CE 374 K – Hydrology

Infiltration

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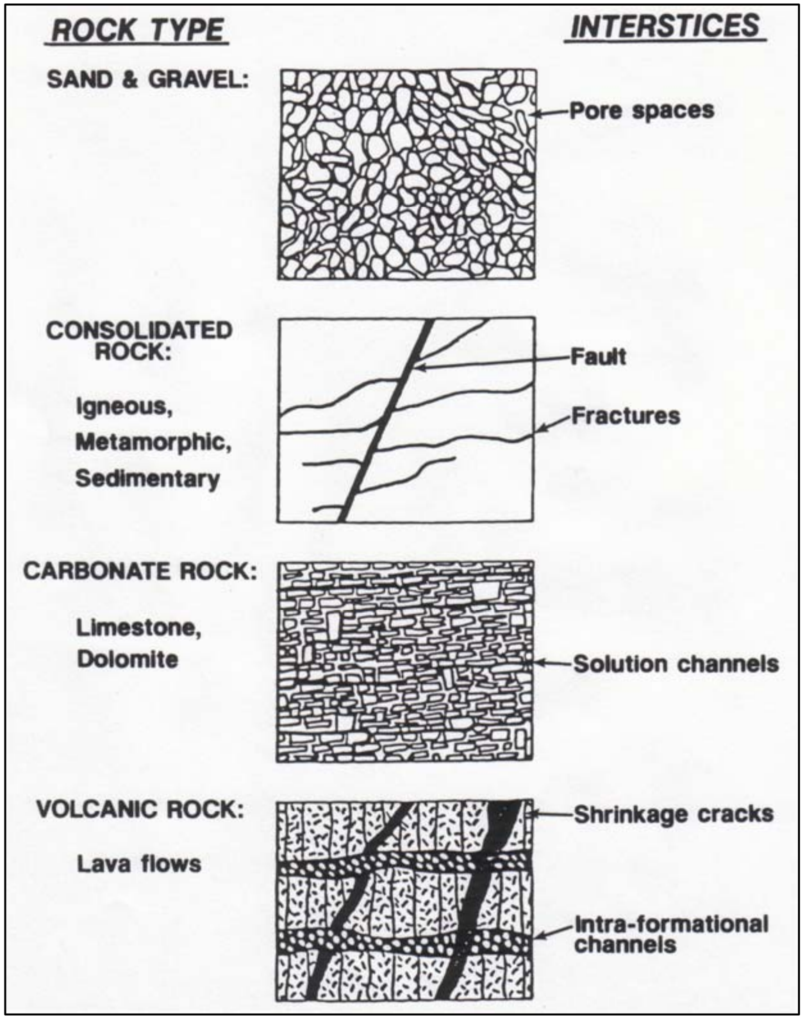
Porous Medium

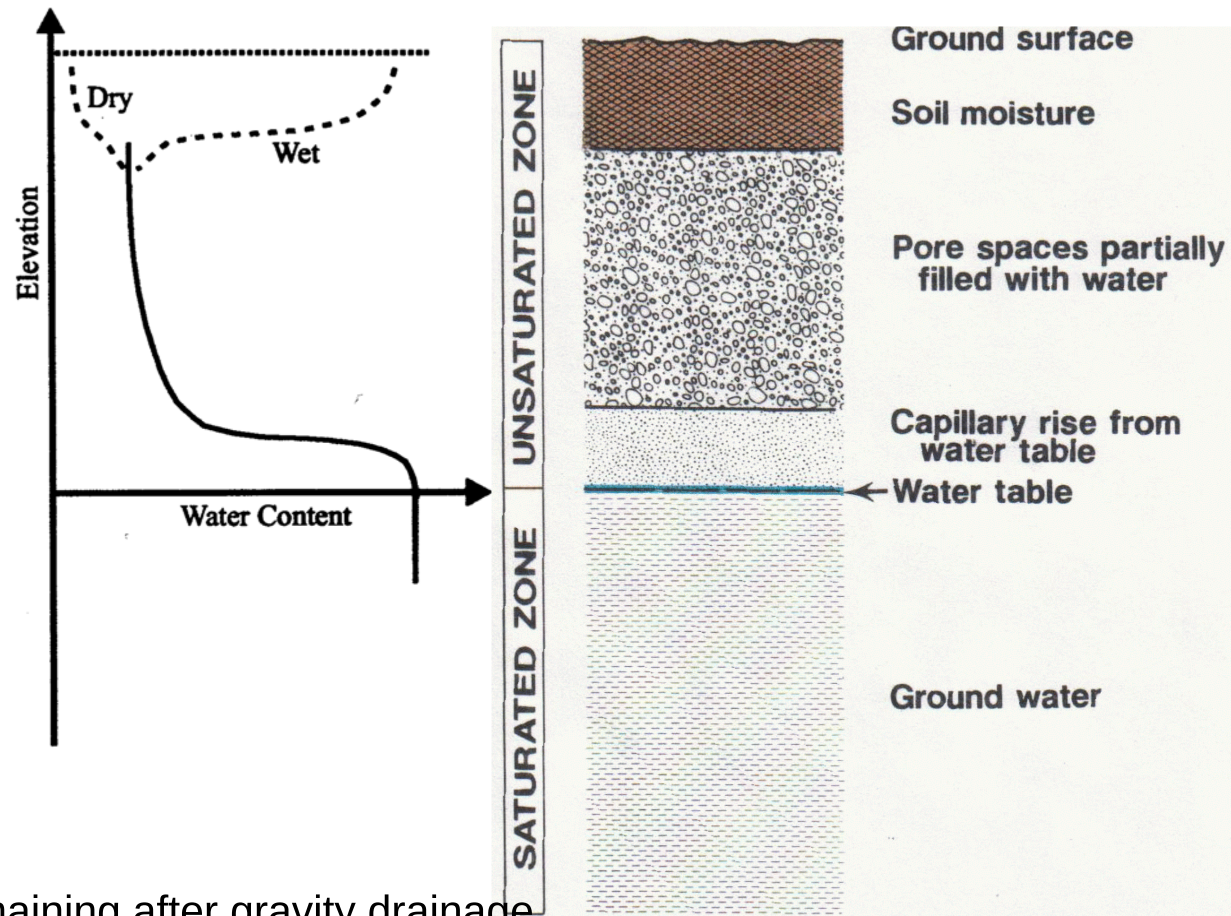
- **Groundwater**
  - All waters found beneath the ground surface
  - Occupies pores (void space space not occupied by solid matter)
- **Porous media**
  - Numerous pores of small size
  - Pores contain fluids (e.g., water and air)
  - Pores act as conduits for flow of fluids
- **Type of rocks in a formation and their**
  - Number, size, and arrangement of pores
  - Affect the storage and flow through a formation.
- **Pores shapes are irregular because**
  - differences in the minerals making up the rocks
  - geologic processes experienced by them.



