CE 374 K – Hydrology

Stream and Reservoir Routing

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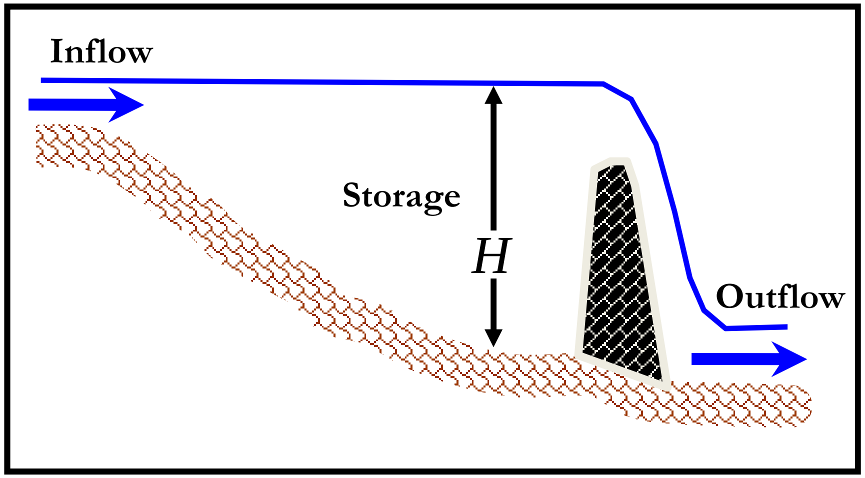
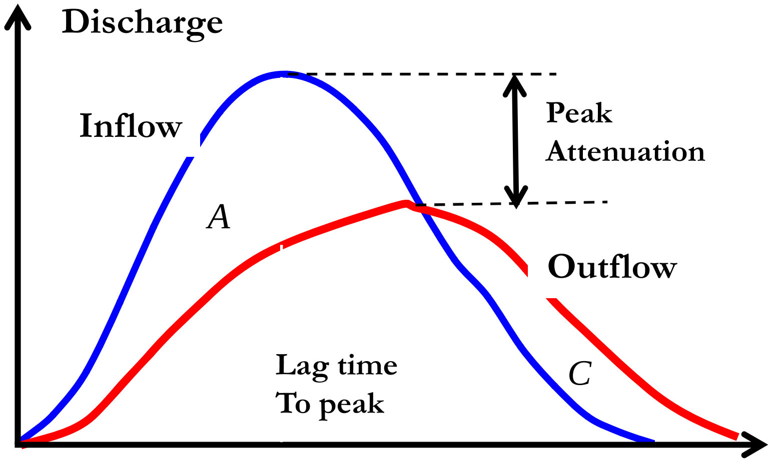
Reservoir Routing





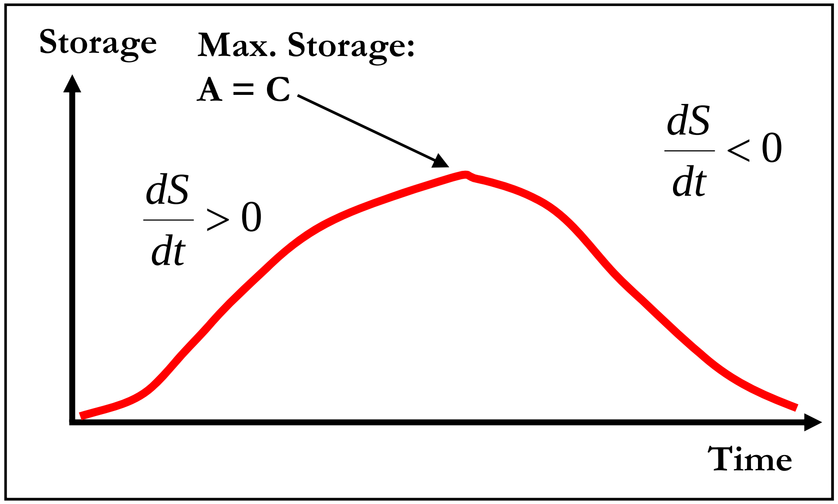
Inflow and outflow hydrographs for a small, level-surface reservoir

A: I > O, filling

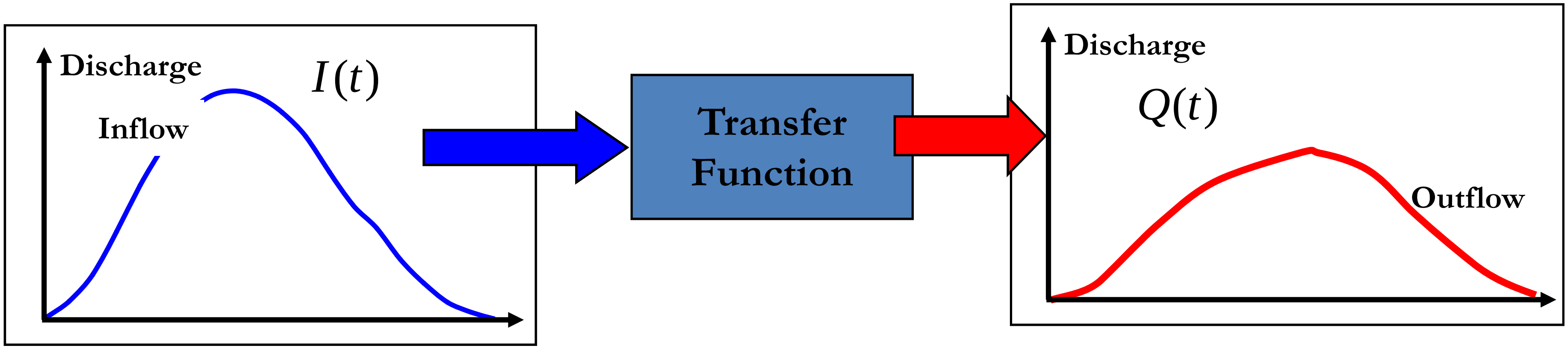
C: I < O, emptying

$$\frac{dS}{dt} = I(t) - Q(t)$$

$$Q(t) = f(H)$$



Hydrologic Routing



$I(t) = \text{Inflow}$

$Q(t) = f\left[x, I(t)\right]$

$Q(t) = \text{Outflow}$

Upstream Hydrograph

Channel:

Characteristics: *x*

Routing Method: *f*(..)

