CE 374 K – Hydrology

Momentum and Energy

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Momentum Equation

Reynolds's Transport Theorem $$\frac{d\vec{\boldsymbol{B}}}{dt} = \frac{d}{dt}\iiint_{CV} \vec{\boldsymbol{\beta}}\rho d\forall + \iint_{CS} \vec{\boldsymbol{\beta}}\rho\vec{\boldsymbol{V}} \cdot d\vec{\boldsymbol{A}}$$

$$\vec{\boldsymbol{B}} = M\vec{\boldsymbol{V}} \quad \text{momentum of the system;} \quad \vec{\boldsymbol{\beta}} = d\vec{\boldsymbol{B}}/dm = \vec{\boldsymbol{V}}$$

Newton's Second Law $$\frac{d\vec{\boldsymbol{B}}}{dt} = \frac{dM\vec{\boldsymbol{V}}}{dt} = \sum \vec{\boldsymbol{F}}$$

$$\sum \vec{\boldsymbol{F}} = \frac{d}{dt}\iiint_{CV} \vec{\boldsymbol{V}}\rho d\forall + \iint_{CS} \vec{\boldsymbol{V}}\rho\vec{\boldsymbol{V}} \cdot d\vec{\boldsymbol{A}}$$ **Unsteady, nonuniform flow**

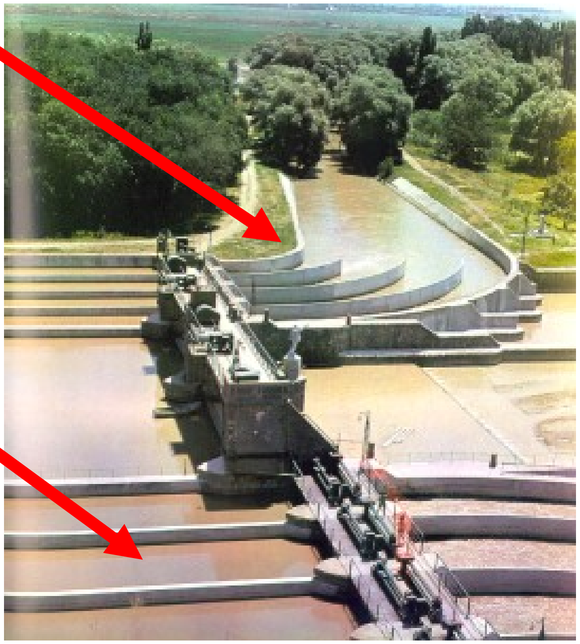
Nonuniform flow – velocity varies in space
Uniform flow - velocity constant in space
Unsteady flow – velocity varies in time
Steady flow – velocity constant in time

Momentum Equation

$$\sum \vec{F} = \frac{d}{dt}\iiint_{CV} \vec{V}\rho d\forall + \iint_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$

$$\sum \vec{F} = \iint_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$ **Steady, nonuniform flow**

$$\sum \vec{F} = 0$$ **Steady, uniform flow**



Energy Equation

Internal E_u

Kinetic $\frac{MV^2}{2}$

Potential Mgz

Reynolds Transport Equation

$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{CV} \beta\rho d\forall + \iint_{CS} \beta\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

Energy of the System

Extensive

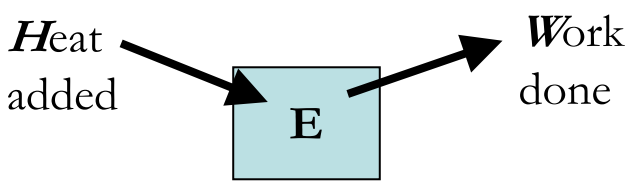
$$B = E = E_u + \frac{1}{2}MV^2 + Mgz$$

Intensive

$$\beta = dB/dm = e_u + \frac{V^2}{2} + gz$$

First Law

$$\frac{dB}{dt} = \frac{dE}{dt} = \frac{dH}{dt} - \frac{dW}{dt}$$



Combining

$$\frac{dH}{dt} - \frac{dW}{dt} = \frac{d}{dt}\iiint_{CV} \left(\frac{V^2}{2} + e_u + gz\right)\rho d\forall + \iint_{CS} \left(\frac{p}{\rho} + \frac{V^2}{2} + e_u + gz\right)\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_f = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

