(m\vec{\boldsymbol{V}})}{dt}$$

- Energy
  - If heat is added to system or work is done by system, energy will change

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Control Volumes

- Solid Mechanics
  - Follow the system, determine what happens to it
- Fluid Mechanics
  - Consider the behavior in a specific region or Control Volume
- Convert System approach to CV approach
  - Look at specific regions, rather than specific masses
- Reynolds Transport Theorem
  - Relates time derivative of system properties to rate of change of property in CV



$$B = \int_{CV} b\,dm = \int_{CV} b\rho\,d\forall$$

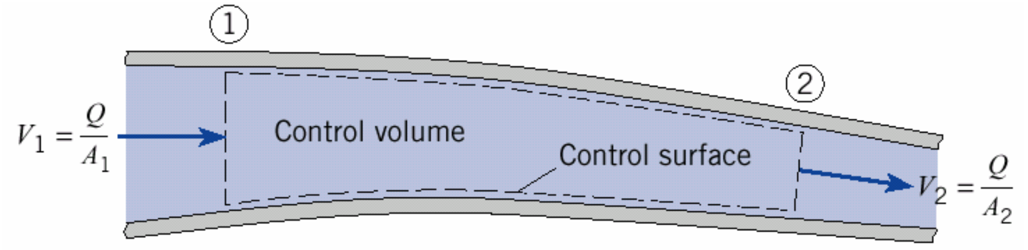
$$= \text{mass, momentum, energy (extensive)}$$

$$b = \frac{dB}{dm}$$

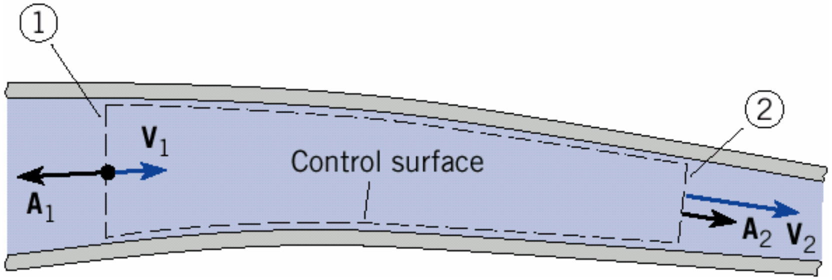
$$= \text{amount of } B \text{ per unit mass (intensive)}$$

CV Inflow & Outflow

$$Q = \vec{V} \cdot \vec{A}$$



Area vector always points *outward* from CV



$$\begin{aligned} Q_{out} - Q_{in} &= V_2 A_2 - V_1 A_1 \\ &= \vec{V}_2 \vec{A}_2 - \vec{V}_1 \cdot \vec{A}_1 \\ &= \sum_{CS} \vec{V} \cdot \vec{A} \end{aligned}$$

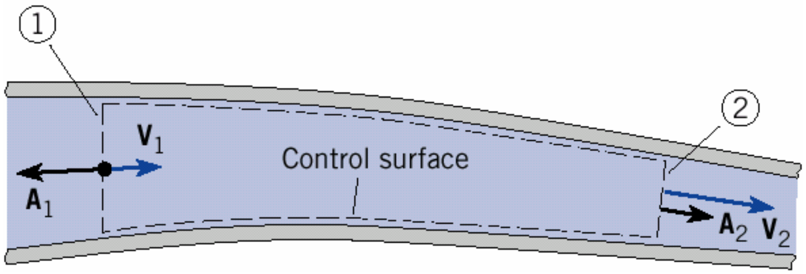
CV Inflow & Outflow

$$b\dot{m} = \dot{B}$$

$$\dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in} = b\dot{m}_{out} - b\dot{m}_{in} = \sum_{CS} b\rho\vec{\boldsymbol{V}} \cdot \vec{\boldsymbol{A}} = \sum_{CS} b\dot{m}$$