Downstream Hydrograph

Input, output, storage are related

$$\frac{dS}{dt} = I(t) - Q(t)$$

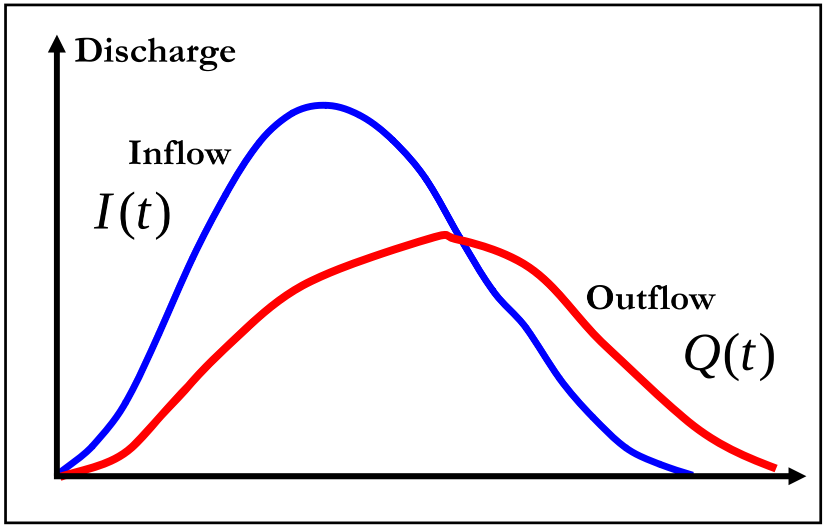
Storage function is needed

$$S = f(I, Q)$$

Relations Between Discharge and Storage

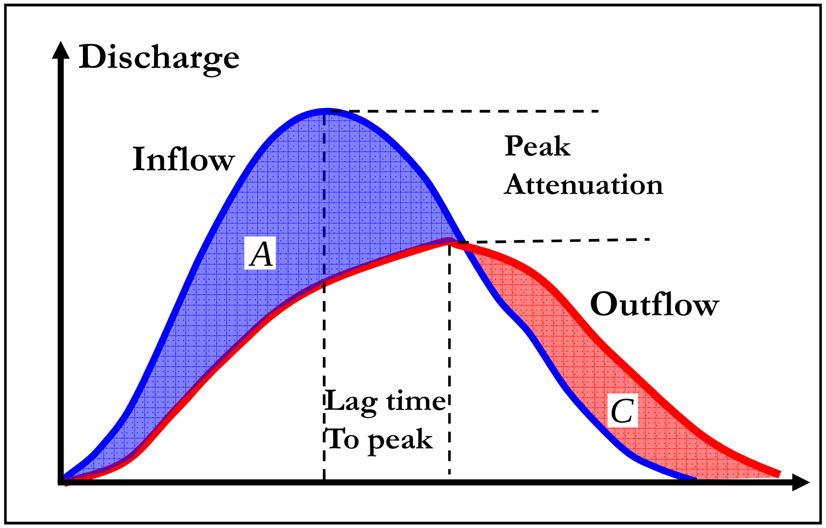
- Flow routing
  - Find a downstream hydrograph given an upstream hydrograph
  - Lumped or Distributed
  - Lumped $\frac{dS}{dt} = I(t) - Q(t)$
  - Storage function needed

$$S = f(I, \frac{dI}{dt}, \cdots, Q, \frac{dQ}{dt}, \cdots)$$



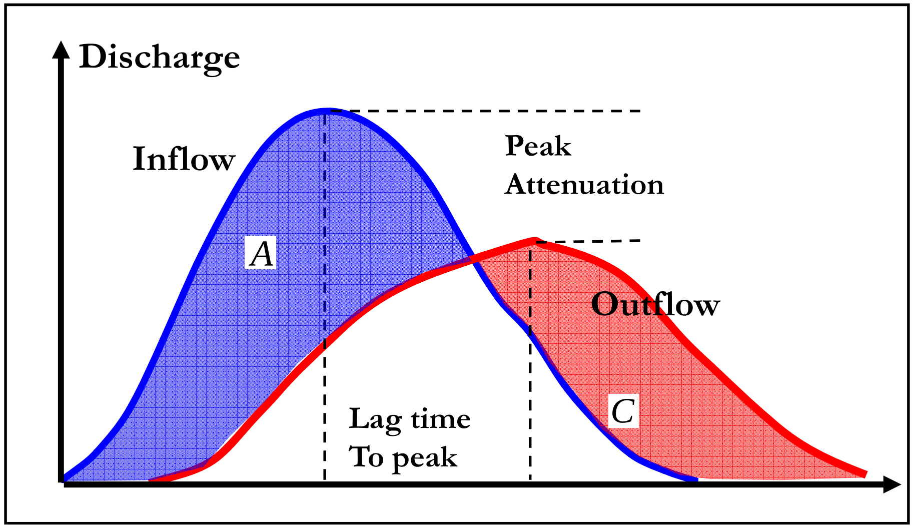
Transformation of Hydrographs: Redistribution and Translation