Internal Energy

- Sensible Heat – related to temperature

$$de_u = C_p dT$$

Specific heat C_p

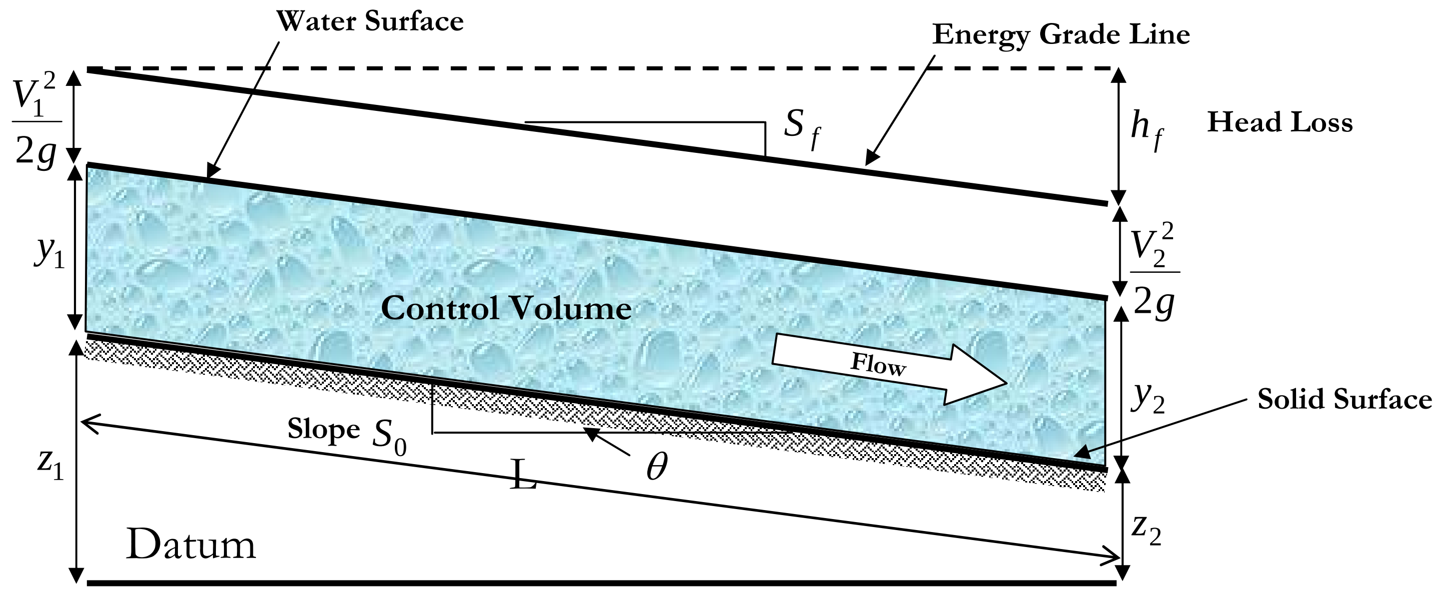
- Latent Heat – related to phase changes
  - Fusion/Melting
    - ice – water, 0.33×10^6 J/kg
  - Vaporization/Condensation
    - water – water vapor, 4.2×10^3 J/kg
  - Sublimation
    - ice – water vapor, 2.5×10^6 J/kg
  - Main internal energy change in hydrology

