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ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF A BRACED EXCAVATION

J.G. Zornberg and R.F. Azevedo
Pontifical Catholic University of Rio de Janeiro, PUC/RJ,
Rio de Janeiro, Brazil

SUMMARY

This paper deals with a finite element program developed to analyze plane strain and axissymetric geotechnical problems. Lade's work hardening model was implemented into this code originating non-linear problems with non-symmetric stiffness matrices. Both the non-linearity and the non-symmetry were solved by different approaches. The program represents not only the removal (exacavation) but also the adding (embankments) of elements to the original finite element mesh. Bar elements with a bi-linear behavior are available to represent struts and tie-rods. A macro-programming language is used to facilitate the simulation of sequential construction steps. The paper first describes all these features of the developed program. Subsequently it shows the utilization of the program to analyze a strutted excavation made to allow the construction of a subway structure in São Paulo city, Brazil.

1. INTRODUCTION

Nowadays, as a consequence of the expansion of the public transportation metropolitan networks and the construction of large buildings, a notably increase in number and dimensions of soil excavations can be observed in densely populated urban regions. In these circumstances, the ground movement becomes an indispensable foresight to the successful performance of these projects.

A promising way of estimating both ground and retaining structure movements is by means of the finite element technique. In fact, the finite element method is probably the most general tool presently available to analyze the stress and displacement fields associated with the proper simulation of the different exacavation phases. The results of these analyses depend predominantly on the stress-strain and strength theory employed to model the soil's properties nearby the exacavation. Soils constitutive characteristics, such as non-linearity, dilatancy and deformations irreversibility, are aspects that can be well represented by means of elasto-plastic theories. Among them, Lade's model [1,2] has been found appropriate to represent the behavior of cohesionless soils and normally consolidated clays and was used in this work.

2. PROGRAM CHARACTERISTICS

ANLOG is a finite element program developed at Pontifical Catholic University of Rio de Janeiro with the objective of analyzing geotechnical problems and, in particular, excavation works [3]. Two questions were mainly considered in the code development. First, the implementation of Lade's model to represent the stress-strain behavior of the soil, and second, the appropriate simulation of the sequential steps inherent in the particular constructive process of each problem.

2.1. Representation of stress-strain behavior of the soil

As it is well known, Lade's elasto-plastic model [1,2] consists on a two yield surfaces model in which the total strain increments are divided into an elastic component, a plastic collapse component and a plastic expansive component. Each of these strains is calculated separately, the elastic strains increments are determined by Hooke's law, the plastic collapsive strains by a plastic stress-strain theory involving a cap-type surface with an associated flow rule, and the plastic expansive strains by a stress-strain theory that involves a conical yield surface with a non-associated flow rule. The non-associated properties of the theory have some implications in the finite element code implementation. On the one hand, non-symmetric solvers are required as the tangential stiffness will be expressed in a non-symmetric form. On the other, the necessary machine storage capacity will be nearly doubled. In the developed program, both symmetric and non-symmetric systems of algebraic equations are solved by a direct procedure. Furthermore, the required storage and computational effort are reduced by storing the necessary parts of the upper triangular portion of the stiffness matrix by columns and the lower triangular portion by rows (Skyline scheme) [4].

With the purpose of avoiding the shortcomings of non-symmetric approach, numerical schemes have been developed which enable the use of the symmetric equation solvers in tangential stiffness programs for non-associated materials [5,6]. In this work, three different techniques were implemented, besides the non-symmetric one, in order to lead to a symmetric stiffness matrix. While using these approaches, stress integration continues to be performed with the real non-symmetric constitutive matrix. in the first implemented procedure, the real non-associated material is transformed into an equivalent associated material by forcing a few parameters of Lade's model to assume predetermined values, compelling the plastic potential function to be identical to the yield function and making the normality criterion to be applied to the soil. A second approach consists on defining a symmetric matrix $[D_{ep}^{m}]$, such that:

$$[D_{ep}^{m}] = \frac{1}{2}\,[[D_{ep}] + [D_{ep}]^{T}] \tag{1}$$

where $[D_{ep}]$ is the real non-symmetric elasto-plastic matrix and $[D_{ep}]^{T}$ his transpose. This very simple technique turned out very efficient in the analyzed problems, showing rapid convergence. Finally, the third approach consists on assembling the global stiffness matrix only considering the elastic strains component. In this case, the convergence rate proved to be very slow.