Redistribution



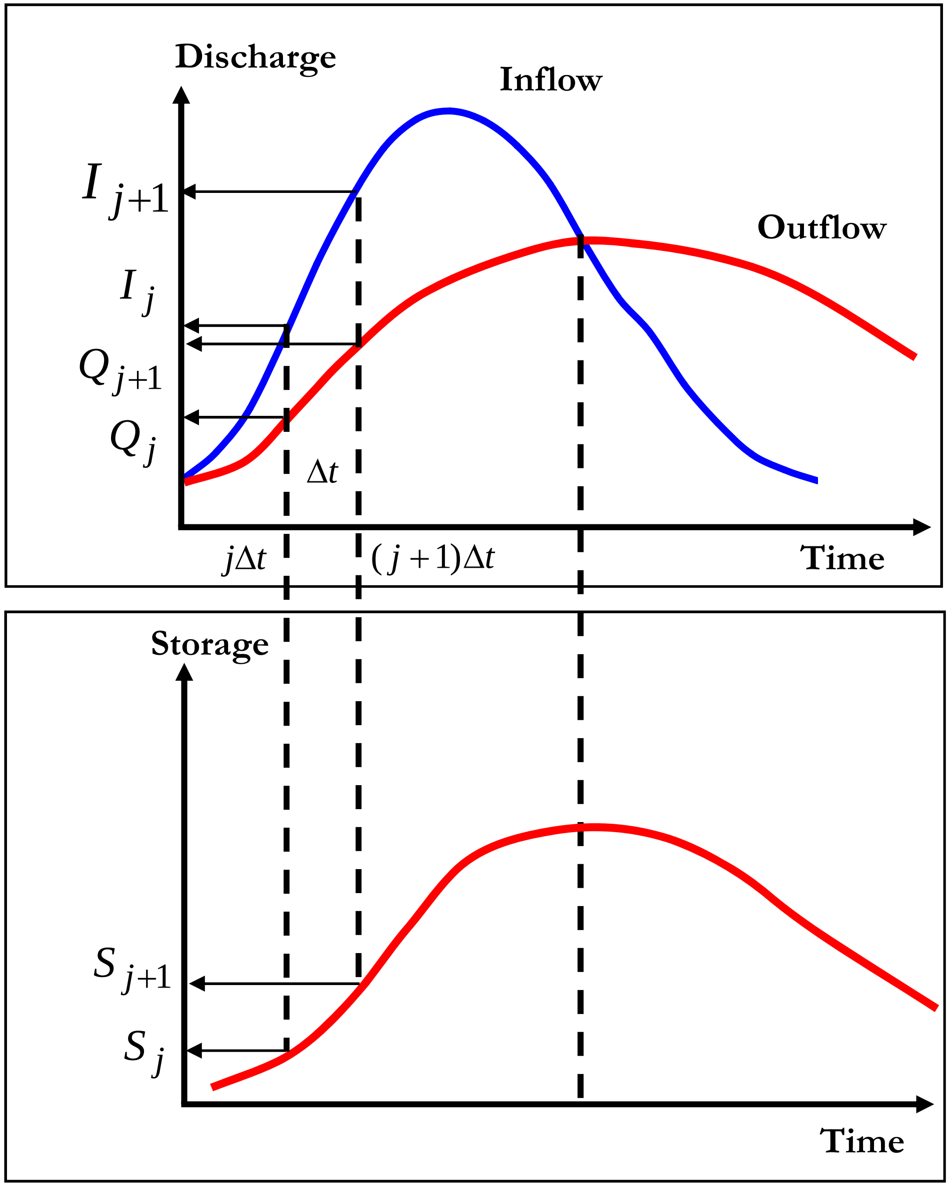
Reservoir Storage

Translation



Channel Storage

Reservoir Routing (Level Pool)



$$\frac{dS}{dt} = I(t) - Q(t)$$

$$\int_{S_j}^{S_{j+1}} dS = \int_{j\Delta t}^{(j+1)\Delta t} I\,dt - \int_{j\Delta t}^{(j+1)\Delta t} Q\,dt$$

$$\frac{S_{j+1} - S_j}{\Delta t} = \frac{I_{j+1} + I_j}{2} - \frac{Q_{j+1} + Q_j}{2}$$

$$\underbrace{\frac{2S_{j+1}}{\Delta t} + Q_{j+1}}_{\text{Unknown}} = \underbrace{I_{j+1} + I_j + \frac{2S_j}{\Delta t} - Q_j}_{\text{Known}}$$

Need a function relating

$$\frac{2S}{\Delta t} + Q, \text{and}\ Q$$

Level Pool Routing (Cont.)

$$\underbrace{\frac{2S_{j+1}}{\Delta t}+Q_{j+1}}_{\textbf{Unknown}}=\underbrace{I_{j+1}+I_{j}+\frac{2S_{j}}{\Delta t}-Q_{j}}_{\textbf{Known}}$$

Compute

$$\frac{2S_{j+1}}{\Delta t}+Q_{j+1}$$

From

$$\frac{2S}{\Delta t}+Q,\text{and}\,Q$$

Then compute

$$Q_{j+1}$$

Then compute

$$\frac{2S_{j+1}}{\Delta t}-Q_{j+1}=\frac{2S_{j+1}}{\Delta t}+Q_{j+1}-2Q_{j}$$

Level Pool Example

H (ft)	A (ft2)	S (ft3)	2S/Dt+Q (cfs)	Q (cfs)
0	43,560	0	0	0
1	43,560	43,560	158	12.8
2	43,560	87,120	308.4	18
3	43,560	130,680	457.7	22.1
4	43,560	174,240	606.3	25.5
5	43,560	217,800	754.5	28.5
6	43,560	261,360	902.5	31.3
7	43,560	304,920	1050.2	33.8
8	43,560	348,480	1197.7	36.1
9	43,560	392,040	1345.1	38.3
10	43,560	435,600	1492.4	40.4

$$Q_{orifice} = C_d A_0 \sqrt{2gh}$$

Detention pond

Given:

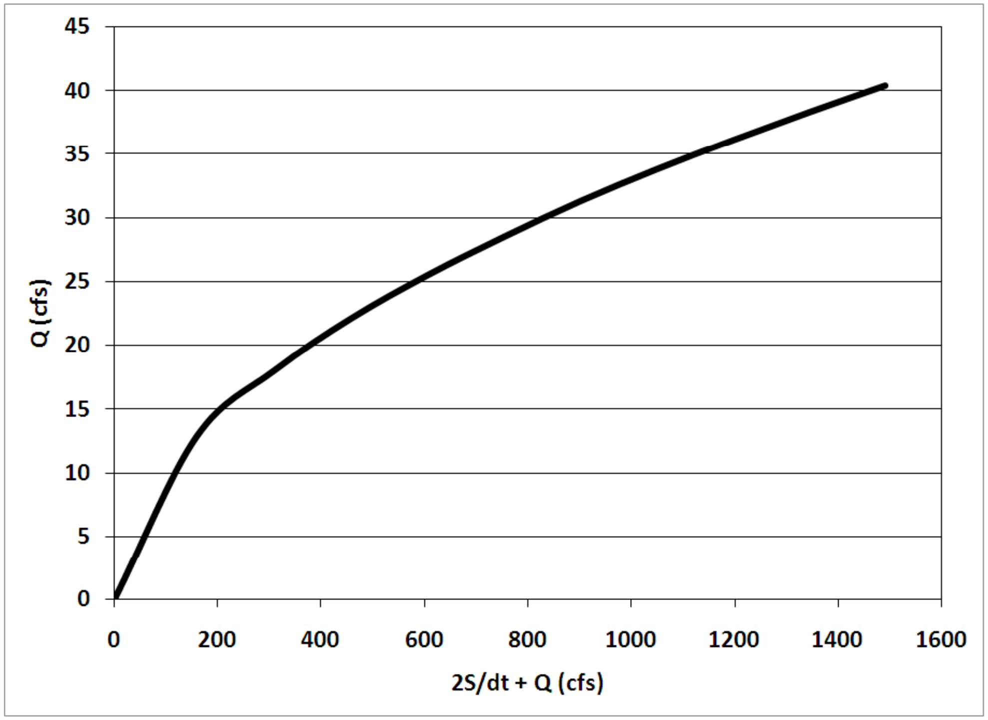
Area = 1 acre

Outlet = 18 inch diameter concrete pipe

C = 0.9

Find:

Outlet hydrograph



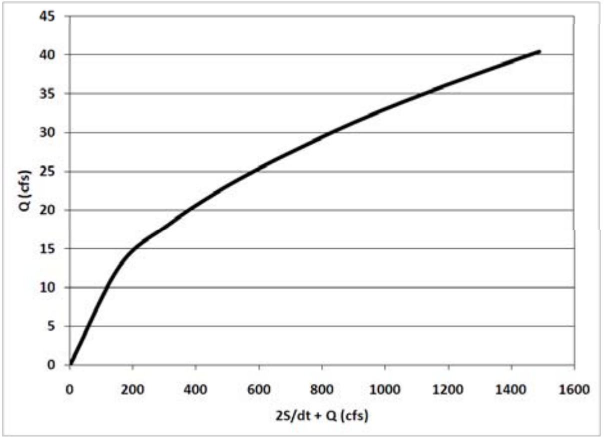
Level Pool Example (Cont.)