Steady Uniform Flow in an Open Channel

- **Continuity** $\iint_{CS} \vec{V} \cdot d\vec{A} = 0 \quad \longrightarrow$

Steady flow	**Uniform flow**
$Q_1 = Q_2;$	$V_1 = V_2$
$A_1 = A_2;$	$y_1 = y_2$

Uniform channel



Steady Uniform Flow in an Open Channel

- **Energy**

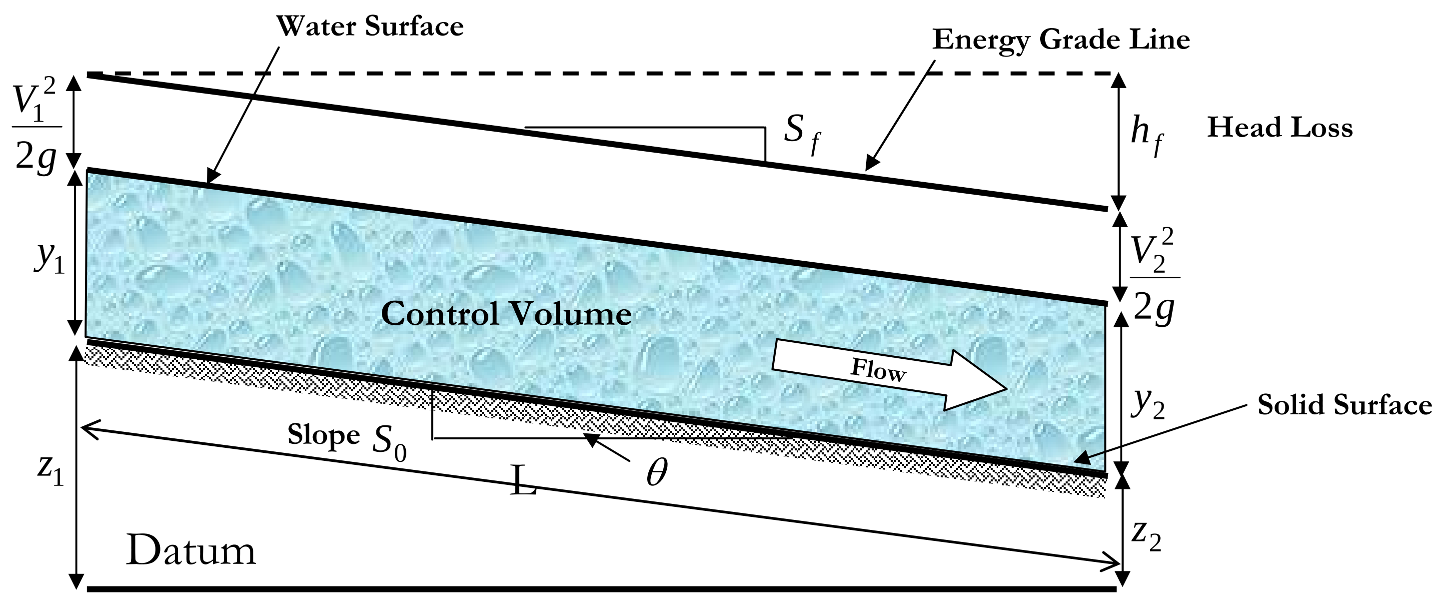
$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_f = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$V_1 = V_2$$

$$y_1 = y_2$$

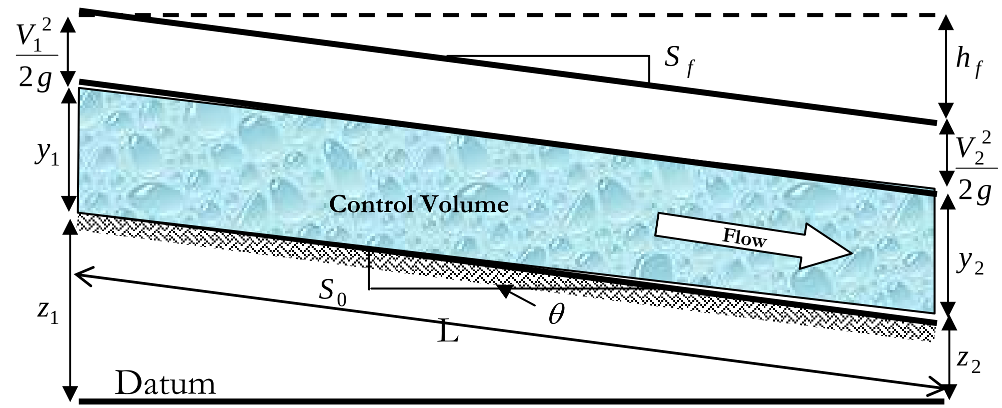
$$z_1 - z_2 = h_f$$

$$\frac{h_f}{L} = \frac{z_1 - z_2}{L} = S_f = S_0$$



Flow in an Open Channel

Steady, uniform flow



- **Momentum** $\sum \vec{\mathbf{F}} = 0$
- **3 forces on CV:**
  - **Pressure: cancels**
  - **Friction:** $\vec{\mathbf{F}}_f = -\tau_0 (PL)$
  - **Gravity:** $\vec{\mathbf{F}}_g = \gamma AL \sin\theta = \gamma ALS_f$
  - **Sum:** $\sum \vec{\mathbf{F}} = 0 = -\tau_0 (PL) + \gamma ALS_f$