Distribution of Subsurface Water

Moisture Profile Soil Profile Description

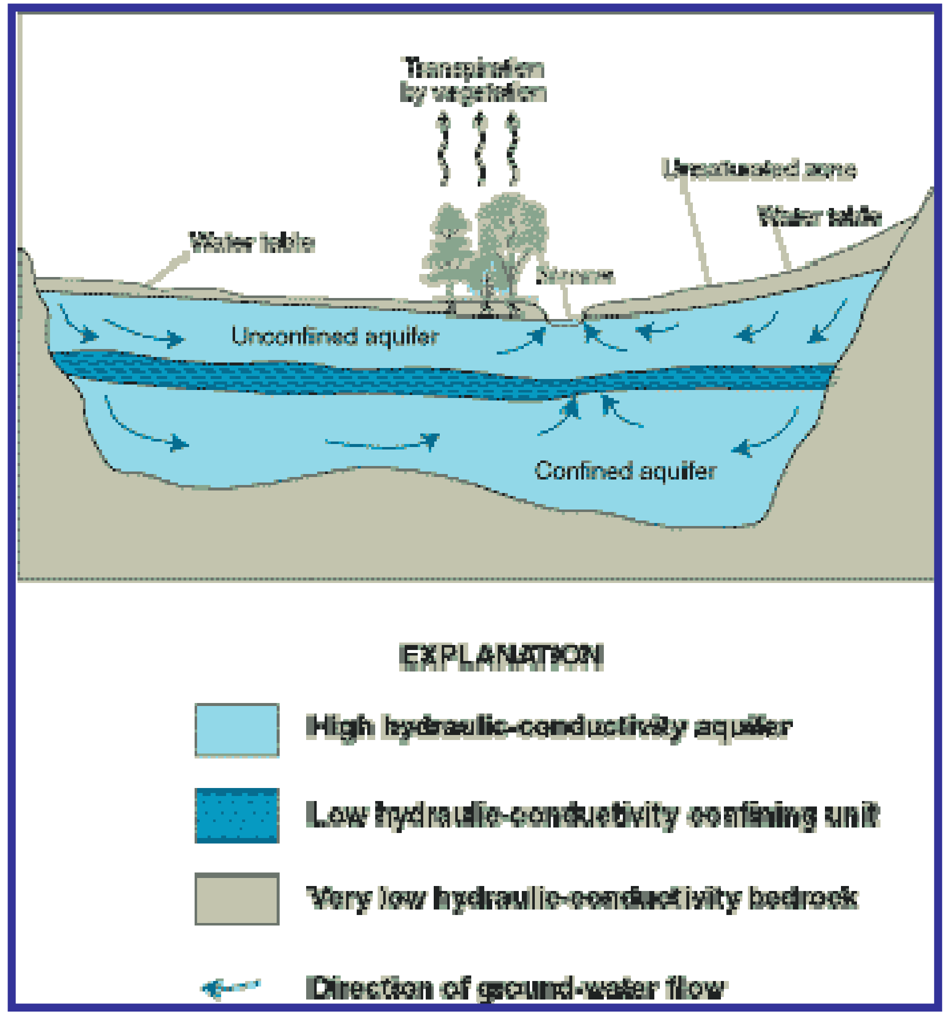
- **Different zones**
  - depend on % of pore space filled with water
- **Unsaturated Zone**
  - Water held by capillary forces, water content near field capacity except during infiltration
- **Soil zone**
  - Water moves down (up) during infiltration (evaporation)
- **Capillary fringe**
  - Saturated ar base
  - Field capacity at top
- **Saturated Zone**
  - Fully saturated pores



Field capacity - Water remaining after gravity drainage
Wilting point - Water remaining after gravity drainage & evapotranspiration

Aquifer Types

- **Aquifer** - store & transmit
  - Unconsolidated deposits sand and gravel, sandstones etc.
- **Aquiclude** – store, don’t transmit
  - Clays and less shale
  - Impervious boundaries of aquifers
- **Aquitard** – transmit don’t store
  - Shales and less clay
  - Leaky confining layers of aquifers
- **Confined aquifer**
  - Under pressure
  - Bounded by impervious layers
- **Unconfined aquifer**
  - Phreatic aquifer, water table aquifer
  - Bounded above by water table
  - Atmospheric pressure at water table



Porosity

Porosity – total volume that can be filled with water

Moisture content – actual volume filled with water

Saturation – fraction of pore volume filled with water

$$\text{Porosity} \qquad \phi = \frac{V_i}{V}; \qquad 0 \le \phi \le 1$$

$$\text{Moisture content} \qquad \theta = \frac{V_w}{V}; \qquad 0 \le \theta \le n$$

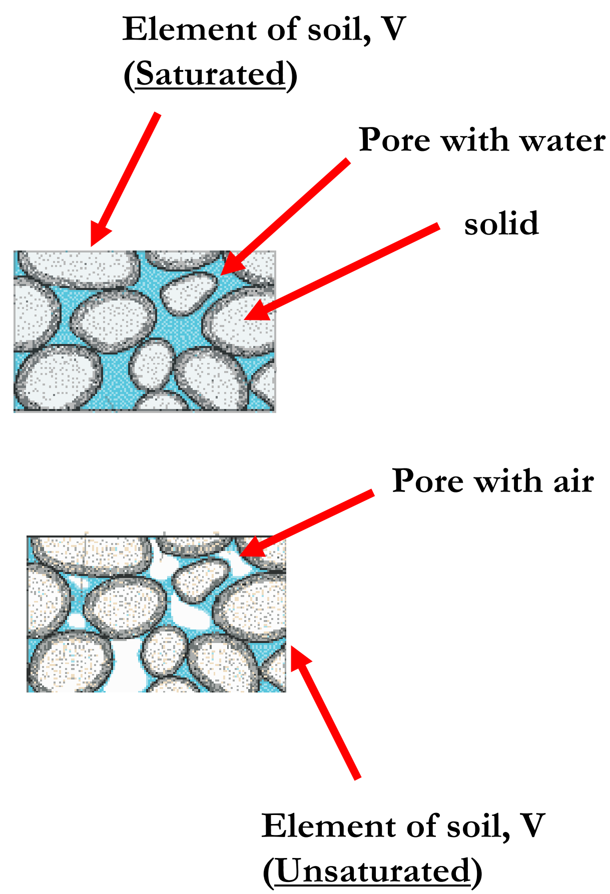
$$\text{Saturation} \qquad S = \frac{V_w}{V_i} = \frac{\theta}{\phi}; \qquad 0 \le S \le 1$$

$V =$ gross volume of element

$V_i =$ volume of pores

$V_s =$ volume of solids

$V_w =$ volume of water



Aquifer Properties

- **<u>Porosity</u>** (*n*)
  - Percent of total pore space occupied by voids
- **<u>Hydraulic conductivity</u>** (*K*)
  - Ability of a formation to transmit water
- **<u>Storativity</u>** (*S*)
  - Ability of a formation to store water

Sedimentary Material	Porosity (%)
Peat Soil	60-80
Soils	50-60
Clay	45-55
Silt	40-50
Med. to Coarse Sand	35-40
Uniform Sand	30-40
Fine to Med Sand	30-35
Gravel	30-40
Gravel and Sand	30-35
Sandstone	10-20
Shale	1-10
Limestone	1-10