Time (min)	I_j (cfs)	I_j+I_{j+1} (cfs)	$2S_j/dt$ - Q_j (cfs)	$2S_{j+1}/dt$ + Q_{j+1} (cfs)	Q_j (cfs)
0	0	0	0	0	0
10	20	20	11.8	20.0	4.1
20	40	60	54.6	71.8	8.6
30	60	100	129.0	154.6	12.8
40	50	110	207.2	239.0	15.9
50	40	90	261.6	297.2	17.8
60	30	70	294.2	331.6	18.7
70	20	50	306.2	344.2	19.0
80	10	30	299.2	336.2	18.5
90	0	10	273.2	309.2	18.0
100	0	0	239.2	273.2	17.0
110	0	0	207.4	239.2	15.9
120	0	0	178.0	207.4	14.7
130	0	0	151.0	178.0	13.5
140	0	0	126.4	151.0	12.3
150	0	0	104.0	126.4	11.2
160	0	0	83.8	104.0	10.1

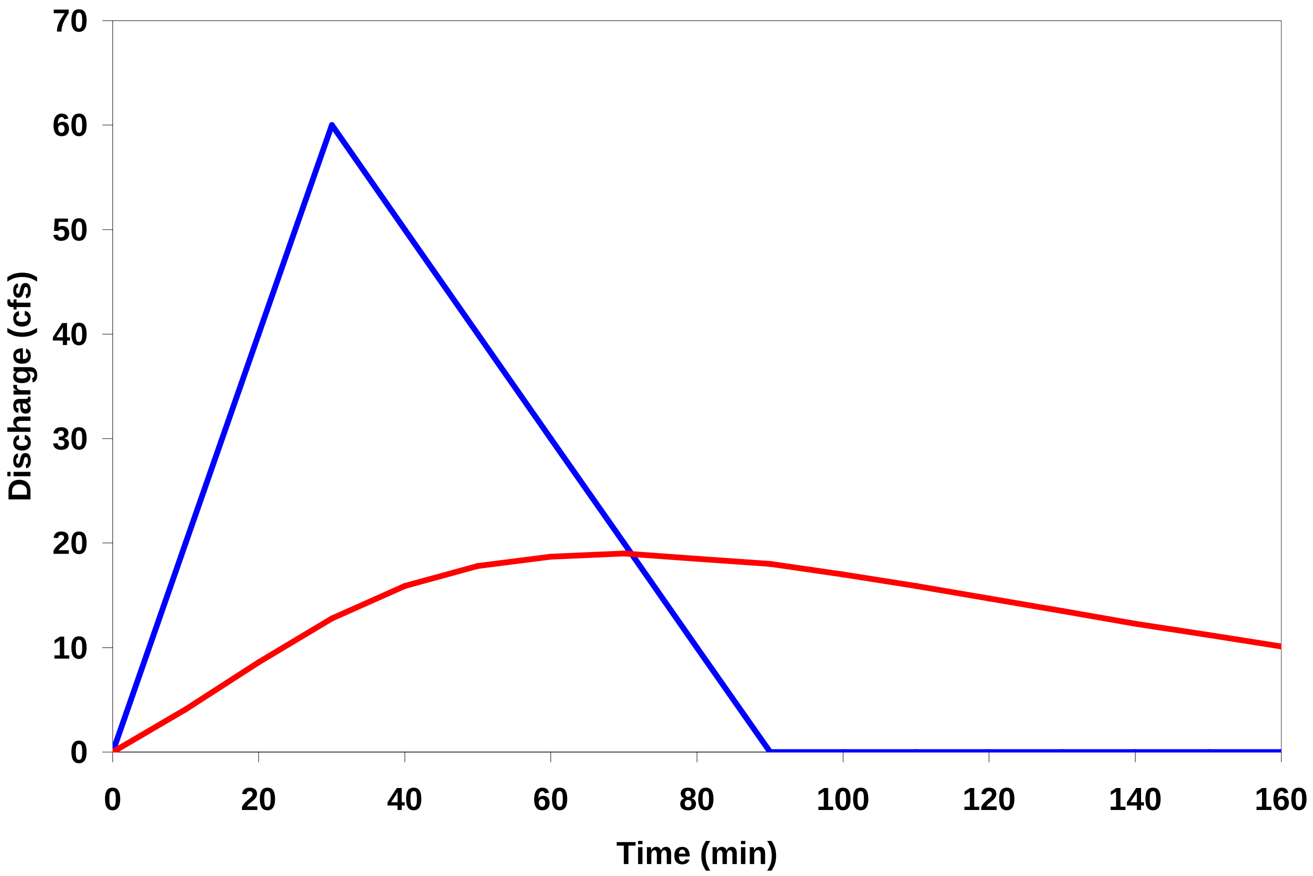
$$\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)=\left(I_{j+1}+I_j\right)+\left(\frac{2S_j}{\Delta t}-Q_j\right)$$

$$\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)\Rightarrow Q_j$$

$$\left(\frac{2S_j}{\Delta t}-Q_j\right)=\left(\frac{2S_{j+1}}{\Delta t}+Q_{j+1}\right)-2Q_j$$



Level Pool Example (Cont.)