$$S_0 = S_f$$

$$R = \frac{A}{P}$$

$$\tau_0 = \gamma RS_f$$

Open Channel Flow

- **Darcy – Weisbach Equation: head loss due to wall friction**

$$h_f = f\,\frac{L}{D}\frac{V^2}{2g} \qquad S_f = \frac{h_f}{L} \qquad D = 4R = 4\frac{A}{P}$$

- **Chezy's Equation for open channel flow**

$$V = C\sqrt{RS_f} \qquad C = \sqrt{\frac{8g}{f}}$$

- **Manning's Equation for open channel flow**

$$\boxed{V = \frac{1}{n}R^{2/3}S_f^{1/2}} \qquad C = \frac{1}{n}R^{1/6}$$

Manning's Equation

- **Manning's Equation for open channel flow**

$$V = \frac{1}{n} R^{2/3} S_f^{1/2} \qquad V = \frac{1.49}{n} R^{2/3} S_f^{1/2} \qquad R = \frac{A}{P}$$

- **Valid for fully turbulent flow**

$$n^6 \sqrt{RS_f} \geq 1.1x10^{-13}$$

$$\text{As } n \uparrow, V \downarrow$$

- **Laminar flow: use Chezy with *f* from Moody diagram**

Manning, Robert, "On the Flow of Water in Open Channels and Pipes,"
Transactions of the Institution of Civil Engineers of Ireland, 1891

Manning’s n

Material	**Manning *n***	**Material**	**Manning *n***
Natural Streams		***Excavated Earth Channels***	
Clean and Straight	**0.030**	**Clean**	**0.022**
Major Rivers	**0.035**	**Gravelly**	**0.025**
Sluggish with Deep Pools	**0.040**	**Weedy**	**0.030**
		Stony, Cobbles	**0.035**
Floodplains	**0.035**	***Non-Metals***	
Pasture, Farmland	**0.050**	**Finished Concrete**	**0.012**
Light Brush	**0.075**	**Unfinished Concrete**	**0.014**
Heavy Brush	**0.15**	**Gravel**	**0.029**
Trees		**Earth**	**0.025**

Ethics Question

- (http://www.lmnoeng.com/manningn.htm)
- *Is it ethical to use an engineering software program to solve a problem if you cannot complete the calculations manually?*

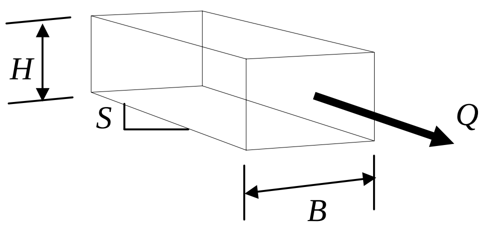
Manning's *n*

- Hydraulic computations related to discharge require an evaluation of the roughness of the channel.
- This is an art developed through experience.
- The appearance of some typical channels whose roughness coefficients are known can be studied on the web page:

wwwrcamnl.wr.usgs.gov/sws/fieldmethods/Indirects/nvalues/

Example

- Manning's equation: Steady, uniform flow in an open channel. Find velocity and flow rate



Given:

H = 5 ft

S = 0.03 %

B =200 ft; and

n = 0.015

$$P = B + 2xH = 200 + 2x5 = 210\ ft$$

$$R = \frac{A}{P} = \frac{200x5}{210} = 4.76\ ft$$

$$V = \frac{1.49}{n} R^{2/3} S_f^{1/2}$$
$$= \frac{1.49}{0.015} (4.76)^{2/3} (0.0003)^{1/2}$$
$$= 4.87\ ft/s$$

$$Q = VA = 4.87x200x5 = 4870\ ft^3/s$$

You can check that flow is, indeed, turbulent