Piezometric Head

- Pressure distribution

$$\frac{\partial p}{\partial z} = -\gamma$$

- Piezometric head - energy per unit weight of the fluid

$$h = \frac{p}{\gamma} + z$$

Elevation head

Pressure head

$h =$ piezometric head

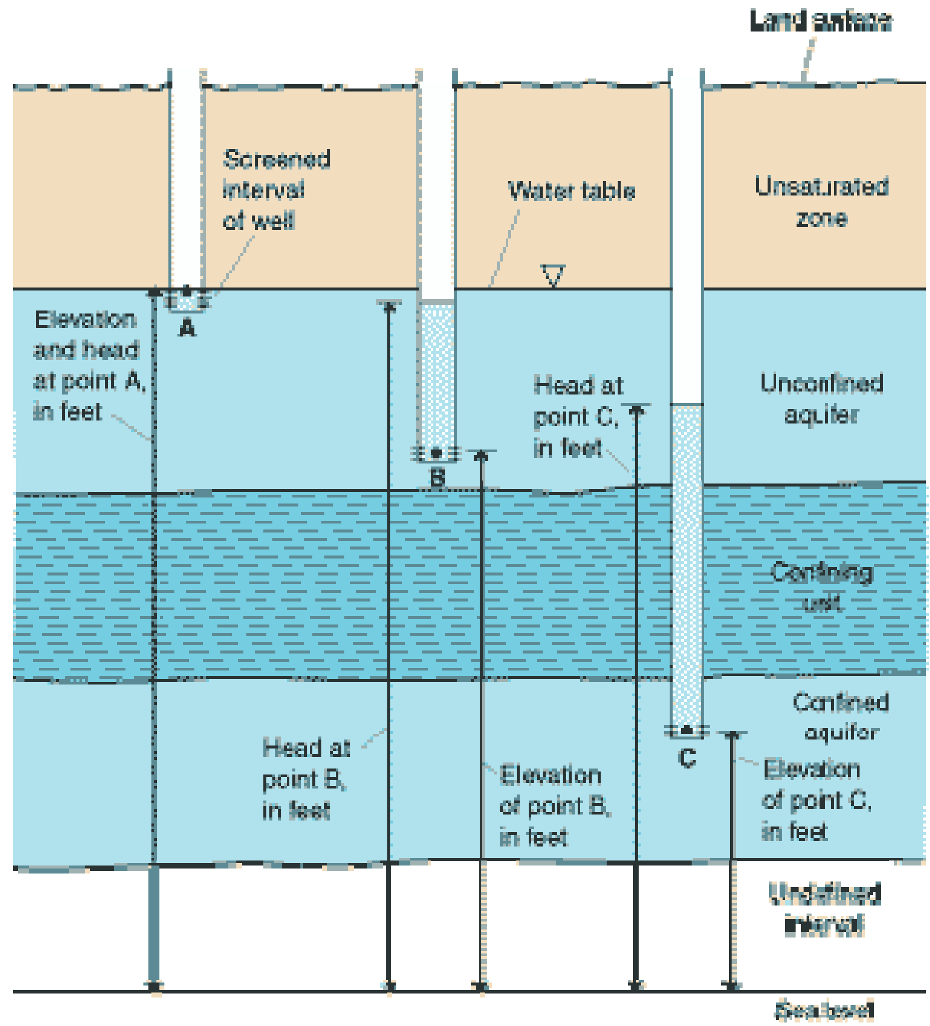
$p =$ fluid pressure

$\gamma = \rho g =$ fluid specific weight

$\rho =$ fluid density

$g =$ gravitational constant

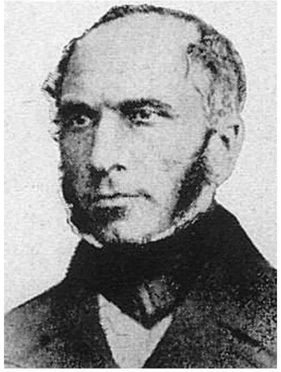
$z =$ elevation



Dijon Fountain

Darcy’s Law

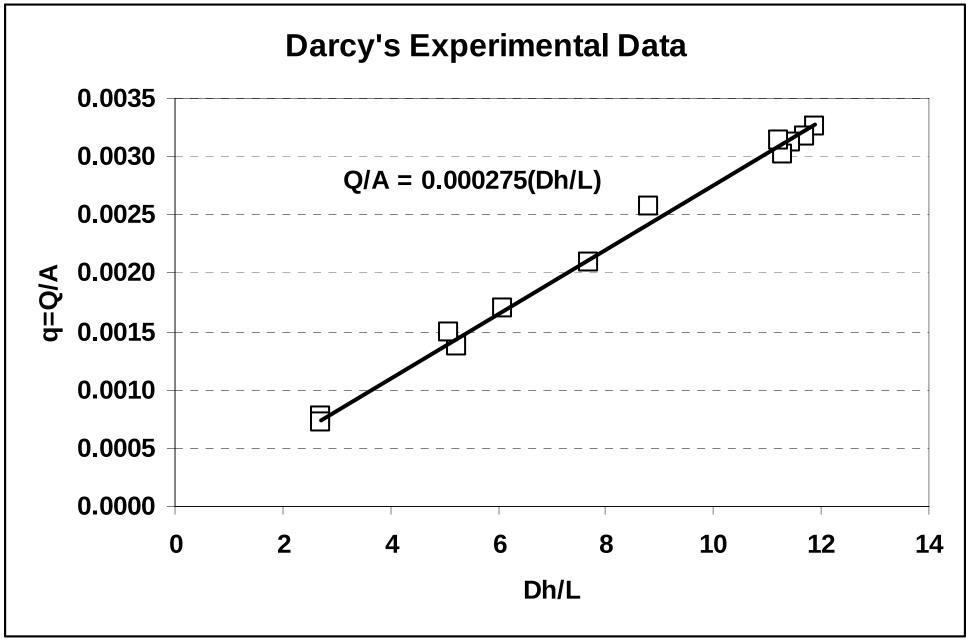
- Output is proportional to the head and inversely related to the length traversed