$$M_{sys,t+\Delta t} = M_{CV,t+\Delta t} + \Delta M_{out} - \Delta M_{in}$$

$$B_{sys,t+\Delta t} = B_{CV,t+\Delta t} + \Delta B_{out} - \Delta B_{in}$$



Reynolds Transport Theorem

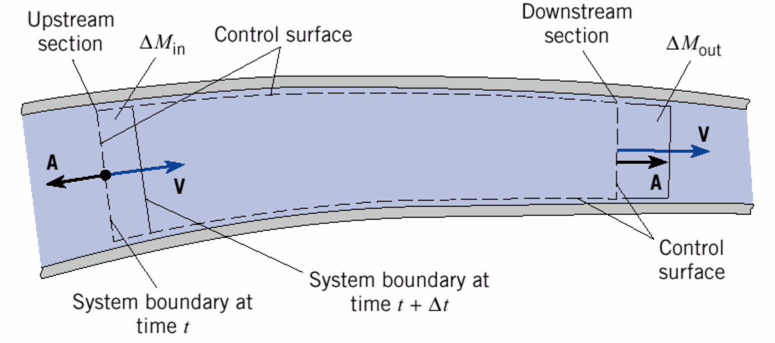
$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{B_{t+\Delta t} - B_{CV,t}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{B_{CV,t+\Delta t} + \Delta B_{out} - \Delta B_{in} - B_{CV,t}}{\Delta t}$$

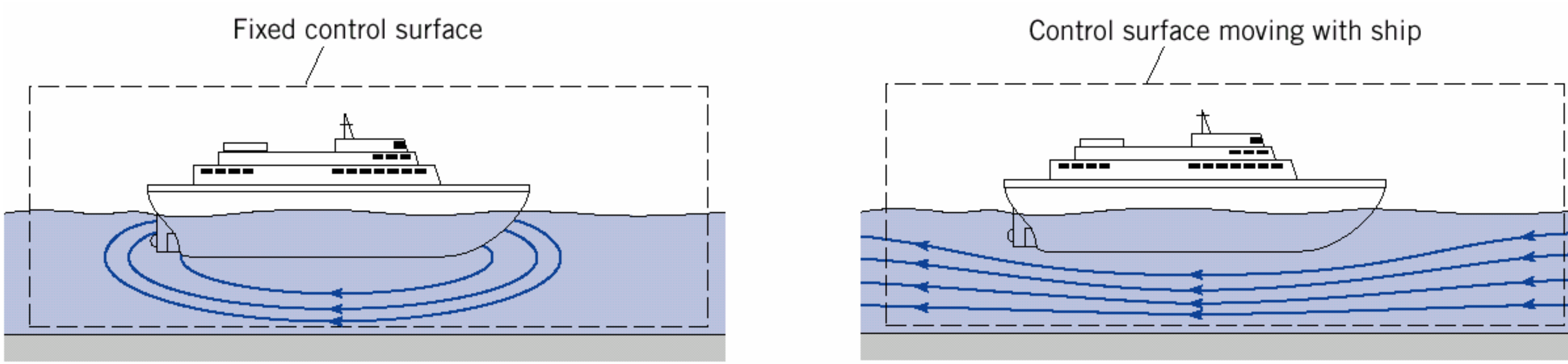
$$= \lim_{\Delta t \to 0} \frac{B_{CV,t+\Delta t} - B_{CV,t}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta B_{out} - \Delta B_{in}}{\Delta t}$$

$$= \frac{dB_{CV}}{dt} + \dot{B}_{net}$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} b\rho d\forall + \sum_{CS} b\rho \vec{V} \cdot \vec{A}$$



Steady vs. Unsteady CV



$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho d\forall + \sum_{CS} b\rho \vec{\boldsymbol{V}} \cdot \vec{\boldsymbol{A}} \quad \Rightarrow \quad \frac{dB_{sys}}{dt} = \sum_{CS} b\rho \vec{\boldsymbol{V}} \cdot \vec{\boldsymbol{A}}$$

