CE 319 F

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Elementary Mechanics of Fluids

Continuity Equation

Continuity Equation

- Reynolds Transport Theorem

$$B = M_{sys} \text{ (extensive)}$$

$$b = \frac{dB}{dm} = \frac{dM_{sys}}{dm} = 1 \text{ (intensive)}$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} b\rho d\forall + \sum_{CS} b\rho \vec{V} \cdot \vec{A}$$

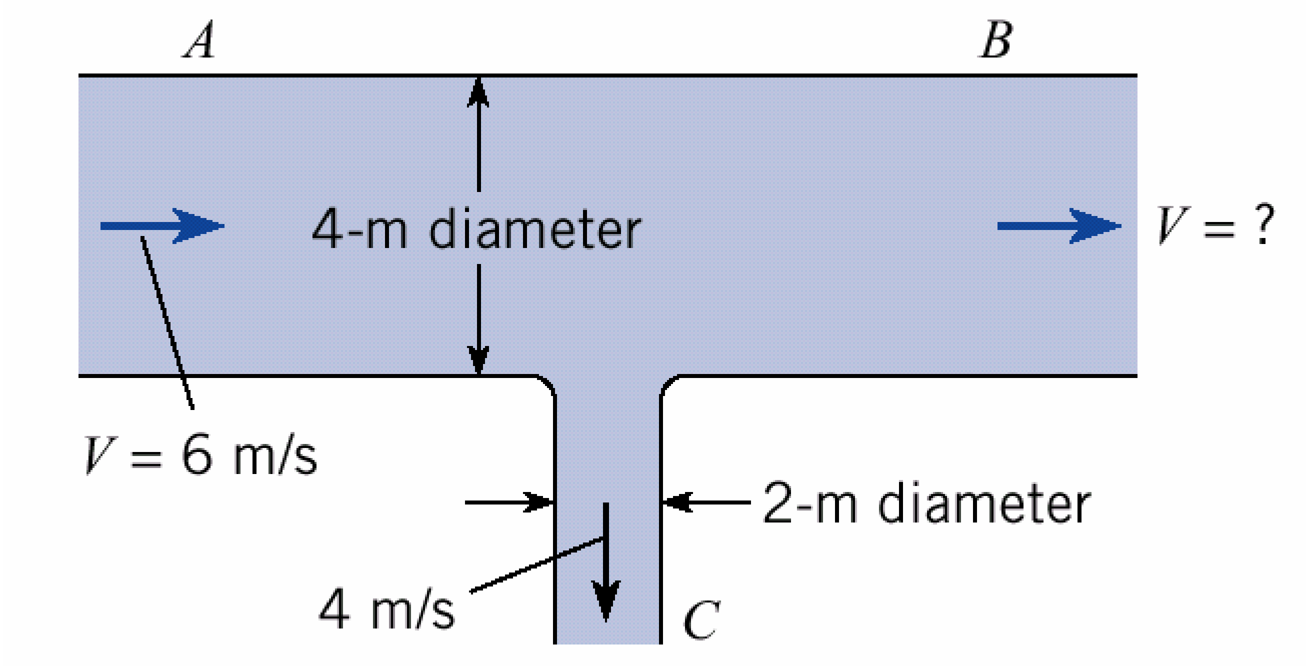
$$0 = \frac{d}{dt} \int_{CV} \rho d\forall + \sum_{CS} \rho \vec{V} \cdot \vec{A}$$

Unsteady Case

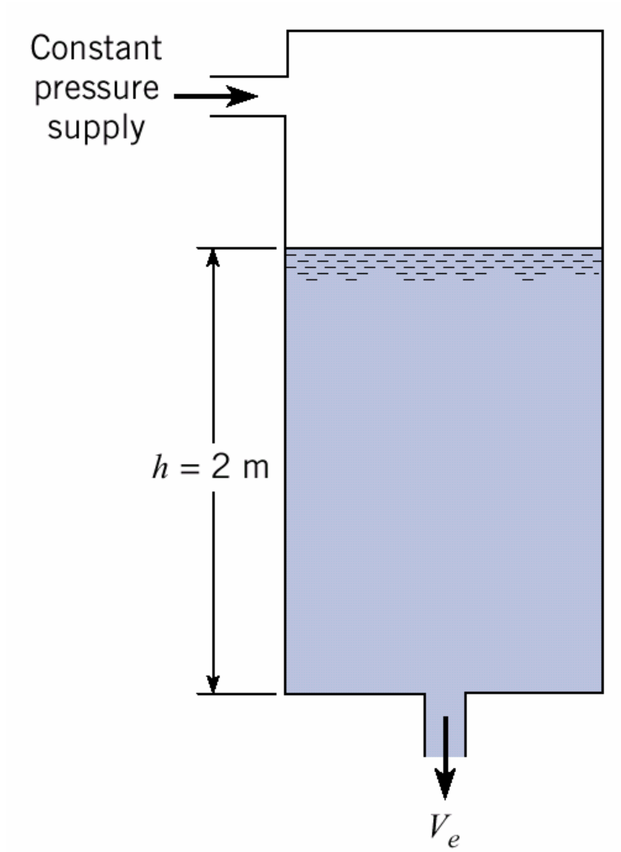
$$0 = \sum_{CS} \rho \vec{V} \cdot \vec{A}$$