Level Pool (Book Example)

H (ft)	Q (cfs)	S (ft3)	2S/Dt+ Q (cfs)
0	0	0	0
0.5	3	21,780	76
1	8	43,560	153
1.5	17	65,340	235
2	30	87,120	320
2.5	43	108,900	406
3	60	130,680	496
3.5	78	152,460	586
4	97	174,240	678
4.5	117	196,020	770
5	137	217,800	863
5.5	156	239,580	955
6	173	261,360	1044
6.5	190	283,140	1134
7	205	304,920	1221
7.5	218	326,700	1307
8	231	348,480	1393
8.5	242	370,260	1476
9	253	392,040	1560
9.5	264	413,820	1643
10	275	435,600	1727

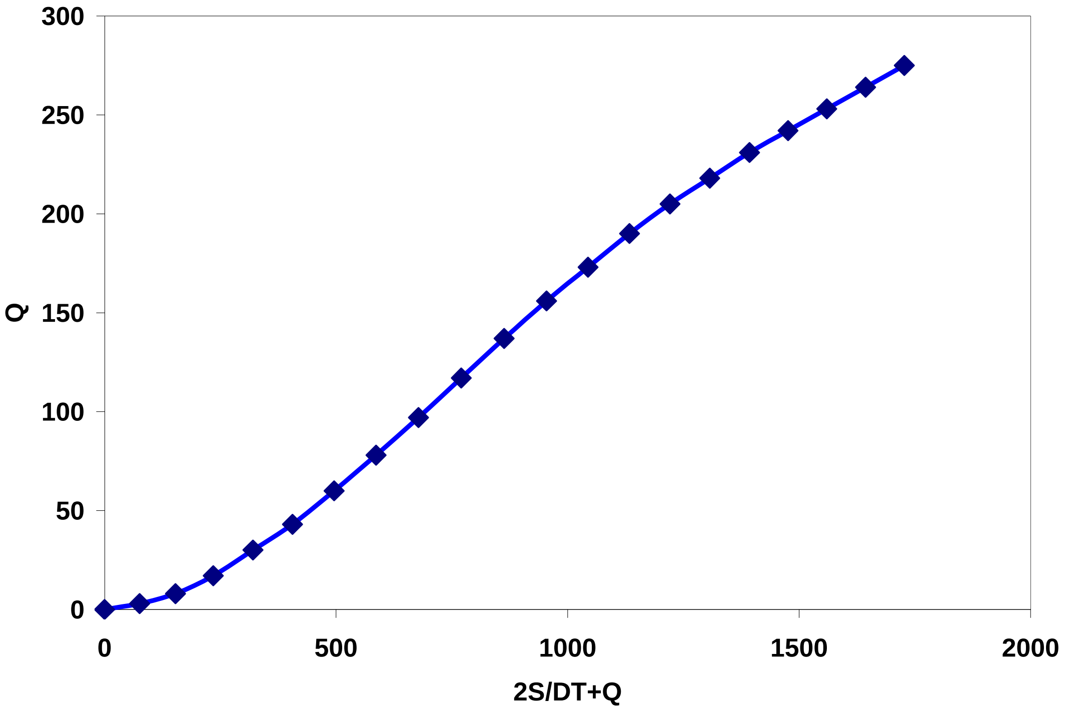
Detention pond

Given:

- **Area = 1 acre**
- **Outlet = 5-ft diameter concrete pipe**

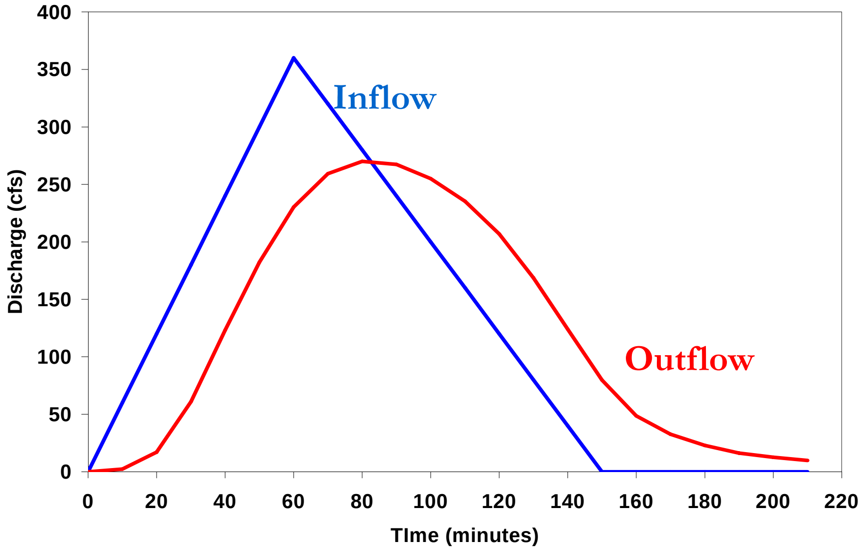
Find:

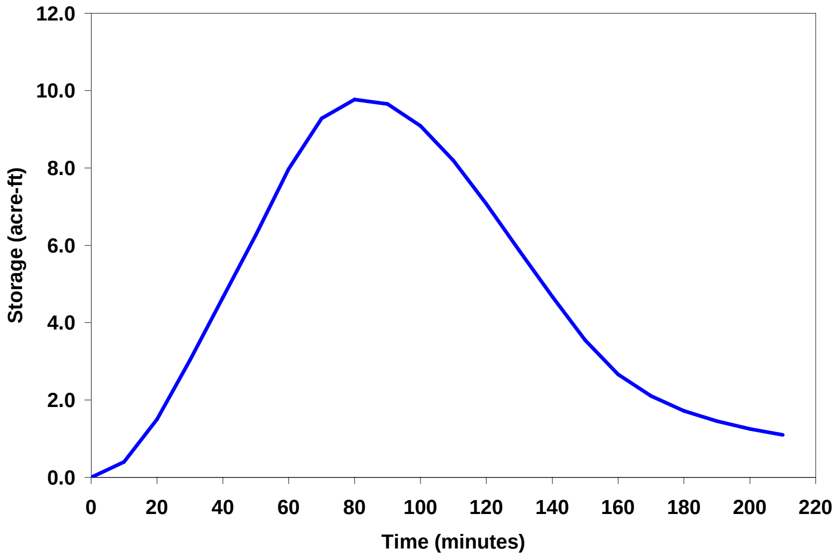
- **Outlet hydrograph**



Level Pool Example (Cont.)

Time (min)	Inflow (cfs)	Ij+Ij+1 (cfs)	2Sj/dt-Qj (cfs)	2Sj+1/dt-Qj+1 (cfs)	Outflow (cfs)
0	0	0	0	0	0
10	60	60	55	60	2
20	120	180	201	235	17
30	180	300	379	501	61
40	240	420	552	799	123
50	300	540	728	1092	182
60	360	660	927	1388	230
70	320	680	1089	1607	259
80	280	600	1149	1689	270
90	240	520	1134	1669	267
100	200	440	1064	1574	255
110	160	360	954	1424	235
120	120	280	820	1234	207
130	80	200	683	1020	169
140	40	120	555	803	124
150	0	40	435	595	80
160	0	0	338	435	49
170	0	0	273	338	33
180	0	0	227	273	23
190	0	0	195	227	16
200	0	0	169	195	13
210	0	0	150	169	10