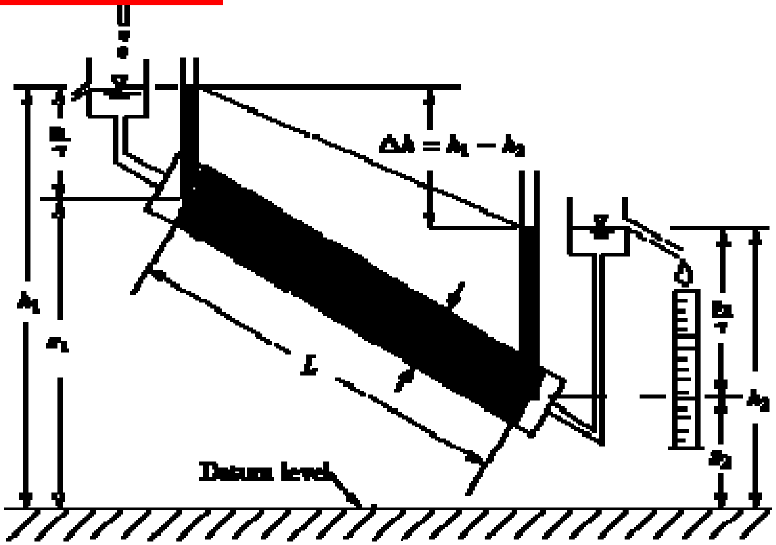


Henry Darcy

$$Q \propto A\frac{h_1 - h_2}{L}$$

$$\boxed{q = \frac{Q}{A} = -K\frac{dh}{dz}}$$





K = hydraulic conductivity

q = Q/A = specific discharge

Continuity Equation

$$V = dxdydz = \text{volume of element}$$

$$V_w = \theta dxdydz = \text{volume of water}$$

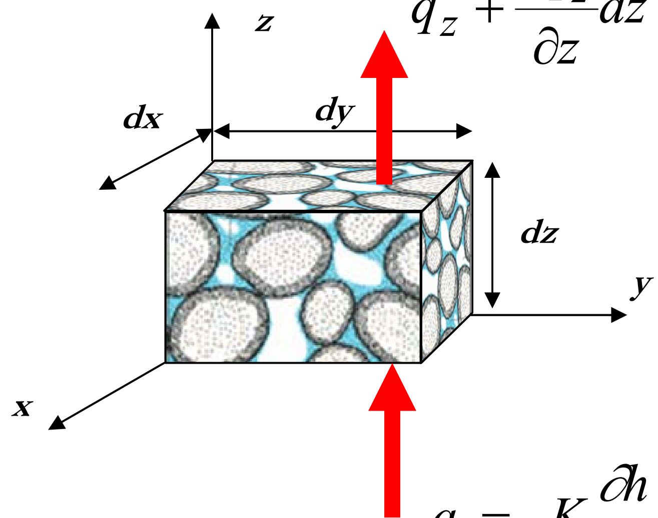
$$\frac{d}{dt}\iiint_{CV}\rho_w d\forall + \iint_{CS}\rho_w \mathbf{V}\cdot\mathbf{dA} = 0$$

$$\rho_w dxdydz\frac{d\theta}{dt} + \rho_w dzdxdy\frac{\partial q}{\partial z} = 0$$

$$\boxed{\frac{\partial\theta}{\partial t} + \frac{\partial q}{\partial z} = 0}$$

Continuity Equation

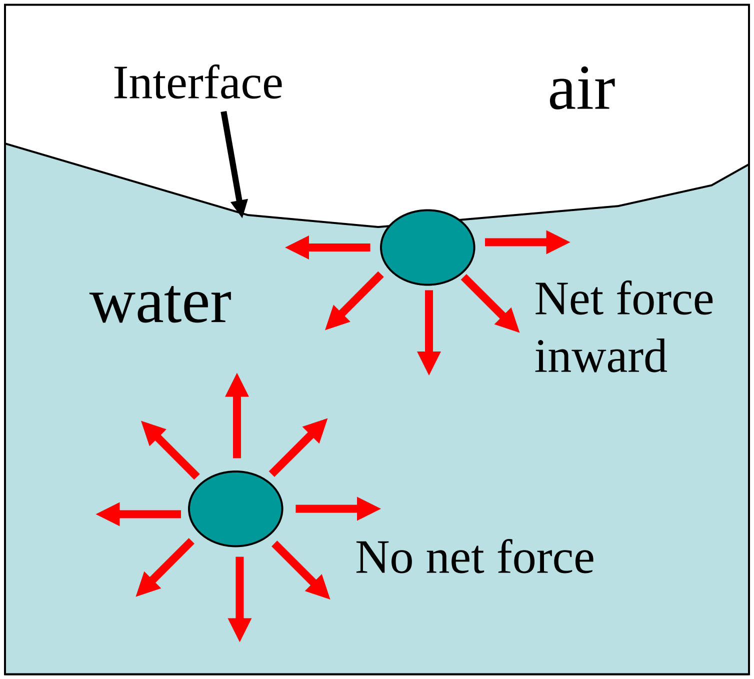
$$q_z + \frac{\partial q_z}{\partial z}dz$$



$$q_z = -K\frac{\partial h}{\partial z}$$

Surface Tension

- Below interface
  - forces act equally in all directions
- At interface
  - some forces are missing
  - pulls molecules down and together
  - like membrane exerting *tension* on the *surface*
- If interface is curved
  - higher pressure will exist on concave side
- Pressure increase
  - balanced by surface tension, σ
- σ = 0.073 N/m (@ 20^{o}C)