The stiffness matrix will be dependent of both the stress level and the stress history of the materials. Techniques for solving non-linear stiffness equations are quite widespread. In this work, different incremental-iterative approaches based on the Newton-Raphson scheme were introduced. The stress integration carried out during each equilibrium iteration is performed by using an explicit integration technique, where the solution is obtained by forward integrating over a sufficient number of sub-increments in order to obtain the required accuracy. Considering that the used model involves two simultaneous yield surfaces and with the purpose, on the other hand, of defining a criterion that allow a physical visualization of the sub-increments magnitude, the number of sub-increments is defined by limiting the maximum strain increment in each integrating point.

2.2. Simulation of Geotechnical Problems

In order to define a versatile code that allow the simulation of different phases of construction in a sequential geotechnical work, a macro programming language was introduced. The macro programming language is associated with a set of compact subprograms each designed to compute one or, at most, a few basic steps in a finite element solution process. In this way, a specific macro command is used to remove elements from the original finite element mesh, performing an excavation phase [7]. Bar elements are used to numerically simulate struts and tie-rods. In order to prevent a strut to be submitted to a tension force, a bi-linear constitutive behavior, that introduces a minimal bar stiffness at tension, was considered. In a similar way, a bar element simulating a tie-rod will not be allowed to yield compression forces. A two-node bar element that connects the wall to the anchor would lead to great increase in the stiffness matrix bandwidth an hence the storage required. However, the increase in needed storage will be minimal as the skyline storage scheme was used to store the stiffness matrix. Installation, prestressing and removal of bar elements are all performed with specific macro commands.

Appropriate commands allow the data input as well as the creation of plot data files used to produce the displacements and stress fields. A proper macro command is used to solve the finite element problem itself using one of the implemented algorithms according to control parameters supplied by the user. Specification of initial stresses (either geostatic or isotropic) as well as specification of the initial loci of the yield surfaces can also be provided. Other commands are presently available in the code and many other ones could easily be implemented to represent the necessary steps to simulate specific phases in a geotechnical work.

3.STRESS-STRAIN PARAMETERS OF THE SOILS INVOLVED IN THE ANALYSIS

In order to analyze a strutted excavation made to allow the construction of a subway structure in São Paulo city, Brazil, the elasto-plastic parameters of the surrounding soils were calculated. The city of São Paulo is located in a tertiary sedimentary basin, of fluvial-lacustrine origin, located along the Atlantic Coast. The sediments that fill up the basin suffered a weathering process which left signs such as mottling and precompression by drying. A wide laboratory research has been developed in order to characterize the stress-strain-strength behavior of these materials [8]. Some of these experimental results were used in this work during the calibration process of Lade's model.

3.1. "Red Clay" Modelling

The more superficial materials, known as "Red Clays", are a porous soil whose void ratio is typically grater than 1.0, and whose degree of saturation is less than 100.0%. This layer is composed by an upper horizon, the "Porous Red Clays", that suffered a more intense weathering and a lower one, the "Stiff Red Clays". Both horizons show the same strength envelope that identify them as a unique layer. However, the mechanical behavior was highly influenced by the laterization process. Elasto-plastic parameters were obtained for the "Stiff Red Clay". Although the material is slightly over consolidated, it could be considered as normally consolidated during the calibration process. Laboratory tests were developed with partially saturated samples at natural moisture content. The total stresses obtained from the tests were considered identical to the effective stresses. Three drained conventional triaxial compression (CTC) tests corresponding to the confining pressures of 49 KPa, 98 KPa and 196 KPa were used for calibrating this material. Furthermore, an hydrostatic compression (HC) test was necessary for determining collapsive parameters. Table 1 shows

				RED CLAY	VARIEGATED SOIL
Elastic Parameters			K_{ur}	153.9736	1051.4634
			n	0.568546	0.111901
			ν	0.25	0.29
Plast. Param.	Colapsive		ρ	0.01985998	—
	Expansive	Failure	η_1	133.6332	561.1292
			m	0.884275	1.2048
		Plastic Potential	S	0.4054	—
			t	3.679376	—
			R	-17.46982	—
		Work Hardening	P	0.414041	—
			l	0.446428	—
			α	1.65	—
			β	0.0	—

Table 1 - Summary of soil Parameters

the elasto-plastic nondimensional parameters obtained for the "Red Clay". The parameters notations are according to Lade [1,2].