Hydrologic River Routing
Muskingum Method

- Wedge storage in reach

$$S_{\text{Prism}} = KQ$$

$$S_{\text{Wedge}} = KX(I - Q)$$

K = travel time of peak through the reach

X = weight on inflow versus outflow

X = 0 ➔ Reservoir, storage depends on outflow,

no wedge

X = 0.0 - 0.3 ➔ Natural stream

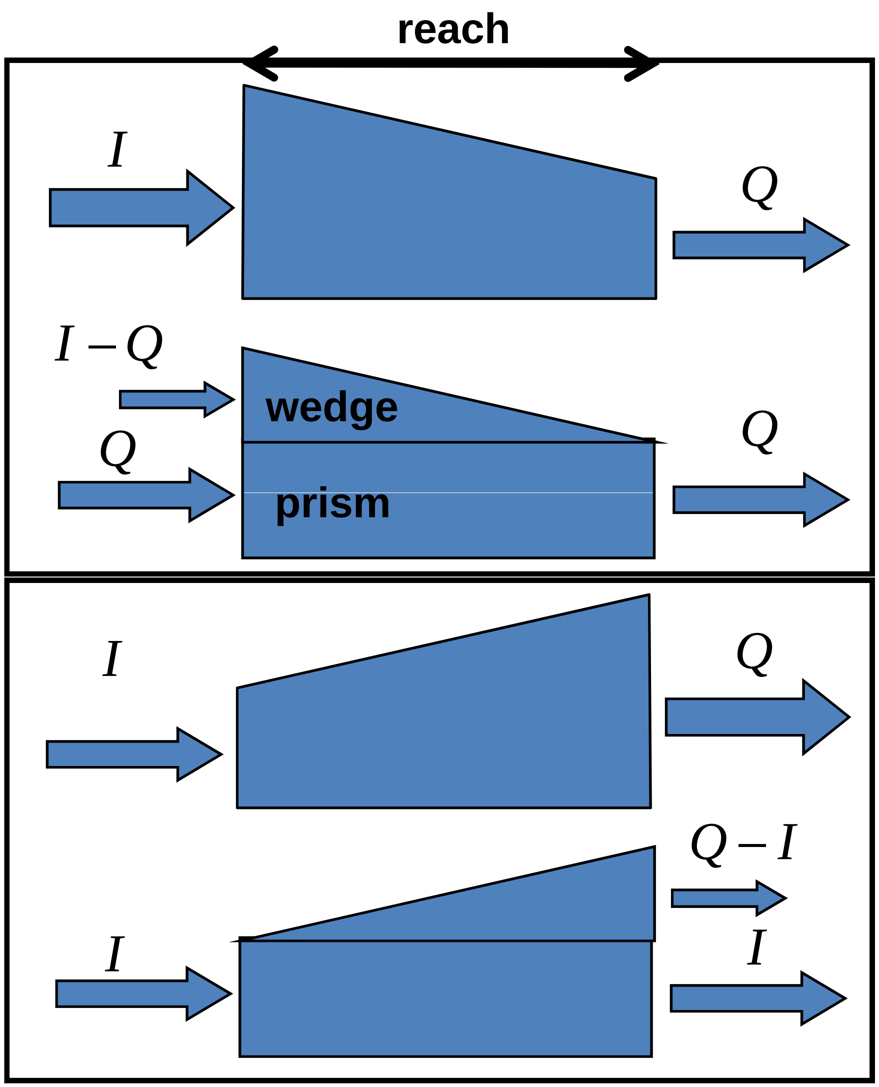
$$S = S_{wedge} + S_{prism}$$

$$= KQ + KX(I - Q)$$

$$\boxed{S = K[XI + (1 - X)Q]}$$

Advancing Flood Wave
$I > Q$

Receding Flood Wave
$Q > I$



Muskingum Method (Cont.)

$$S = K[XI + (1-X)Q]$$

$$S_{j+1} - S_j = K[XI_{j+1} + (1-X)Q_{j+1}] - K[XI_j + (1-X)Q_j]$$

Recall:

$$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_{j+1} + Q_j}{2}\Delta t$$

Combine:

$$\boxed{Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j}$$

$$\boxed{\begin{aligned} C_1 &= \frac{\Delta t - 2KX}{2K(1-X) + \Delta t} \\ C_2 &= \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} \\ C_3 &= \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} \end{aligned}}$$

Muskingum Example

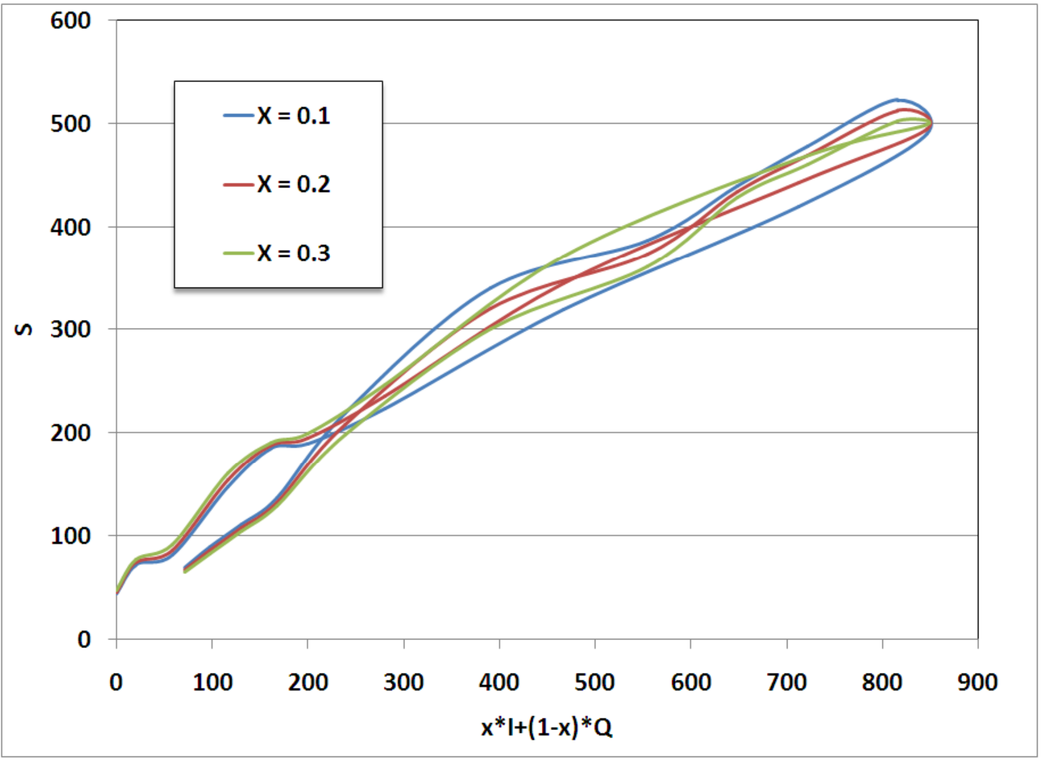
- Estimate K from observed inflow and outflow hydrographs