Capillary Action

- Capillary Pressure
  - Related to surface tension and size of pores

$$P_c = \frac{2\sigma}{R} = P_{air} - P$$

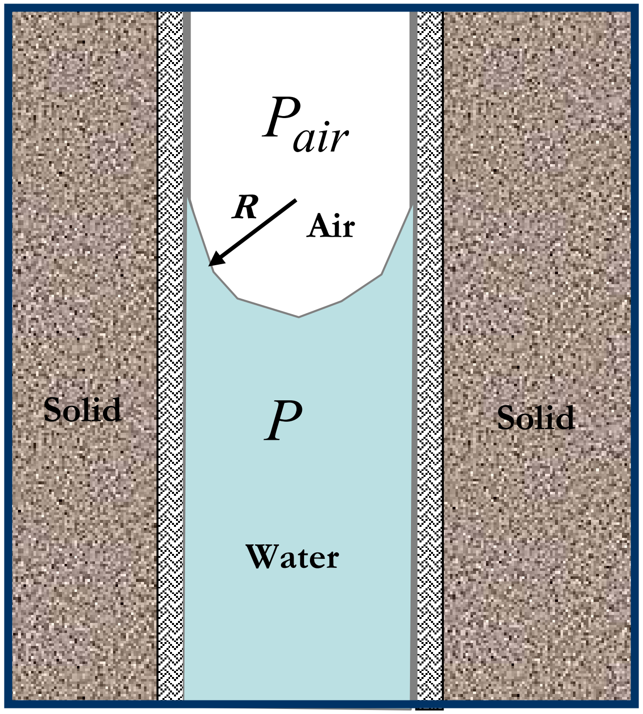
$$P_{air} = 0$$

$$P_c = -P$$

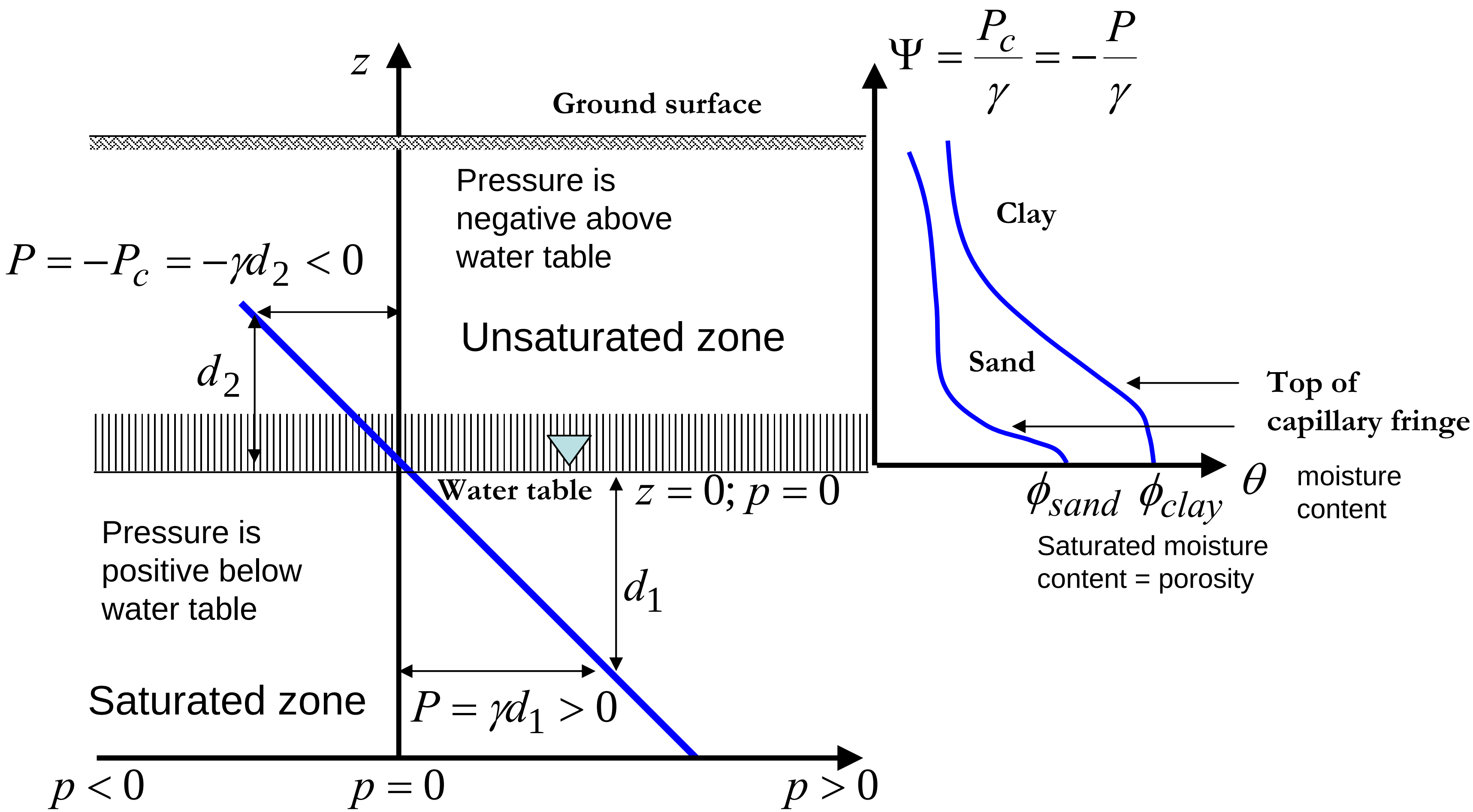
  - Related to water content

$$P_c = P_c(\theta)$$

- Suction Head $\Psi = \frac{P_c}{\gamma}$
- Total Head $h = \Psi + z$



Pressure Distribution in Subsurface



Richard's Equation

- Darcy's Law becomes

$$h = \Psi + z$$

$$q_z = -K\frac{\partial h}{\partial z} = -K\frac{\partial(\Psi + z)}{\partial z} = -\left(K\frac{\partial \Psi}{\partial \theta}\frac{\partial \theta}{\partial z} + K\right)$$

$$= -\left(D\frac{\partial \theta}{\partial z} + K\right)$$

$$D = K\frac{\partial \Psi}{\partial \theta}$$

Soil water diffusivity

- Continuity becomes

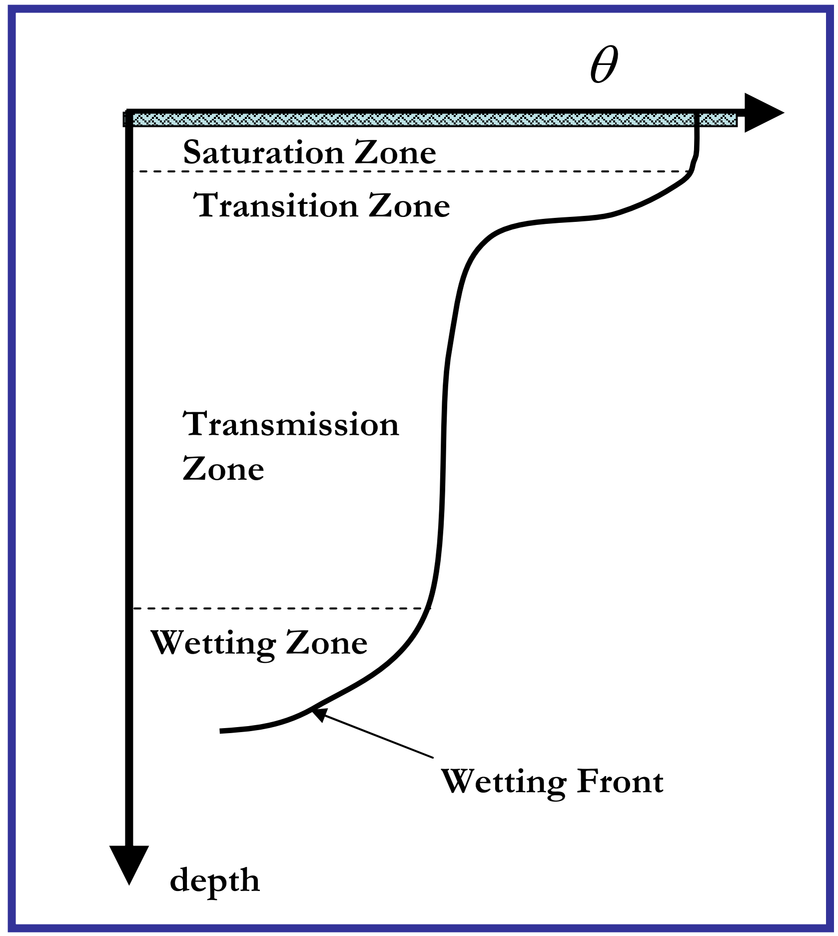
$$\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0$$

$$\boxed{\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z} + K\right)}$$

Richard's Equation

Infiltration

- General
  - Process of water penetrating from ground into soil
  - Factors affecting
    - Condition of soil surface, vegetative cover, soil properties, hydraulic conductivity, antecedent soil moisture
  - Four zones
    - Saturated zone
    - Transmission zone
    - Wetting zone
    - Wetting front



Infiltration

- Infiltration rate
  - Rate at which water enters the soil at the surface

$$f(t)$$

- Cumulative infiltration
  - Accumulated depth of water infiltrating during given time period

$$\boxed{F(t) = \int_0^t f(\tau)d\tau}$$

$$\boxed{f(t) = \frac{dF(t)}{dt}}$$

Infiltration Methods

- Horton and Phillips
  - Infiltration models developed from
  - approximate solutions to exact theory (Richard's Equation)
- Green – Ampt
  - Infiltration model developed from
  - exact solution to approximate theory