"Red Clay" parameters listed in Table 1 were used for the prediction of the behavior of this material for CTC tests by specifying discrete stress points for such test and calculating the strain increments from the theory. Fig.1 shows observed and predicted behavior of the material for the developed tests. The points represent the measured soil behavior and the solid line represents the calculations from the theory. The agreement is very good in both the stress differences and volumetric strains. Prediction of stress-strain relations for HC test was also performed. The comparison between measured and calculated values is shown in fig.2. The behavior of the soil is represented with good accuracy using the stress-strain theory. The initial calculated hydrostatic pressure is in agreement with suction values corresponding to this material.

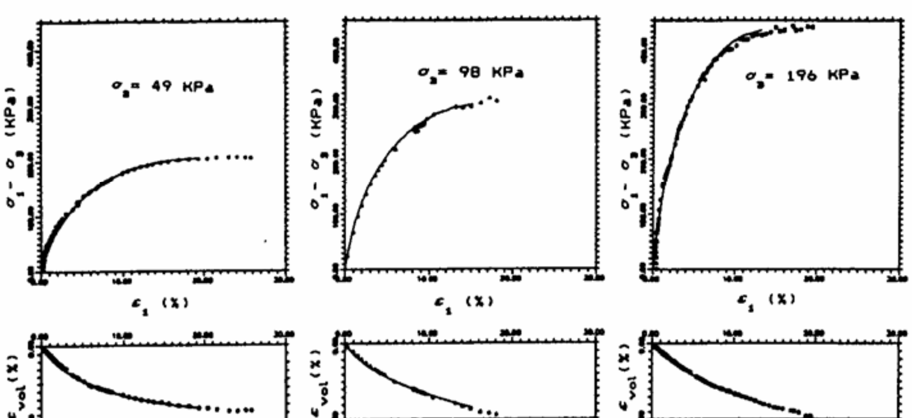

Fig.1 - Measured and Predicted "Red Clay" behavior in CTC tests

A typical stress path of an excavation process is the one that corresponds to a Reduced Triaxial Compression (RTC) test where the axial stress is held constant while the confining stress is reduced from an initial hydrostatic state. Fig.3 shows that the measured deviatoric stresses obtained in these tests [8] are well represented by the theory. Considering that all the parameters had been obtained from HC and CTC tests, the agreement observed in fig.3 stands out the ability of the model to predict stress paths different from those used during calibration. Volumetric strains are not showed because of the difficulty of determining these values in a partially saturated soil during a RTC test. These results emphasize the importance of using this model to analyze an excavation, where the developed stress paths differ from the conventional paths traditionally used in laboratory tests.

3.2. "Variegated Soil" Modelling

The "Variegated Soils" are found below the "Red Clay" layer, near the water table level. This material presents engineering properties varying in a wide range due to the occurrence of very different soils, from sandy soils to fat clays. The pre-consolidation pressure in the studied samples was approximately 800 KPa. This highly over consolidated material was then considered as an elastic, perfectly plastic material. Both failure and elastic strains parameters were still defined with Lade's expressions and, as plastic deformations will not occur, the corresponding plastic parameters needed not to be defined. Table 1 also shows the parameters obtained for the "Variegated Soil".

4. FINITE ELEMENT ANALYSIS OF A BRACED EXCAVATION

The developed program together with the calculated elasto-plastic parameters were used to analyze a strutted excavation made to allow the construction of a subway structure executed in downtown São Paulo city, Brazil. The work consisted on an excavation of 31.0 m depth, adjacent to which there exists