Steady Case

HW (4.72)



HW (4.88)



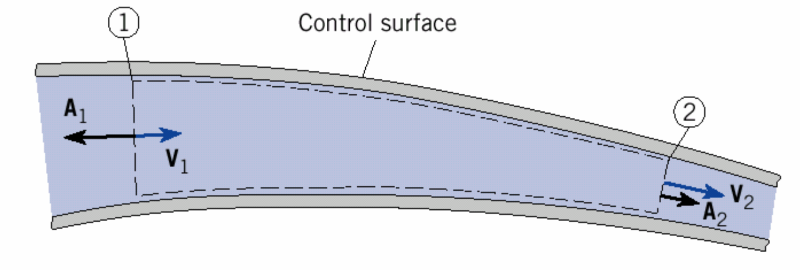
1-D Flow in a Conduit

- Continuity Eq.

$$0 = \frac{d}{dt}\int_{CV} \rho d\forall + \sum_{CS} \rho \vec{\boldsymbol{V}} \cdot \vec{\boldsymbol{A}}$$

- Steady flow

$$0 = \sum_{CS} \rho \vec{\boldsymbol{V}} \cdot \vec{\boldsymbol{A}}$$



- Incompressible fluid

$$0 = -V_1 A_1 + V_2 A_2$$

$$V_1 A_1 = V_2 A_2$$

$$Q_1 = Q_2$$

EX (4.76)

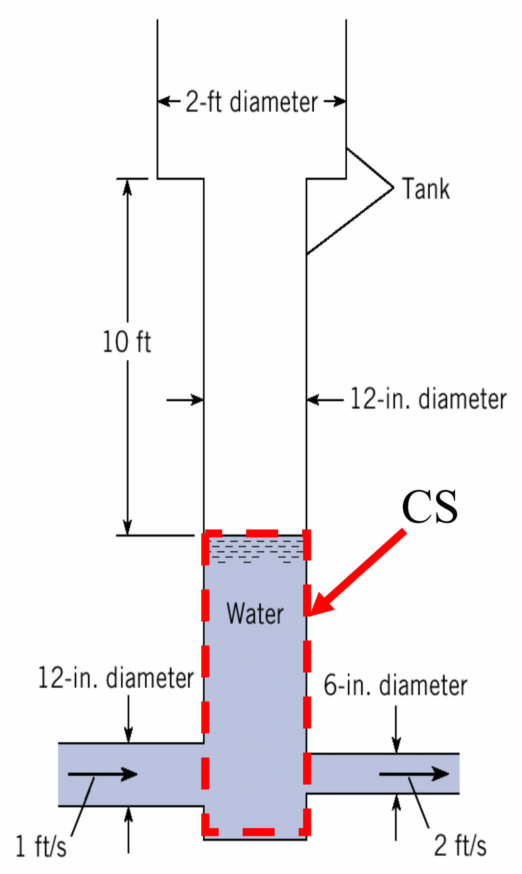
Find: At 22 s, will water surface be rising or falling?
Solution: Define CV, CS moves with water surface. Continuity Eq.

$$0 = \frac{d(Ah)}{dt} - (VA)_{in} + (VA)_{out}$$

$$0 = \frac{d(Ah)}{dt} - \frac{\pi}{4} * 1^2 * 1 + \frac{\pi}{4} * 0.5^2 * 2$$

$$0 = \frac{d(Ah)}{dt} - \frac{\pi}{8}$$

$$A\frac{dh}{dt} = \frac{\pi}{8} > 0$$ Water is rising



EX (4.76)

In the 12-in. section

$$\frac{dh}{dt} = \frac{1}{A}\frac{\pi}{8}$$

$$= \frac{1}{\frac{\pi}{4}*1^2}\frac{\pi}{8}$$

$$= \frac{1}{2} ft/s$$

Time to move up 10 ft in 12-in section

$$10 = \frac{dh}{dt}*t$$

$$= \frac{1}{2}*t$$

$$t = 20s$$

In the 24-in section after 22 s

$$\frac{dh}{dt} = \frac{1}{A}\frac{\pi}{8}$$

$$= \frac{1}{\frac{\pi}{4}*2^2}\frac{\pi}{8}$$

$$= \frac{1}{8} ft/s$$