Hortonian Infiltration

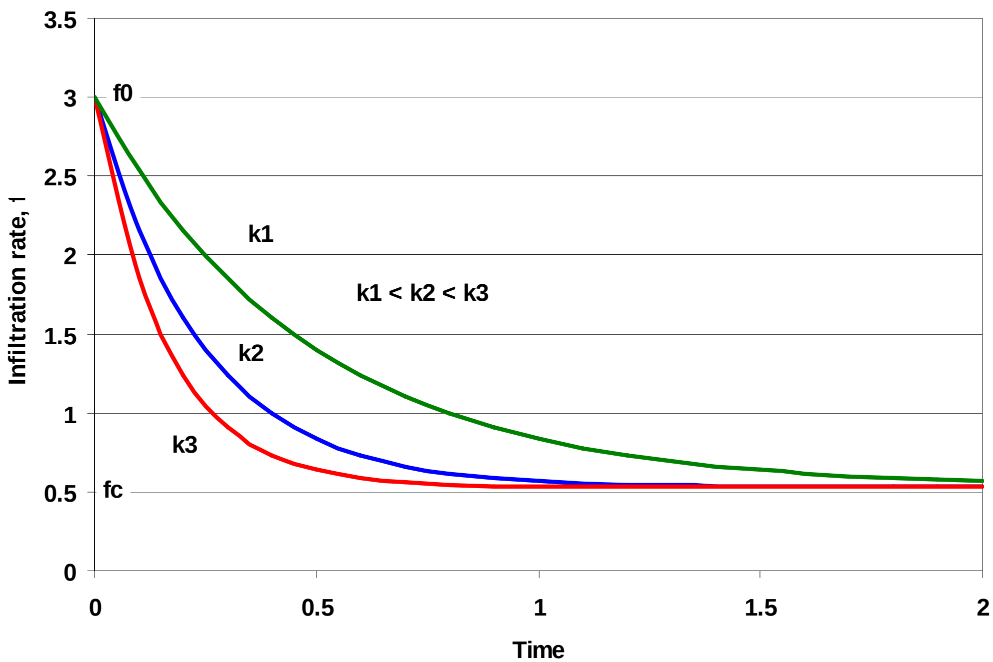
- Recall Richard's Equation
  - Assume K and D are constants, not a function of θ or *z*
- Solve for moisture diffusion at surface

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z} + K \right)$$

$$\frac{\partial \theta}{\partial t} = D \frac{\partial^2 \theta}{\partial z^2} + \cancel{\frac{\partial K}{\partial z}}$$

$$0$$

$$\frac{\partial \theta}{\partial t} = D \frac{\partial^2 \theta}{\partial z^2}$$



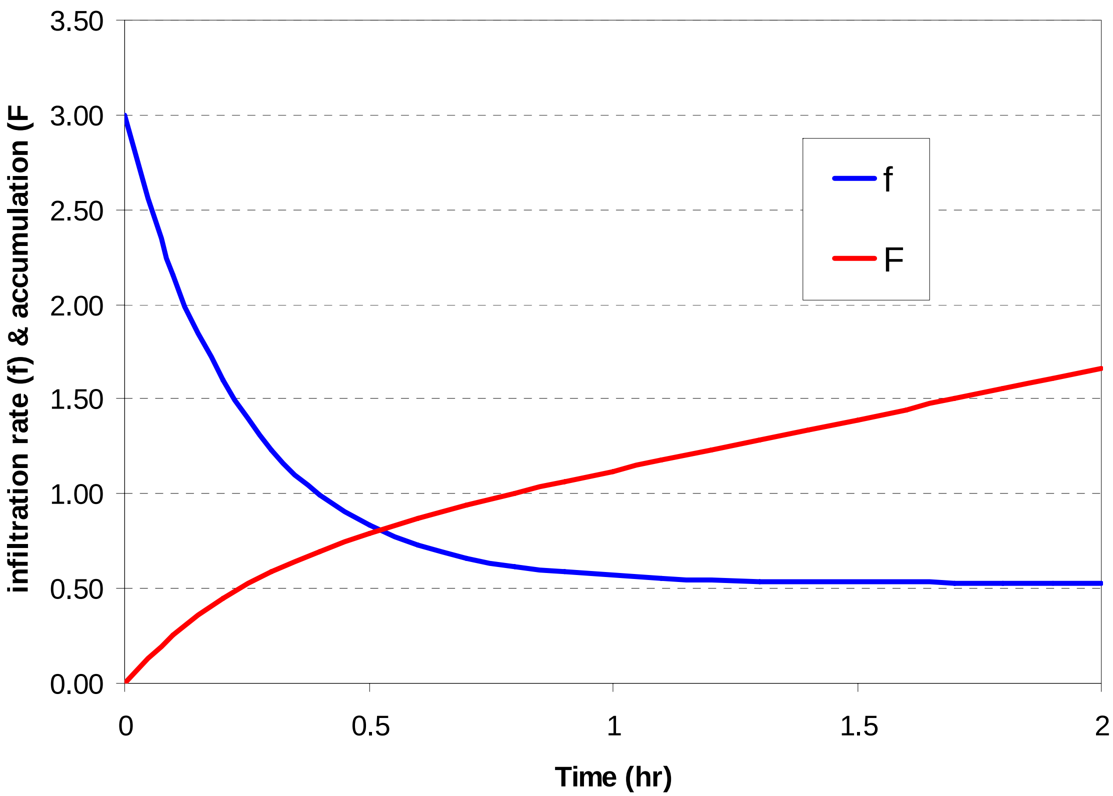
$$\boxed{f(t) = f_c + (f_0 - f_c)e^{-kt}}$$

Horton's Equation

Trapezoid Rule

$$F(t) = \int_0^t f(\tau)\,d\tau$$

$$F(t_n) = \int_0^{t_n} f(\tau)d\tau \approx \frac{\Delta t}{2}(f_0 + 2f_1 + \cdots + 2f_{n\text{-}1} + f_n) = \frac{\Delta t}{2}(f_0 + 2\sum_{i=1}^{n-1} f_i + f_n)$$



t	f	F
0	3.00	0.00
0.1	2.16	0.26
0.2	1.60	0.45
0.3	1.23	0.59
0.4	0.99	0.70
0.5	0.84	0.79
~	~	~
1.8	0.53	1.55
1.9	0.53	1.61
2	0.53	1.66

Philip's Equation

- Recall Richard's Equation
  - Assume K and D are functions of θ, not *z*
- Solution
  - Two terms represent effects of
    - Suction head
    - Gravity head
- S – Sorptivity
  - Found from experiment