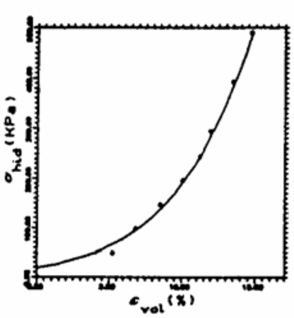

Fig.2."Red Clay" behavior in HC test

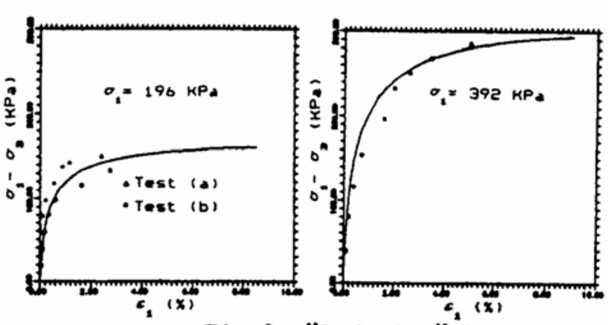

Fig.3. "Red Clay" behavior in RTC tests

a 17 floor building with 2 underground floors. This building deserved a special consideration in relation to the probable settlements that would occur as a consequence of the excavation. The retaining structure consisted of a system of soldier piles with lagging. The water table was initially lowered from its initial level, approximately at the interface between the "Red Clay" and the "Variegated Soil", down to 4.0 m under the future excavation bottom. Three strut levels with no prestressing forces were considered in the retaining structure.

The used finite element mesh consisted of 481 nodal points and 147 elements, 3 of which were bar elements and the rest consisted on 8-node isoparametric elements. As indicated in fig.4.a, the initial state of stresses of the problem was characterized by a geostatic state defined from the knowledge of the coefficient of earth pressure at rest of each material. The first analysis consisted on simulating the excavation of the two underground floors of the building adjacent to the excavation under study. The loading applied by the adjacent building foundation was further represented numerically. Afterwards, the stress field was obtained by simulating the lowering of the water table and getting, in this way, the state of the soil mass previous to the beginning of the excavation execution. Four finite element analyses, corresponding to the four constructive phases of the excavation under study, were subsequently developed. The installation of the struts was performed in the corresponding stages of the process. Fig.4.b illustrates the last phase of the excavation process. These analyses were carried out using the Standard Newton-Raphson Method and considering either the non-symmetric or the symmetric stiffness matrix during the non-linear solution process.

The stress states, displacements fields and safety factors of the soil mass, as well as the efforts on the different components of the retaining structure, were obtained from these analyses. The minimal settlements of the adjacent building obtained numerically were confirmed with the insignificant values measured in situ. The displacements evolution of the wall during the four constructive phases of the excavation can be

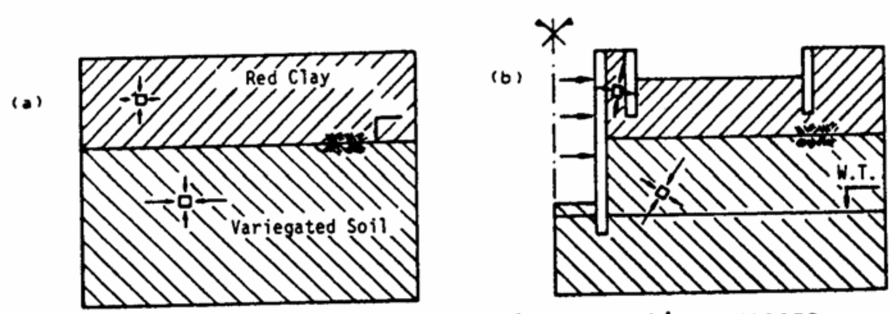

Fig.4. Initial and final stages of the excavation process

observed in fig.5. These displacements are influenced by the high force exerted by the last strut level. The maximum lateral calculated movement (3.69 cm) is in coincidence with the development, in the soil mass adjacent to the wall, of a localized active failure area.

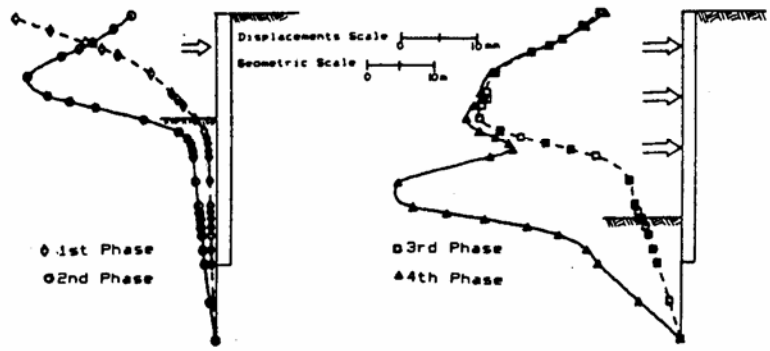

Fig.5. Wall displacements evolution during excavation

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