Continuity Equation

- Reynolds Transport Theorem

$$B = M_{sys} \text{ (extensive)}$$

$$b = \frac{dB}{dm} = \frac{dM_{sys}}{dm} = 1 \text{ (intensive)}$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho d\forall + \sum_{CS} b\rho\vec{\boldsymbol{V}}\cdot\vec{\boldsymbol{A}}$$

$$0 = \frac{d}{dt}\int_{CV} \rho d\forall + \sum_{CS} \rho\vec{\boldsymbol{V}}\cdot\vec{\boldsymbol{A}}$$

Unsteady Case

$$0 = \sum_{CS} \rho\vec{\boldsymbol{V}}\cdot\vec{\boldsymbol{A}}$$

Steady Case

Example (4.57)

- Continuity equation

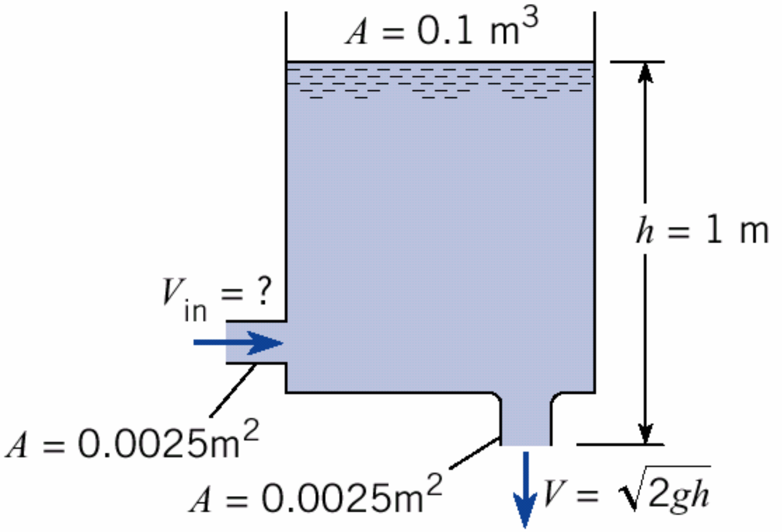
$$0=\frac{d}{dt}\int_{CV}\rho d\forall+\sum_{CS}\rho\vec{V}\cdot\vec{A}$$

$$=\frac{d}{dt}(\rho A_{tank}h)-\rho V_{in}A_{in}+\rho V_{out}A_{out}$$

$$=A_{tank}\frac{dh}{dt}-V_{in}A_{in}+V_{out}A_{out}$$

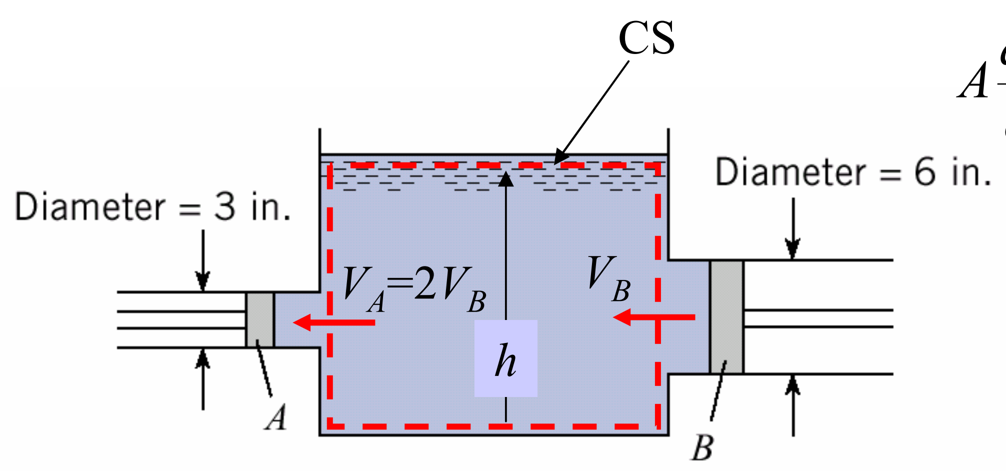
$$=0.1*0.1x10^{-2}-V_{in}(0.0025)+\sqrt{2g*1}(0.0025)$$

$$V_{in}=4.47\,m/s$$



Example (4.61)