- Plot $[XI+(1-X)Q]$ vs. S

- For various values of X

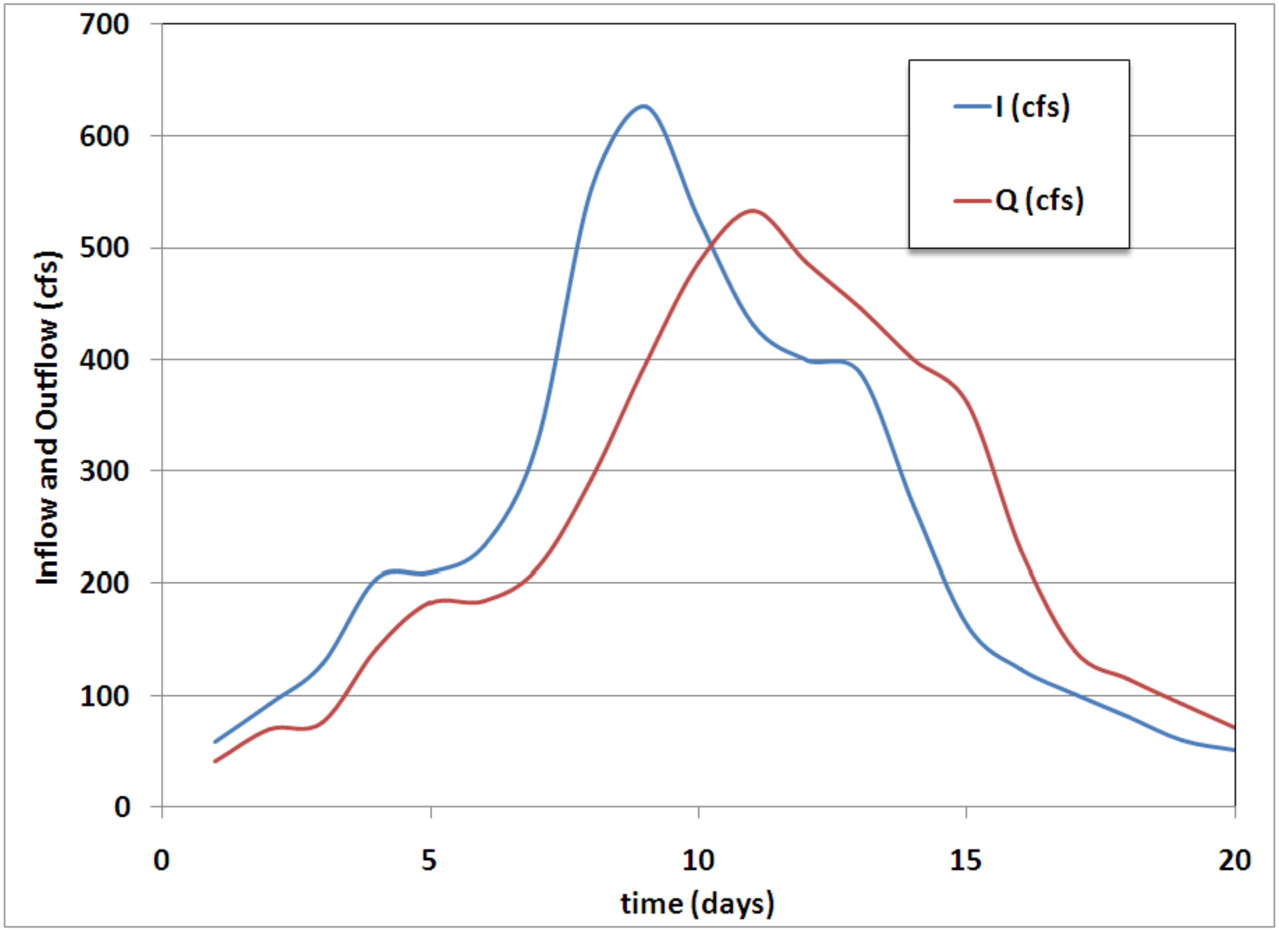
- Choose the one which gives a loop closest to a straight line

- The Slope of this line is K



Muskingum Example

- River reach - Know inflow and outflow hydrographs
- Estimate Muskingum Parameters



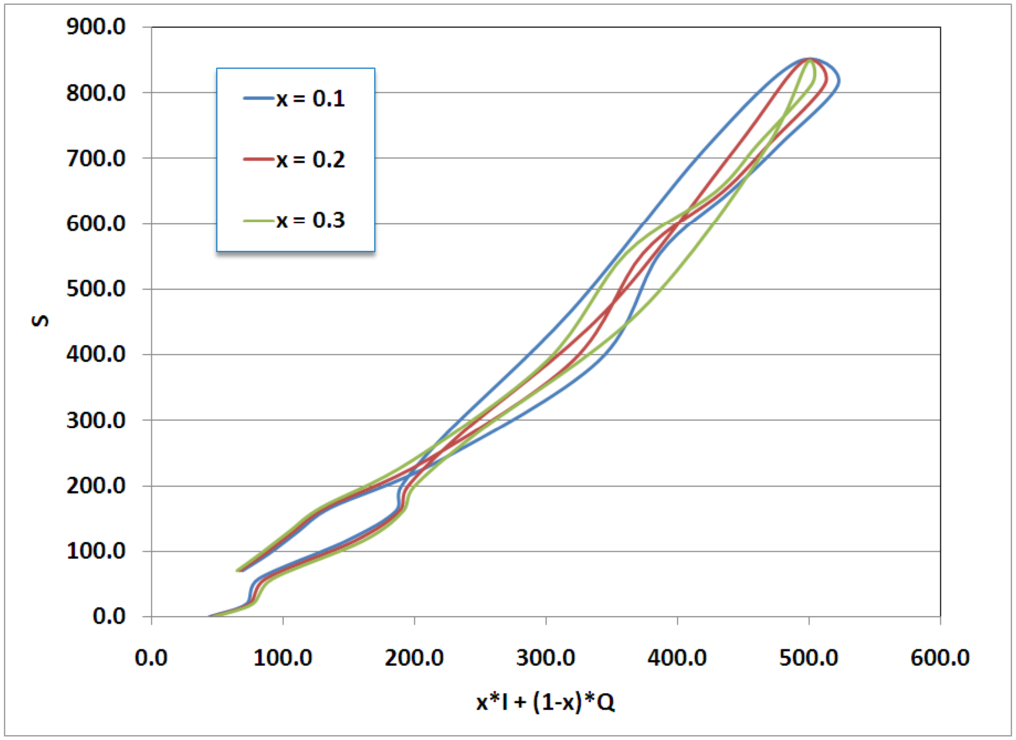
Time (day)	I (cfs)	Q (cfs)
1	59	42
2	93	70
3	129	76
4	205	142
5	210	183
6	234	185
7	325	213
8	554	293
9	627	397
10	526	487
11	432	533
12	400	487
13	388	446
14	270	400
15	162	360
16	124	230
17	102	140
18	81	115
19	60	93
20	51	71

Muskingum Example (Cont.)