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z} + K \right)$$

$$\boxed{f(t) = \frac{1}{2} S \sqrt{t} + Kt}$$

Philip's Equation

Green – Ampt Infiltration

L = Depth to Wetting Front

θ_i = Initial Soil Moisture

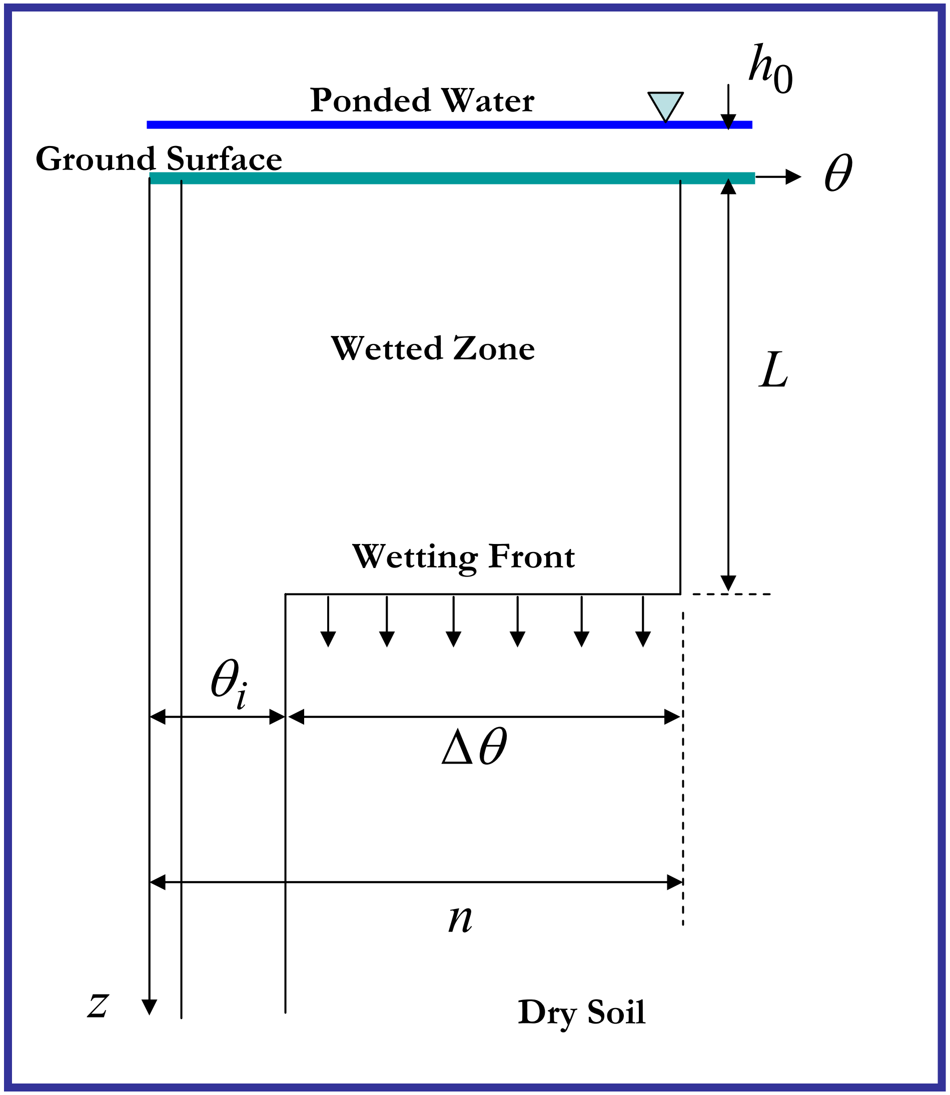
$$F(t) = L(n - \vartheta_i) = L\Delta\theta$$

$$f = \frac{dF}{dt} = \Delta\theta \frac{dL}{dt}$$

$$q_z = -K\frac{\partial h}{\partial z} = -f$$

$$h = \Psi + z$$

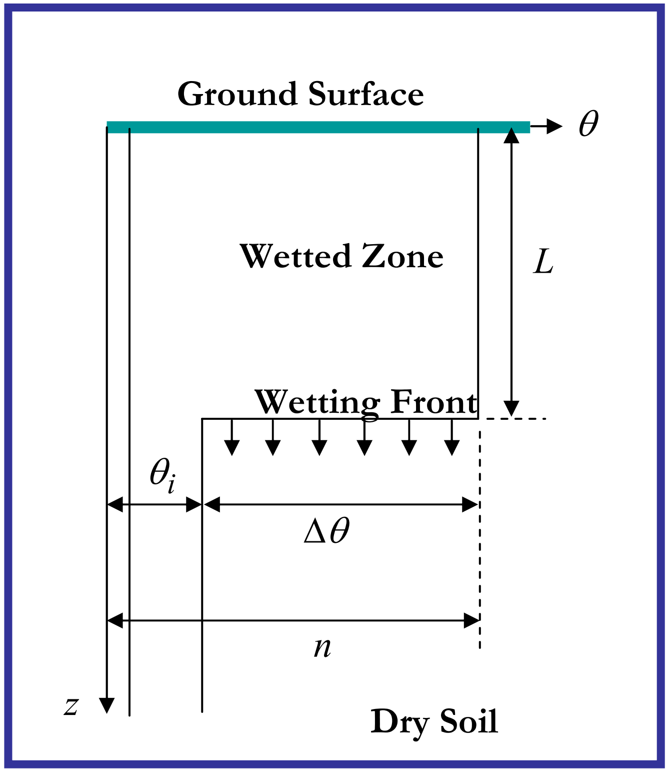
$$f = K\frac{\partial \psi}{\partial z} + K$$



Green – Ampt Infiltration (Cont.)

$$f = K\frac{\partial \psi}{\partial z} + K$$

- Apply finite difference to the derivative, between
  - Ground surface $\quad z = 0, \psi = 0$
  - Wetting front $\quad z = L, \psi = \psi_f$



$$f = K\frac{\partial \psi}{\partial z} + K = K\frac{\Delta \psi}{\Delta z} + K = K\frac{\psi_f - 0}{L - 0} + K$$

$$F(t) = L\Delta\theta$$

$$L = \frac{\Delta\theta}{F}$$

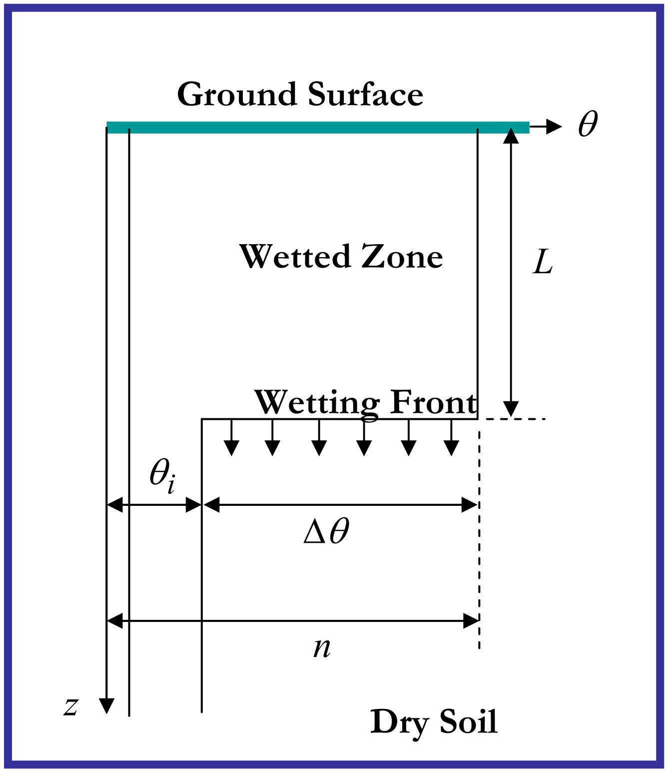
$$\boxed{f = K\left(\frac{\Delta\theta\psi_f}{F} + 1\right)}$$

Green – Ampt Infiltration (Cont.)