- Select a CV that moves up and down with the water surface
- Continuity Equation



$$0 = \frac{d}{dt}\int_{CV} \rho d\forall + \sum_{CS} \rho \vec{V} \cdot \vec{A}$$

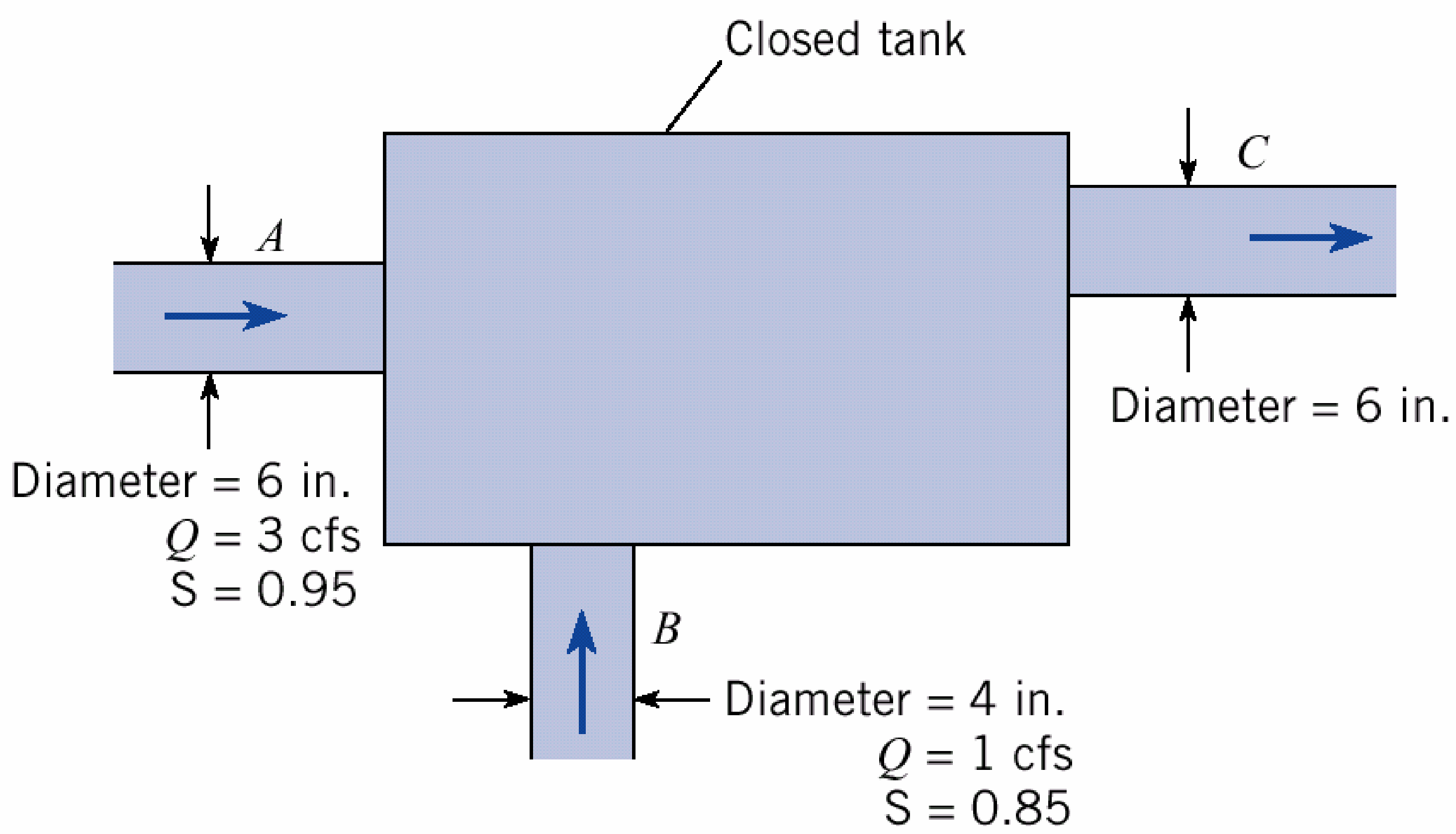
$$0 = \frac{d}{dt}\int_{CV} \rho d\forall + \rho 2V_B A_A - \rho V_B A_B$$