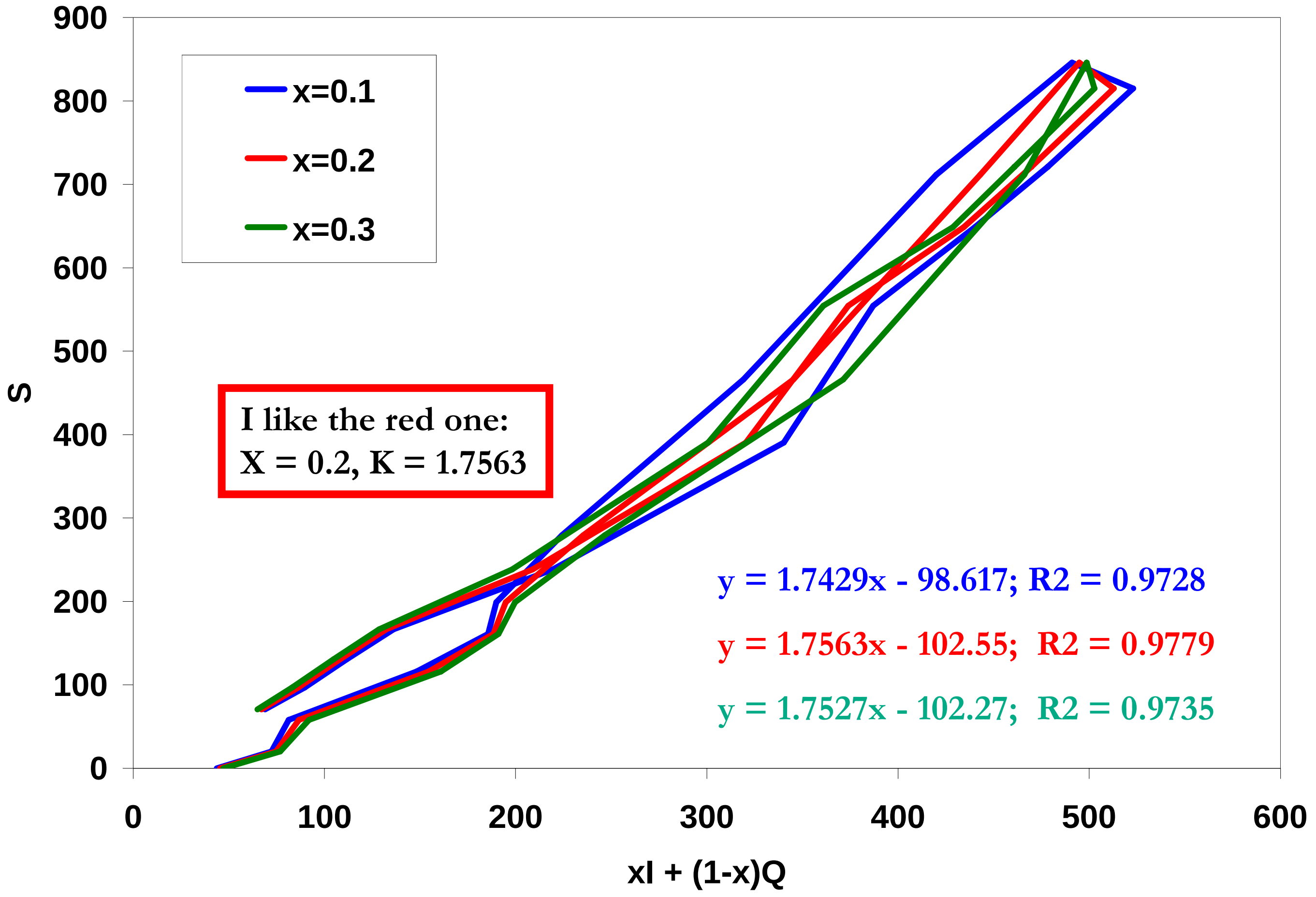
- **Know inflow and outflow hydrograph**
- **Compute storage**
- **Compute** [$XI+(1-X)Q$] **for various values of** X

$$S_{j+1} = S_j + \frac{I_{j+1}+I_j}{2}\Delta t - \frac{Q_{j+1}+Q_j}{2}\Delta t$$

Time	S	x*I+(1-x)*Q	x*I+(1-x)*Q	x*I+(1-x)*Q
t (day)	S (ft3)	X = 0.1	X = 0.2	X = 0.3
1	0	44	45	47
2	20	72	75	77
3	58	81	87	92
4	116	148	155	161
5	161	186	188	191
6	199	190	195	200
7	280	224	235	247
8	466	319	345	371
9	712	420	443	466
10	846	491	495	499
11	815	523	513	503
12	721	478	470	461
13	649	440	434	429
14	555	387	374	361
15	391	340	320	301
16	239	219	209	198
17	167	136	132	129
18	131	112	108	105
19	97	90	86	83
20	71	69	67	65



Muskingum Example (Cont.)



I like the red one:
X = 0.2, K = 1.7563

y = 1.7429x - 98.617; R2 = 0.9728

y = 1.7563x - 102.55; R2 = 0.9779

y = 1.7527x - 102.27; R2 = 0.9735

Muskingum Example (Cont.)

$$\boxed{Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j}$$

t (day)	I (cfs)	C1*Ij+1	C2*Ij	C3*Qj	Q (cfs)
1	59	0	0	0.0	42
2	93	7.3	26.4	20.0	53.6
3	129	10.1	41.6	25.5	77.1
4	205	16.0	57.6	36.6	110.3
5	210	16.4	91.6	52.4	160.4
6	234	18.3	93.8	76.2	188.3
7	325	25.4	104.6	89.5	219.4
8	554	43.3	145.2	104.2	292.7
9	627	49.0	247.6	139.1	435.6
10	526	41.1	280.2	206.9	528.2
11	432	33.7	235.0	250.9	519.7
12	400	31.2	193.0	246.9	471.2
13	388	30.3	178.7	223.8	432.9
14	270	21.1	173.4	205.6	400.1
15	162	12.6	120.6	190.1	323.4
16	124	9.7	72.4	153.6	235.7
17	102	8.0	55.4	112.0	175.3
18	81	6.3	45.6	83.3	135.2
19	60	4.7	36.2	64.2	105.1
20	51	4.0	26.8	49.9	80.7