$$f = K\left(\frac{\Delta\theta\psi_f}{F} + 1\right) \rightarrow f = \Delta\theta\frac{dL}{dt}$$

$$F(t) = L\Delta\theta$$

$$\Delta\theta\frac{dL}{dt} = K\left(\frac{\psi_f}{L} + 1\right)$$



$$\frac{K}{\Delta\theta}dt = dL - \frac{\psi_f dL}{\psi_f + L}$$

Integrate

$$\frac{K}{\Delta\theta}t = L - \psi_f \ln(\psi_f + L) + C$$

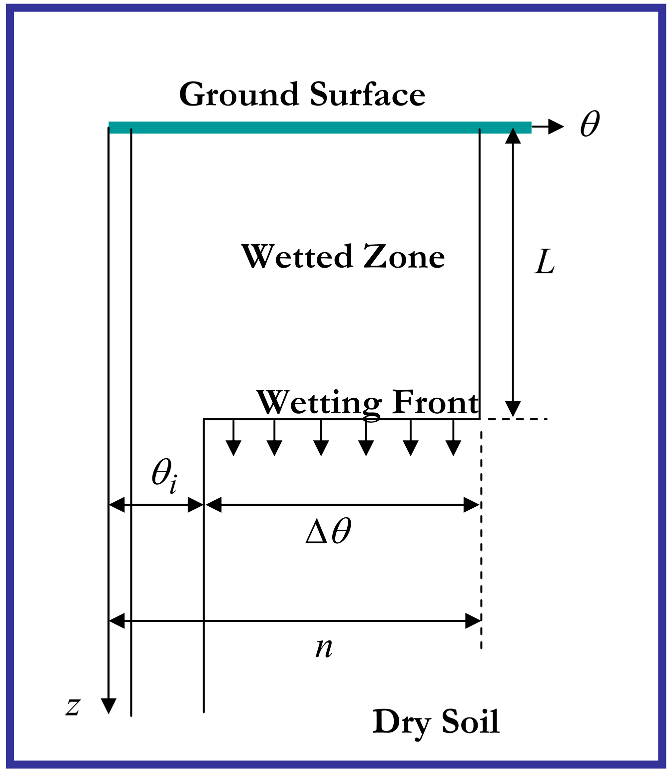
Evaluate the constant of integration

$$L = 0 \; @ \; t = 0$$

$$C = \psi_f \ln(\psi_f + L)$$

$$\boxed{Kt = L\Delta\theta - \Delta\theta\psi_f \ln\left(\frac{\psi_f}{\psi_f + L}\right)}$$

Green – Ampt Infiltration (Cont.)



$$Kt = L\Delta\theta - \Delta\theta\psi_f \ln\left(\frac{\psi_f}{\psi_f + L}\right)$$

$$\boxed{F = Kt + \Delta\theta\psi_f \ln\left(1 + \frac{F}{\Delta\theta\psi_f}\right)}$$

$$\boxed{f = K\left(\frac{\Delta\theta\psi_f}{F} + 1\right)}$$

Nonlinear equation, requiring iterative solution.

See: http://www.ce.utexas.edu/prof/mckinney/ce311k/Lab/Lab8/Lab8.html
And: http://www.ce.utexas.edu/prof/mckinney/ce311k/homework/Solutions-F06/Lab8.pdf

Soil Parameters

- Green-Ampt model requires
  - Hydraulic conductivity
  - Porosity
  - Wetting Front Suction Head

Soil Class	**Porosity**	**Effective Porosity**	**Wetting Front Suction Head**	**Hydraulic Conductivity**
	n	θ_e	ψ	K
			(cm)	(cm/h)
Sand	0.437	0.417	4.95	11.78
Loam	0.463	0.434	9.89	0.34
Clay	0.475	0.385	31.63	0.03

$$\theta_e = n - \theta_r \qquad \Delta\theta = (1 - s_e)\theta_e \qquad s_e = \frac{\theta - \theta_r}{\theta_e}$$

Ponding Time

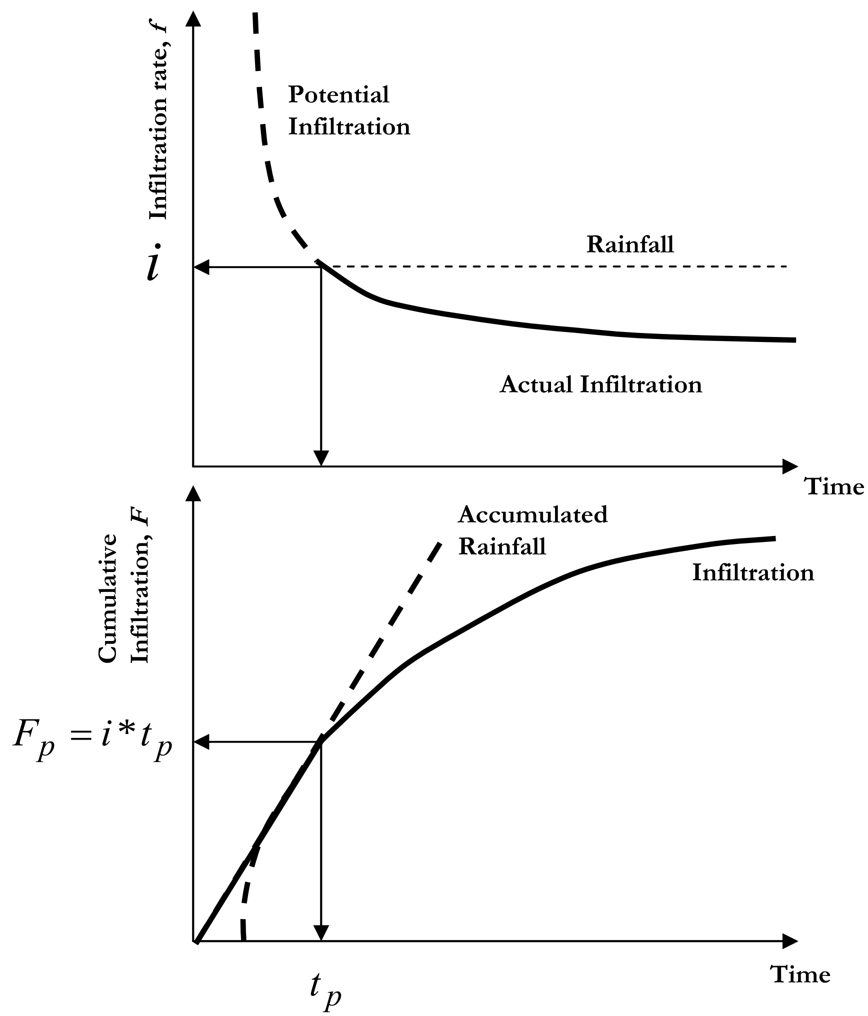
- Up to the time of ponding
  - all rainfall has infiltrated
  - i = rainfall rate

$$f = i \qquad F = i * t_p$$

$$f = K\left(\frac{\Delta\theta\psi_f}{F} + 1\right)$$

$$i = K\left(\frac{\Delta\theta\psi_f}{i * t_p} + 1\right)$$

$$\boxed{t_p = K\frac{\Delta\theta\psi_f}{i(i-K)}}$$



Example

- Silty-Loam soil
- 30% effective saturation
- 5 cm/hr rainfall intensity

$$\theta_e = 0.486$$
$$\psi = 16.7\,cm$$
$$K = 0.65\,cm/hr$$
$$s_e = 0.30$$

$$\Delta\theta = (1 - s_e)\theta_e = (1 - 0.3)(0.486) = 0.340$$

$$\psi\Delta\theta = 16.7 * 0.340$$

$$t_p = K\frac{\Delta\theta\psi_f}{i(i-K)} = 0.65\frac{5.68}{5.0(5.0-0.65)(i-K)} = 0.17\,\mathrm{hr}$$