$$A_A = \tfrac{\pi}{4}3^2; \;\; A_B = \tfrac{\pi}{4}6^2; \;\; A_A = \tfrac{1}{4}A_B$$

$$0 = A\frac{dh}{dt} + 2V_B(\tfrac{1}{4}A_B) - V_B A_B$$

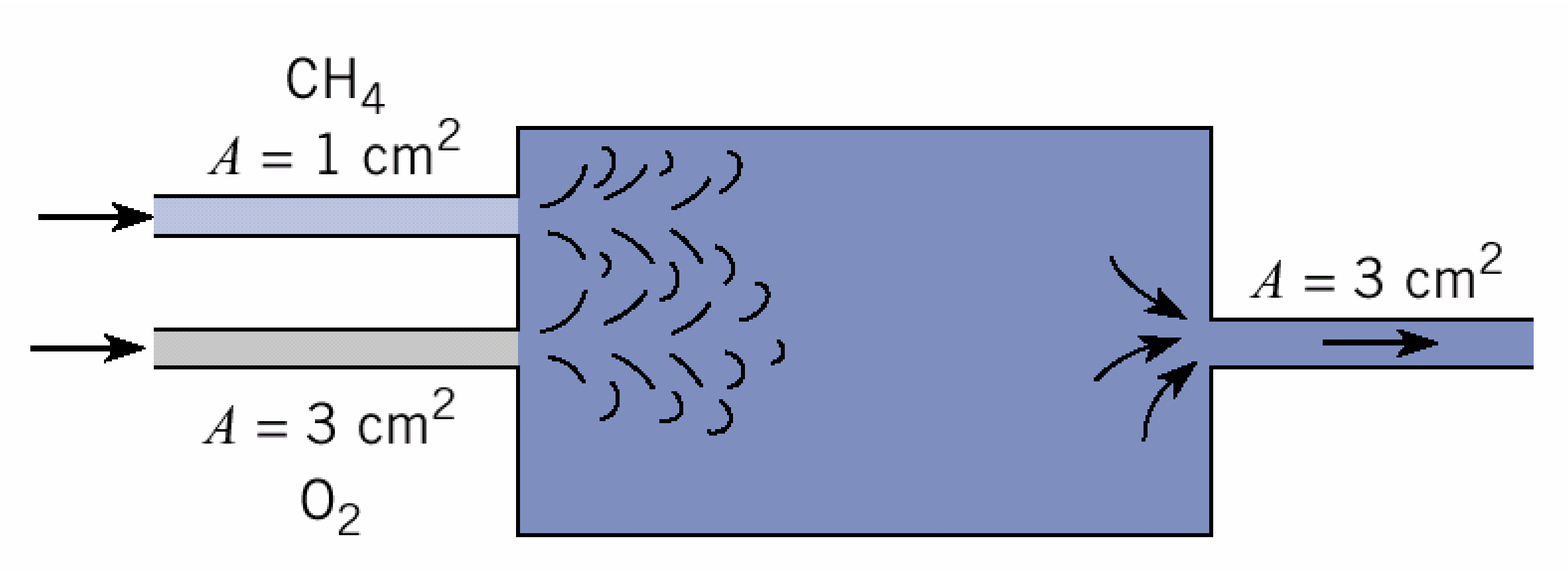
$$A\frac{dh}{dt} = \tfrac{1}{2}V_B A_B > 0 \quad \therefore \text{surface is rising}$$

HW (4.80)



Closed tank
A
C
Diameter = 6 in.
Q = 3 cfs
S = 0.95
Diameter = 6 in.
B
Diameter = 4 in.
Q = 1 cfs
S = 0.85

HW (4.81)



HW (4.82)